Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2015

HIGHER SCHOOL CERTIFICATE PRELIMINARY COURSE

ASSESSMENT TASK 3 – PRELIMINARY FINAL EXAM

Mathematics Extension 1

Time allowed: 2 Hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
P3 PE3	chooses and applies appropriate arithmetic, algebraic, graphical , trigonometric and geometric techniques solves problems involving polynomials	1, 2
P4	chooses and applies appropriate arithmetic, algebraic, graphical and trigonometric techniques	3
P6 P7	relates the derivative of a function to the slope of its graph determines the derivative of a function through routine application of the rules of differentiation	4
PE6	makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations	5

Total Marks 75

Attempt Questions 1-5

Question	Out of	Marks
Q1	/15	
Q2	/15	
Q3	/15	
Q4	/15	
Q5	/15	
	Percent:	

General Instructions:

- Questions 1-5 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-5, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

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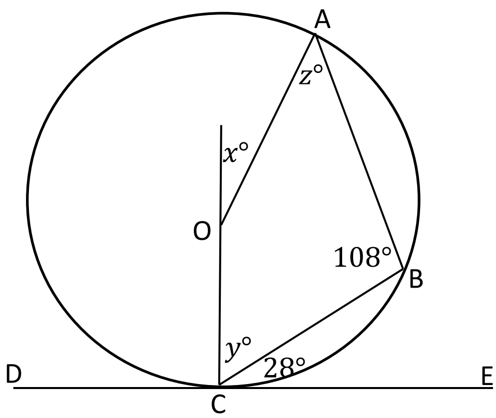
Question One (Start a new booklet) marks

(a) Solve

$$\frac{1}{x-2} > 1$$ **3**

(b)



O is the centre of the circle.

Find the value of each pronumeral giving reasons. **3**

(c) Find the co-ordinates of the point that divides externally the interval joining the points $(-5, 6)$ and $(-2, 3)$ in the ratio $3:2$. **3**

(d) The acute angle between the lines $2x + 2y = 5$ and $y = 3x + 1$ is θ. Find the value of θ, correct to the nearest minute. **3**

(e) Let $P(x) = x^3 - ax^2 + x$ be a polynomial where a is a real number. When $P(x)$ is divided by $(x-3)$ the remainder is 12. Find the remainder when $P(x)$ is divided by $(x+1)$. **3**

Question Two (Start a new booklet) marks

(a) Evaluate $\tan 225°$. **1**

(b) Simplify $\dfrac{1}{1-\sin V}+\dfrac{1}{1+\sin V}$ **3**

(c) The bearing of a ship from lighthouse A is N75°E. The bearing from a second lighthouse B, 44 km south of A, is N40°E. Find the distance of the ship from B to the nearest km. You must draw a neat diagram marking all given information. **3**

(d) *A*, *B* and *C* are three towns such that *B* is $20\,km$ from *A* in a direction $330°$ and *C* is $30\,km$ from *A* in a direction $203°\ 08'$. Find the distance from *B* to *C* to one decimal place. You must draw a neat diagram marking all given information. **3**

(e) The sides of a triangular field have lengths 80*m*, 90*m* and 100*m*.

(i) Calculate the size of the smallest angle to the nearest minute. You must draw a neat diagram marking all given information. **3**

(ii) Calculate the area of the field to the nearest m^2 . **2**

Question Three (Start a new booklet) marks

(a) Write down the expansion for $\tan(\theta+\phi)$. **1**

(b) Simplify $\frac{1}{2}\sin 2\theta \tan\theta$ **2**

(c) If α and β are angles between 0° and 90°, and $\sin\alpha = \frac{3}{5}$ and $\tan\beta = \frac{7}{24}$, find without the use a calculator the exact value of $\sin(\alpha-\beta)$. **3**

(d) Solve for $0 \le \theta \le 2\pi$:

(i) $\sin 2\theta = \frac{-1}{2}$ **2**

(ii) $\sin^2 x - \sin x \cos x = 0$ **3**

(e)

(i) Write $\sqrt{3}\cos x - \sin x$ in form $r\cos(x+\alpha)$ where $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Hence or otherwise, solve $\sqrt{3}\cos x = 1 + \sin x$ for $0 < x < 2\pi$. **2**

Question Four (Start a new booklet) marks

(a) Use the rules of differentiation to differentiate with respect to x:

(i) $y = (3x+4)(x^2-2x)$ **2**

(ii) $y = \sqrt{25-x^2}$ (Answer as a surd) **2**

(iii) $y = \dfrac{x+1}{x^2+1}$ **3**

(b) Differentiate $f(x) = (x+1)^3$ from first principles. **3**

(c) A block is cast into the shape of a right cylinder with a total surface area of $20\pi\, cm^2$. The radius of the base is r cm and the height is h cm. The total surface area of a cylinder is given by $A = 2\pi r^2 + 2\pi rh$.

(i) Express h in terms of r. **1**

(ii) Express the volume V in terms of r. **1**

(iii) Find the value of r for which the volume is the greatest. Answer in exact form. **3**

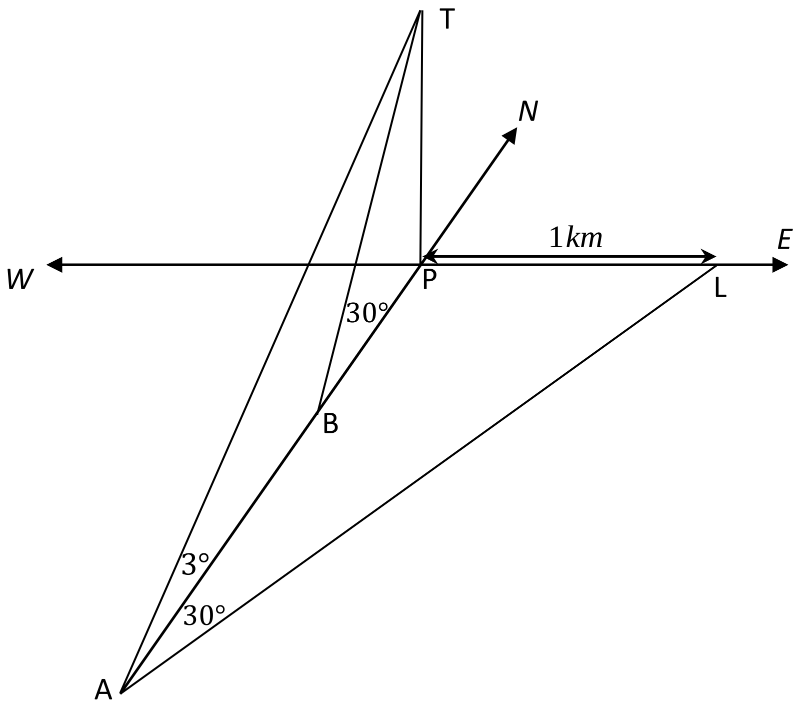
Question Five (Start a new booklet) marks

(a) Find the general solution to $\cos 2\theta = 2 + \cos\theta$. **3**

(b) A boat is sailing due north from a point A towards a point P on the shore line. The shore line runs from west to east.

In the diagram, T represents a tree on a cliff vertically above P, and L represents a landmark on the shore. The distance PL is $1km$.

From A the point L is on a bearing of 030°, and the angle of elevation to T is 3°. After sailing for some time the boast reaches a point B, from which the angle of elevation to T is 30^o.



(i) Show that $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$ **3**

(ii) Find the exact distance AB. **1**

(c) If $t = \tan\left(\dfrac{\theta}{2}\right)$ solve for t the equation $12\tan\theta = 5$ for $\pi \le \theta < \dfrac{3\pi}{2}$ **4**

(d) Sketch $y = x + \dfrac{4}{x-1}$, showing asymptotes, intercepts and stationary points having determined their nature. **4**

Question One (Start a new booklet) marks

Marking **Comments**

(a) Solve $\dfrac{1}{x-2} > 1$ **3**

$$(x-2)^2 \times \frac{1}{x-2} > 1 \times (x-2)^2$$
$$x-2 > (x-2)^2$$
$$0 > (x-2)^2 - (x-2)$$
$$> (x-2)(x-2-1)$$
$$> (x-2)(x-3)$$

❶ correct algebraic processes to quadratic

Mostly well done.

and graphing $y = (x-2)(x-3)$

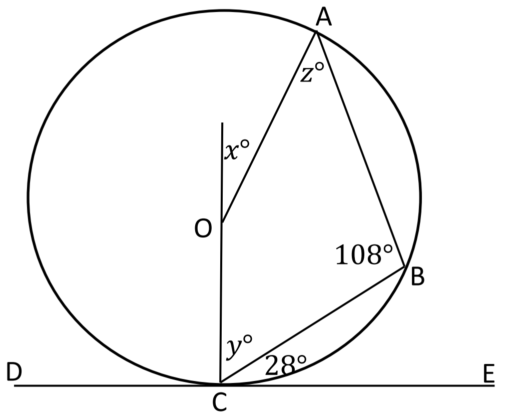
❶ Justifies (diagram or tests)

Some students did not justify their inequality

Then from graph, $(x-2)(x-3) < 0$ when $2 < x < 3$.

❶ solution

(b) O is the centre of the circle. Find the value of each pronumeral giving reasons. **3**



If constructing or adding extra information into a given diagram, you MUST reproduce the diagram with the constructions or additions.

$x + 180^\circ = 2 \times 108^\circ$ ($\angle$ at cent. twice $\angle$ at circum.

$x = 36^\circ$

On same arc – reflex arc AC)

❶ correct with reason

Many gave $z = 28^\circ$ and stated the Angle in Alternate Segment Theorem – this does not apply here!

$y + 28^\circ = 90^\circ$ (tang. and radius are $\perp$)

$y = 62^\circ$

❶ correct with reason

$z + 144^\circ + 62^\circ + 108^\circ = 360^\circ$ ($\angle$ sum of quad).

$z = 46^\circ$

❶ correct with reason

$\angle CAB = 28^\circ$

(c) Find the co-ordinates of the point that divides externally the interval joining the points $(-5,6)$ and $(-2,3)$ in the ratio $3:2$. **3**

Solution	Marking	Comments
$(?,?)$----$(-5,6)$-----$(-2,3)$----$(?,?)$ ---- 3 ----- 2 ---- i.e. point is to the "right" of $(-2,3)$ hence $\left(\dfrac{3\times{}^{-}2+{}^{-}2\times{}^{-}5}{3+{}^{-}2},\dfrac{3\times3+{}^{-}2\times6}{3+{}^{-}2}\right)$ $=\left(4,{}^{-}3\right)$	❶ correct formula or other explanation ❶ –ve for external division correct ❶ solution	Many did not substitute into the formula correctly.

(d) The acute angle between the lines $2x+2y=5$ and $y=3x+1$ is θ. Find the value of θ, correct to the nearest minute. **3**

Solution	Marking	Comments
$2x+2y=5$ and $y=3x+1$ $y=\dfrac{5}{2}-x$ $\quad m_2=3$ $m_1=-1$ Hence $\tan\theta=\left\|\dfrac{m_2-m_1}{1+m_1m_2}\right\|$	❶ correct gradients in formula	Some students stated the formula incorrectly.
$=\left\|\dfrac{3-{}^{-}1}{1+3\times{}^{-}1}\right\|$ $=\left\|\dfrac{4}{{}^{-}2}\right\|$ $=2$	❶ correct tan value	
$\theta=\tan^{-1}(2)$ $=63^\circ26'$	❶ correct angle in degrees & minutes	

(e) Let $P(x)=x^3-ax^2+x$ be a polynomial where a is a real number. When $P(x)$ is divided by $(x-3)$ the remainder is 12. Find the remainder when $P(x)$ is divided by $(x+1)$ **3**

Solution	Marking	Comments
$P(x)=x^3-ax^2+x$ gives $P(3)=3^3-a\times3^2+3$	❶ $x=3$ correctly substituted	Mostly well done.
$12=30-9a$ $a=2$	❶ correct a value	
Hence $x={}^{-}1$ gives $P({}^{-}1)=({}^{-}1)^3-2\times({}^{-}1)^2+({}^{-}1)$ $={}^{-}1-2+{}^{-}1$ $={}^{-}4$	❶ correct remainder	

Question Two (Start a new booklet)

marks

(a) Evaluate $\tan 225°$. **1**

$\tan 225^\circ$

$= +\tan 45^\circ$

$= 1$

❶ correct value

Mostly well done

(b) Simplify $\dfrac{1}{1-\sin V}+\dfrac{1}{1+\sin V}$ **3**

$$\frac{1}{1-\sin V}+\frac{1}{1+\sin V}$$
$$=\frac{1+\sin V+1-\sin V}{(1-\sin V)(1+\sin V)}$$
$$=\frac{2}{1-\sin^2 V}$$
$$=\frac{2}{\cos^2 V}$$
$$=2\sec^2 V$$

❶ numerator correct

Occasional minor errors

❶ resolves denominator

A few could not resolve denominator into cos.

❶ answer

A non-fraction is simpler than a fraction! Many lost this last mark.

(c) The bearing of a ship from lighthouse A is N75°E. The bearing from a second lighthouse B, 44 km south of A, is N40°E. Find the distance of the ship from B to the nearest km. You must draw a neat diagram marking all given information. **3**

44 *km*

❶ diagram correct

Some students did not have North directly up the page for their bearing questions. Several also drew as if it was a 3D trig question.

Quite a few had issues putting A north of B in their diagram.

$\angle SAB = 105^\circ$ (st. ln.), then $\angle ASB = 35^\circ$ (Δ sum), so

$$\frac{BS}{\sin 105^\circ}=\frac{44}{\sin 35^\circ}$$
$$BS=\frac{44\sin 105^\circ}{\sin 35^\circ}$$
$$=74.09777261$$
$$\doteq 74\,km$$

❶ Sine Rule correct

Many simple calculation errors.

❶ answer

READ the requirements of the question! Nearest *km*!

(d) A, B and C are three towns such that B is $20\,km$ from A in a direction $330°$ and C is $30\,km$ from A in a direction $203°\ 08'$. Find the distance from B to C to one decimal place. You must draw a neat diagram marking all given information. **3**

Similar general comments as for (c) above.

20 km

❶ diagram correct

30 km

$$\angle BAC = 330^\circ - 203^\circ 08'$$
$$= 126^\circ 52'$$
$$BC^2 = 20^2 + 30^2 - 2\times 20\times 30\times\cos\left(126^\circ 52'\right)$$
$$= 1300 - 1200\cos\left(126^\circ 52'\right)$$
$$\doteq 2019.945863$$
$$BC \doteq 44.94380784$$
$$\doteq 44.9$$

❶ Cosine Rule correct

READ – to 1 dp. Note: Only one mark deducted in this question for this type of error.

❶ answer

(e) The sides of a triangular field have lengths 80m, 90m and 100m.

(i) Calculate the size of the smallest angle to the nearest minute. You must draw a neat diagram marking all given information. **3**

Many wasted precious exam time on this question by not realizing that the smallest side is opposite the smallest angle!

Otherwise generally well done.

❶ diagram correct

$$\cos\theta = \frac{100^2 + 90^2 - 80^2}{2\times 90\times 100}$$
$$= \frac{117}{180}$$
$$\theta = \cos^{-1}\left(\frac{117}{180}\right)$$
$$= 49^\circ 28'$$

❶ Cosine Rule correct

❶ Answer to nearest minute

(ii) Calculate the area of the field to the nearest m^2 . **2**

$A = \frac{1}{2} \times 100 \times 90 \times \sin\left(49^\circ 28'\right)$ $= 3420.126015$ $\doteq 3420\, m^2$	❶ Area rule correct ❶ Answer	Generally well done, although a few wanted to use cos instead of sin.

Question Three (Start a new booklet)

marks

(a) Write down the expansion for $\tan\left(\theta+\phi\right)$. **1**

$$\tan\left(\theta+\phi\right)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

❶

Well done.

(b) Simplify $\frac{1}{2}\sin 2\theta\tan\theta$ **2**

$$\frac{1}{2}\sin 2\theta\tan\theta$$
$$=\frac{1}{2}\times 2\sin\theta\cos\theta\times\frac{\sin\theta}{\cos\theta}$$
$$=\sin^2\theta$$

❶ correct substitutions
❶ Answer

Generally well done.

(c) If α and β are angles between 0° and 90°, and $\sin\alpha=\frac{3}{5}$ and $\tan\beta=\frac{7}{24}$, find without the use a calculator the exact value of $\sin\left(\alpha-\beta\right)$. **3**

Many did not draw diagrams!

Some wrote $\sin\frac{3}{5}$ instead of $\sin\alpha=\frac{3}{5}$

$$\sin\beta=\frac{7}{25},\cos\beta=\frac{24}{25}$$

$$\sin\alpha=\frac{3}{5},\cos\alpha=\frac{4}{5}$$

❶ setup diagrams and values correct

Then $\sin\left(\alpha-\beta\right)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$
$$=\frac{3}{5}\times\frac{24}{25}-\frac{4}{5}\times\frac{7}{25}$$
$$=\frac{44}{125}$$

❶ correct formula with vaules

❶ Answer

Some students got the formula wrong.

(d) Solve for $0 \le \theta \le 2\pi$:

(i) $\sin 2\theta = \dfrac{-1}{2}$ **2**

If $0 \le \theta \le 2\pi$ then $0 \le 2\theta \le 4\pi$.

$2\theta = \sin^{-1}\left(\dfrac{-1}{2}\right)$

(in quadrants 3,4 – with base angle $\sin^{-1}\left(\dfrac{1}{2}\right)$

$= \dfrac{\pi}{6}$

$2\theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{19\pi}{6}, \dfrac{23\pi}{6}$

$\theta = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}$

➊ Base angle and quadrants resolved

➊ correct answers

About a quarter of the candidates only got 2 of the 4 answers.

(ii) $\sin^2 x - \sin x \cos x = 0$ **3**

$\sin x(\sin x - \cos x) = 0$

Hence $\sin x = 0$ or $\sin x - \cos x = 0$

$x = 0, \pi, 2\pi$ $\quad\quad$ $\sin x = \cos x$

$\tan x = 1 (\Rightarrow Q1, 3)$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

Thus $x = 0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}, 2\pi$

➊ factoring

➊ for cases correct

➊ Answers

Factorizing a problem for some: $\sin x(1 - \cos x)$ was a common error.

Many got the 3 answers for sin but then forgot the other two answers.

(e)

(i) Write $\sqrt{3}\cos x - \sin x$ in form $r\cos(x + \alpha)$ where $0 < \alpha < \dfrac{\pi}{2}$. **2**

$r = \sqrt{\sqrt{3}^2 + 1^2}$ and $\tan \alpha = \dfrac{1}{\sqrt{3}}$

$= \sqrt{4}$

$= 2$ $\quad\quad$ $\alpha = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$= \dfrac{\pi}{6}$

Hence $\sqrt{3}\cos x - \sin x = 2\cos\left(x + \dfrac{\pi}{6}\right)$

➊ correct r & θ values

➊ correct expression

Mostly well done.

Some students had $\left(x - \dfrac{\pi}{6}\right)$

(ii) Hence or otherwise, solve $\sqrt{3}\cos x = 1 + \sin x$ for $0 < x < 2\pi$. **2**

$\sqrt{3}\cos x = 1 + \sin x$

$\sqrt{3}\cos x - \sin x = 1$

$2\cos\left(x + \dfrac{\pi}{6}\right) = 1$ $\quad$ using part (i).

For $0 < x < 2\pi$ then $\dfrac{\pi}{6} < x + \dfrac{\pi}{6} < \dfrac{13\pi}{6}$

➊ correct base angle and quadrants

$\cos\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$ (i.e. Q1 and Q4) $x+\frac{\pi}{6}=\cos^{-1}\left(\frac{1}{2}\right)$ $=\frac{\pi}{3},\frac{5\pi}{3}$ $x=\frac{\pi}{3}-\frac{\pi}{6},\frac{5\pi}{3}-\frac{\pi}{6}$ $=\frac{\pi}{6},\frac{3\pi}{2}$	➊ correct values	Mostly well done.

Question Four (Start a new booklet) marks

(a) Use the rules of differentiation to differentiate with respect to x:

(i) $y=(3x+4)(x^2-2x)$ **2**

$$y=3x^3-2x^2-8x$$
$$y'=9x^2-4x-8$$

❶ correct expansion or rule
❶ correct differentiation

Some who used the product rule also made silly errors

Alternative Method (Product Rule):

$$y'=u.dv+v.du \qquad \text{where} \quad u=3x+4 \quad v=x^2-2x$$
$$du=3 \qquad dv-2x-2$$
$$=(3x+4)(2x-2)+3(x^2-2x)$$
$$=6x^2-6x+8x-8+3x^2-6x$$
$$=9x^2-4x-8$$

(ii) $y=\sqrt{25-x^2}$ (Answer as a surd) **2**

$$y=(25-x^2)^{\frac{1}{2}} \qquad \text{where} \quad u=25-x^2$$
$$du=-2x$$
$$y'=\frac{1}{2}(25-x^2)^{-\frac{1}{2}}\times(-2x)$$
$$=\frac{-x}{\sqrt{25-x^2}}$$

❶ correct Chain Rule
❶ correct differentiation

Some left the negative sign out.

(iii) $y=\dfrac{x+1}{x^2+1}$ **3**

$$y'=\frac{v.du-u.dv}{v^2} \quad \text{where} \quad u=x+1 \quad v=x^2+1$$
$$du=1 \quad dv=2x$$
$$v^2=(x^2+1)^2$$
$$y'=\frac{(x^2+1).1-(x+1).2x}{(x^2+1)^2}$$
$$=\frac{x^2+1-(2x^2+2x)}{(x^2+1)^2}$$
$$=\frac{1-2x-2x^2}{(x^2+1)^2}$$

❶ correct Quotient Rule (or equivalent)
❶ correct algebra
❶ correct differentiation

Students need to <u>state</u> the rule then apply u and v in the correct order.

(b) Differentiate $f(x)=(x+1)^3$ from first principles. **3**

The standard method:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{((x+h)+1)^3 - (x+1)^3}{h}$$

$$= \lim_{h \to 0} \frac{(x+(h+1))^3 - (x+1)^3}{h} ❶$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2(h+1) + 3x(h+1)^2 + (h+1)^3 - (x^3 + 3x^2 + 3x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3x^2 + 3xh^2 + 6xh + 3x + h^3 + 3h^2 + 3h + 1 - (x^3 + 3x^2 + 3x + 1)}{h} ❶$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + 6xh + h^3 + 3h^2 + 3h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + 6x + h^2 + 3h + 3 ❶$$

$$= 3x^2 + 6x + 3$$

(marks for reaching steps indicated above)

Alternative methods:

Some students did realise other (simpler) methods, such as Difference of Two Cubes or Substitution:

Difference of Two Cubes:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h+1)^3 - (x+1)^3}{h} ❶$$

$$= \lim_{h \to 0} \frac{(x+h+1-x-1)\left((x+h+1)^2 + (x+h+1)(x+1) + (x+1)^2\right)}{h}$$

$$= \lim_{h \to 0} \frac{h\left((x+h+1)^2 + (x+h+1)(x+1) + (x+1)^2\right)}{h} ❶$$

$$= \lim_{h \to 0} \left((x+h+1)^2 + (x+h+1)(x+1) + (x+1)^2\right)$$

$$= (x+1)^2 + (x+1)(x+1) + (x+1)^2$$

$$= 3(x+1)^2 ❶$$

Substitution:

$$u = x+1$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(u+h)^3-(u)^3}{h} ❶$$

$$= \lim_{h\to 0}\frac{u^3+3u^2h+3uh^2+h^3-u^3}{h}$$

$$= \lim_{h\to 0}\frac{3u^2h+3uh^2+h^3}{h} ❶$$

$$= \lim_{h\to 0} 3u^2+3uh+h^2$$

$$= 3u^2 ❶$$

$$= 3(x+1)^2$$

Generally (regardless of method used) the notation and setting out was terrible.

Many need to practice cubic expansions.

Some "fudged" their working to make the algebra come out correctly.

(c) A block is cast into the shape of a right cylinder with a total surface area of $20\pi\, cm^2$. The radius of the base is *r* cm and the height is *h* cm. The total surface area of a cylinder is given by $A = 2\pi r^2 + 2\pi rh$.

(i) Express *h* in terms of *r*. **1**

$$A = 2\pi r^2 + 2\pi rh$$

$$20\pi = 2\pi r^2 + 2\pi rh$$

$$10 - r^2 = rh$$

$$h = \frac{10}{r} - r$$

❶ setup and algebra correct

Generally well done.

(ii) Express the volume *V* in terms of *r*. **1**

$$V = \pi r^2 h$$

$$= \pi r^2\left(\frac{10}{r} - r\right)$$

$$= 10\pi r - \pi r^3$$

❶ Answer

Some carry on errors.

(iii) Find the value of *r* for which the volume is the greatest. Answer in exact form. **3**

$$V = 10\pi r - \pi r^3$$

$$\frac{dV}{dr} = 10\pi - 3\pi r^2$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

Stat pts when $\frac{dV}{dr}=0$ $0=10\pi-3\pi r^2$ $3r^2=10$ $r=\pm\sqrt{\frac{10}{3}}$	❶ condition for stat pt correct	Must state for stat. pts. $\frac{dV}{dr}=0$
	❶ r value justified	Should state + and – answers and why –ve rejected. Must test first or second derivative for min/max.
As $r>0$, $\frac{d^2V}{dr^2}<0 \Rightarrow ccd\ or \max t.p.$ Hence $r=\frac{\sqrt{30}}{3}$ for greatest volume.	❶ r correct	Need to conclude and actually answer the question posed.

Question Five (Start a new booklet) marks

(a) Find the general solution to $\cos 2\theta = 2 + \cos\theta$. **3**

$$2\cos^2\theta - 1 = 2 + \cos\theta$$
$$0 = 2\cos^2\theta - \cos\theta - 3$$
$$= 2\cos^2\theta - 3\cos\theta + 2\cos\theta - 3$$
$$= 2\cos\theta(2\cos\theta - 3) + (2\cos\theta - 3)$$
$$= (\cos\theta + 1)(2\cos\theta - 3)$$

Hence

$$\cos\theta + 1 = 0 \qquad 2\cos\theta - 3 = 0$$
$$\cos\theta = -1 \qquad \cos\theta = \frac{3}{2}$$
$$\theta = 2n\pi \pm \cos^{-1}(-1) \qquad no\ soln$$
$$= (2n \pm 1)\pi$$

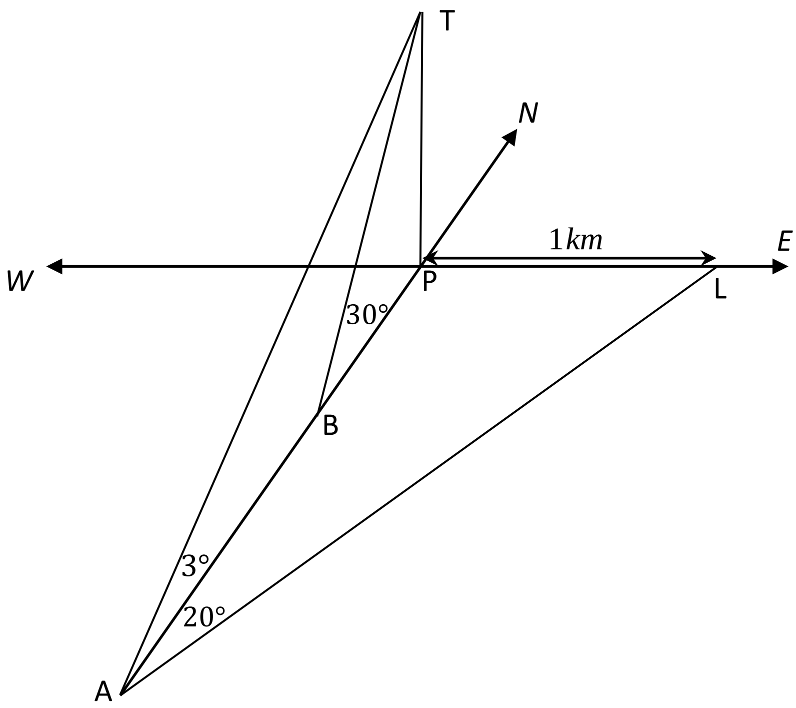
❶ quadratic in cos

❶ cases resolved

❶ general soln correct

Generally well done.

(b) A boat is sailing due north from a point A towards a point P on the shore line. The shore line runs from west to east.
In the diagram, T represents a tree on a cliff vertically above P, and L represents a landmark on the shore. The distance PL is $1km$.
From A the point L is on a bearing of 020°, and the angle of elevation to T is 3°. After sailing for some time the boast reaches a point B, from which the angle of elevation to T is 30^o.



(i) Show that $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$ **3**

In ΔALP: $\tan 20^\circ = \dfrac{1}{AP}$

$$AP = \cot 20^\circ$$

❶ correct AP with Δ identified

This is a SHOW question – the result for BP cannot be used in the working!

In ΔATP: $\tan 3^\circ = \dfrac{PT}{AP}$ $PT = AP\tan 3^\circ$ $= \dfrac{\tan 3^\circ}{\tan 20^\circ}$ In ΔBTP: $\tan 30^\circ = \dfrac{PT}{BP}$ $BP = \dfrac{PT}{\frac{1}{\sqrt{3}}}$ $= \sqrt{3}PT$ $= \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$	❶ correct correct PT with Δ identified ❶ correct substitution and resolution of algebra	AP, PT and BP MUST be derived from the given information! All working had to be shown for these derivations. Many students lacked organisation in their solutions. Also, many quite illegible. This MUST improve!

(ii) Find the exact distance *AB*. **1**

$AB = AP - BP$ $= \dfrac{1}{\tan 20^\circ} - \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$ $= \dfrac{1-\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$	❶ answer (in exact form)	Many lost the mark because their answer was not in EXACT FORM as specified by the question!

(c) If $t = \tan\left(\dfrac{\theta}{2}\right)$ solve for t the equation $12\tan\theta = 5$ for $\pi \le \theta < \dfrac{3\pi}{2}$ **4**

$12 \times \dfrac{2t}{1-t^2} = 5$ $24t = 5 - 5t^2$ $0 = 5t^2 + 24t - 5$ $= 5t^2 + 25t - t - 5$ $= 5t(t+5) - (t+5)$ $= (t+5)(5t-1)$ Hence $t+5=0$ or $5t-1=0$ $t=-5$ $\quad$ $5t=1$ $t = \dfrac{1}{5}$ With $\pi \le \theta < \dfrac{3\pi}{2}$, then $\dfrac{\pi}{2} \le \dfrac{\theta}{2} < \dfrac{3\pi}{4}$ (or $90^\circ \le \dfrac{\theta}{2} < 135^\circ$)leads to: $\dfrac{\theta}{2} = \tan^{-1}(-5)$ $\quad$ $\dfrac{\theta}{2} = \tan^{-1}\left(\dfrac{1}{5}\right)$ $\approx 101^\circ$ $\quad$ $\approx 11^\circ$	❶ quadratic in t ❶ two results correct ❶ testing domain	Most students obtained the first two marks easily. Some need to learn the '*t*' results! For full marks, students needed to refer to the domain in their answer. Most students lost a mark for NOT ANSWERING the question! i.e. to solve for '*t*' values, NOT θ values!

Hence $t=-5$ (reject $t=\frac{1}{5}$ as out of the required domain)	❶ correct t value	

(d) Sketch $y=x+\frac{4}{x-1}$, showing asymptotes, intercepts and stationary points having determined their nature. **4**

$y-x=\frac{4}{x-1}$ $(y-x)(x-1)=4 \quad \Rightarrow y\neq x, x\neq 1$ Hence asymptotes are $y=x$ and $x=1$.	❶ asymptotes correctly identified	$y\neq x, x\neq 1$ are NOT asymptotes.
$\frac{dy}{dx}=1-\frac{4}{(x-1)^2}$ $=\frac{(x-1)^2-4}{(x-1)^2}$ $\frac{d^2y}{dx^2}=\frac{8}{(x-1)^3}$ $\frac{dy}{dx}=0\Rightarrow$ $0=(x-1)^2-4$ $4=(x-1)^2$ $\pm 2=x-1$ $x=1\pm 2$ $=-1,3$		Most students were able to find the correct stationary points. However, testing for nature of stationary points needs to be done correctly – make sure you a using the <u>sign</u> of *y*' or *y*" to determine the nature.
$x=-1$: $y=-3$ $\frac{d^2y}{dx^2}=\frac{8}{(-1-1)^3}$ $<0\Rightarrow ccd$ $\therefore(-1,-3)$ a max t.p. $x=3$: $y=5$ $\frac{d^2y}{dx^2}=\frac{8}{(3-1)^3}$ $>0\Rightarrow ccu$ $\therefore(3,5)$ a min t.p. Also $x=0\Rightarrow y=-4$	❶ Stat pts correct ❶ nature of stat pts correctly done	Graph: Overall, poorly done. In many cases: i) The curve did not approach the asymptotes ii) No relative scale was shown iii) No *y*-intercept shown iv) Asymptotes were drawn solid, not dotted v) The graph size was too small and/or cramped and unclear. vi) Some graphs looked like a string of spaghetti dropped down on the page in roughly the right shape! Take more care with graphing!

❶ diagram (including *y* intercept) correct