Name: ____________________________

Teacher: ____________________________

Class: ____________________________

FORT STREET HIGH SCHOOL

# 2016

**HIGHER SCHOOL CERTIFICATE PRELIMINARY COURSE**

## ASSESSMENT TASK 3 – PRELIMINARY FINAL EXAM

# Mathematics Extension 1

**Time allowed: 2 Hours**
(plus 5 minutes reading time)

| Syllabus Outcomes | Assessment Area Description and Marking Guidelines | Questions |
|---|---|---|
| P3<br>PE3 | chooses and applies appropriate arithmetic, algebraic, graphical , trigonometric and geometric techniques<br>solves problems involving polynomials | 1 |
| P4 | chooses and applies appropriate arithmetic, algebraic, graphical and trigonometric techniques | 3, 4, 5 |
| P6<br>P7 | relates the derivative of a function to the slope of its graph<br>determines the derivative of a function through routine application of the rules of differentiation | 2 |
| PE6 | makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations | 6 |

## Total Marks 75

Attempt Questions 1-6

| Question | Out of | Marks |
|---|---|---|
| Q1 | /13 | |
| Q2 | /13 | |
| Q3 | /11 | |
| Q4 | /13 | |
| Q5 | /13 | |
| Q6 | /12 | |
| Total | /75 | |
| | Percent: | |

## General Instructions:

- Questions 1-6 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-6, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

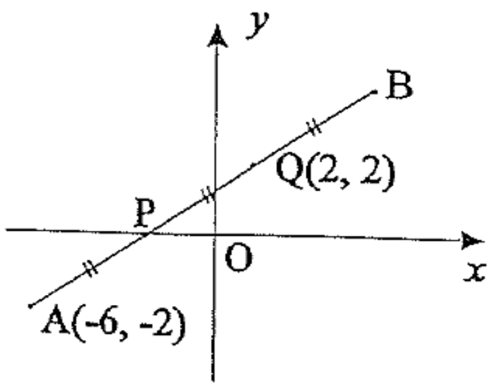
Question 1 13 marks *marks*

a. Find the exact value of $\tan 15°$ 2

b. Solve the inequality $\dfrac{x}{2x-1} \le 5$ 3

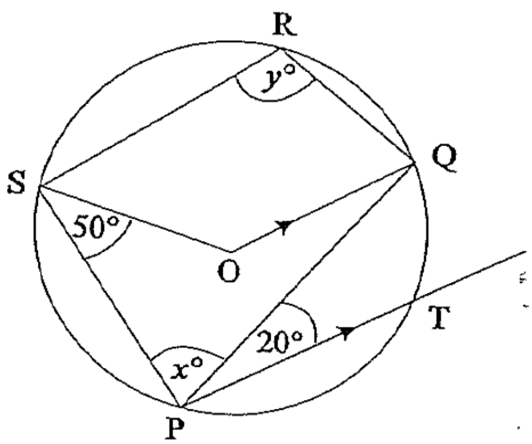
c. On the same axes, sketch the curve $y = x^2$ and $y = |x|$ 2

d. The diagram shows the line interval $AB$ trisected at $P$ and $Q$. The coordinates of $A$ are $(-6,-2)$ and $Q$ are $(2,2)$. Find the coordinates of $B$. 2




e. In the diagram, $O$ is the centre of the circle and $PT \parallel OQ$. Find the values of $x$ and $y$, giving reasons.

4



Question 2 13 marks *marks*

a. Given that $f\left(x\right)=\dfrac{a^{x}+a^{-x}}{2}$, show that $f\left(2x\right)=2\left[f\left(x\right)\right]^{2}-1$ *3*

b. Differentiate $2-5x+2x^{2}$ using first principles. *2*

c. Differentiate each of the following *4*

i. $\dfrac{1}{2}x^{4}-3x+7$

ii. $\dfrac{2x}{x^{2}+4}$

iii. $x^{2}\left(5x^{3}-\dfrac{1}{2x^{2}}\right)^{3}$

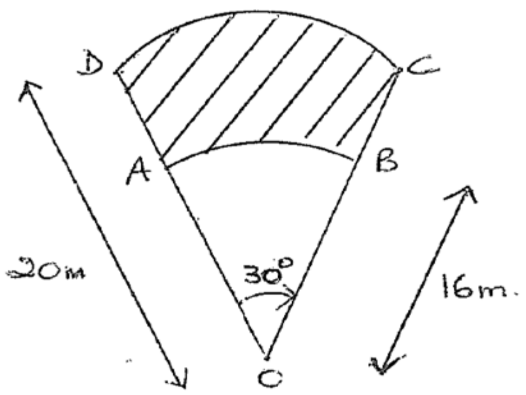
d. Find the equation of the tangent to the curve $y=2x\sqrt{x+1}$ at the point where $x=3$. *4*

Question 3 11 marks *marks*

a.

If $\tan B = -0.2$ and $0° < B < 270°$, find the value of $\operatorname{cosec} B$ in surd form. *2*

b. The diagram below shows the field laid out for the shotput finals at Homebush. In order to qualify for the finals, the shotput must land in the shaded area $ABCD$. *4*



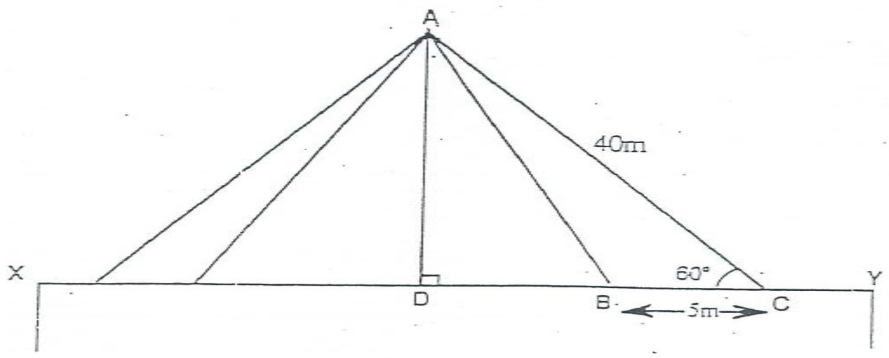

Find the exact perimeter and area of the shaded area $ABCD$.

c. Graph the curve $y = \sqrt{2}\sin x + \cos x$ for $0 \le x \le 2\pi$ *3*

d. The lines $3x - y + 2 = 0$ and $mx - y - 1 = 0$ intersect at $45°$. Find the possible value(s) of $m$. *2*

Question 4 13 marks *marks*

a. A horizontal bridge joins points $X$ and $Y$. Cables were used to support the bridge as shown in the diagram. The distance between the cables $AB$ and $AC$ was $5m$ while $AC$ was $40m$ and $\angle ACB = 60°$ *4*




i. Find the exact height of $A$ above the bridge.

ii. Use the cosine rule to find the length of the cable $AB$.

b. Prove that $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$ and hence evaluate $\tan\dfrac{\pi}{8}$ in exact form. *3*

c. Find the general solutions for $\cos 2\theta = \cos\theta$ *3*

d. Solve $\cos x - \sqrt{3}\sin x = 1$, where $0 \le x \le 2\pi$ by using the $t$ method *3*

Question 5 13 marks *marks*

a. *2*

i. Expand $\cos(\alpha+\beta)$

ii. Show that $\cos 2\alpha = 1 - 2\sin^2 \alpha$

b. If $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ and $\beta = \cos^{-1}\left(\frac{4}{5}\right)$, calculate the exact value of $\tan(\alpha-\beta)$ *2*

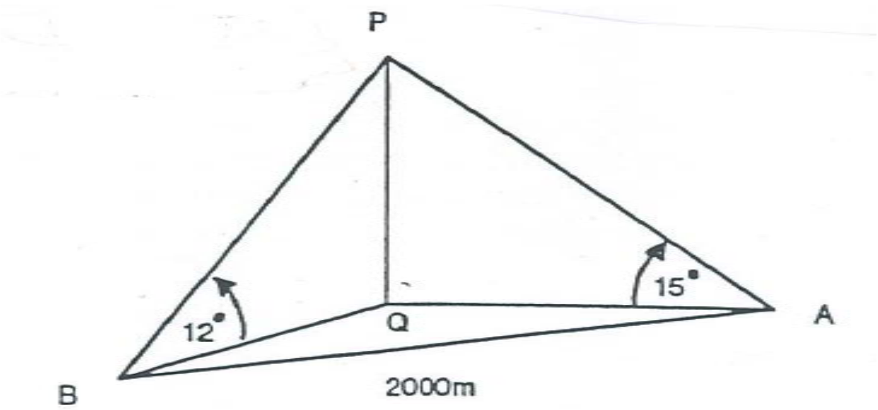
c. $ABCD$ is a cyclic quadrilateral.
Show that $\tan A + \tan B + \tan C + \tan D = 0$ *2*

d.

i. Show that $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$ *3*

ii. Hence, or otherwise, solve $\frac{\sin^3\theta}{\cos\theta} = 1 - \sin\theta\cos\theta$ for $0 \le \theta \le 360$

e. *4*




the angle of elevation of a tower $PQ$ of height $h$ metres at a point $A$ due east of it is $15°$. From another point $B$, due south of the tower the angle of elevation is $12°$. The point $A$ and $B$ are $2000m$ apart on ground level. Find the height $h$ of the tower to the nearest metre.

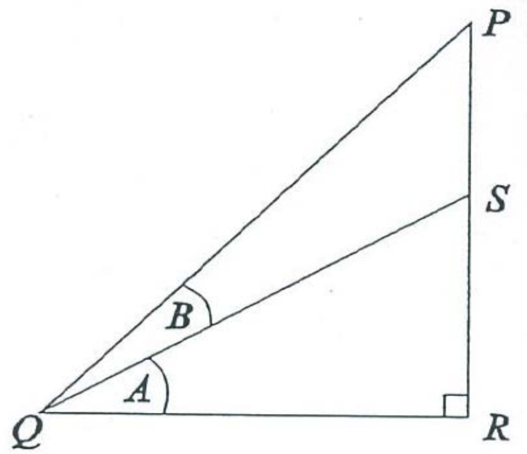
Question 6 12 marks *marks*

a. A man, $M$ , is standing in a horizontal plane which also contains a tower, $AB$ , which is $10$ metres high, and a building, $XY$ , which is $95$ metres high. The man is $210$ metres from the tower and his bearing from it is $42°$.His bearing from the building is $110°$. The bearing of the tower from the building is $150°$.
Put all this information into a diagram. *4*

Find the angle of elevation from the top of the tower to the top of the building. (to the nearest degree)

b.

In the diagram $\angle A > \angle B > 0$ and $\dfrac{\cos(A+B)}{\cos(A-B)} = \dfrac{4}{5}$ *8*




i. Show that $\tan A \times \tan B = \dfrac{1}{9}$

ii. If $PR = QR$, show that $9(\tan A + \tan B) = 8$

iii. Hence, find $A$ and $B$.

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| Question 1 13 marks | *marks* |
|---|---|

**a.** **Find the exact value of** $\tan 15°$ — **2**

$$\tan 15° = \tan\left(45-15\right)$$
$$=\frac{\tan 45-\tan 30}{1+\tan 45\tan 30}$$
$$=\frac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}}$$
$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}+1}$$

**b.**

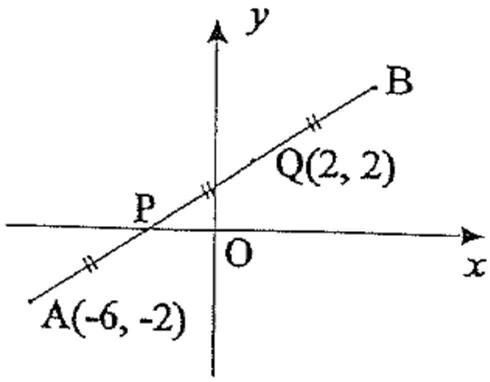
**Solve the inequality** $\frac{x}{2x-1}\le 5$ — **3**

$$\left(2x-1\right)^2\frac{2}{2x-1}\le 5\left(2x-1\right)^2$$
$$\left(2x-1\right)x\le 5\left(2x-1\right)^2$$
$$0\le 5\left(2x-1\right)^2-x\left(2x-1\right)$$
$$0\le\left(2x-1\right)\left[5\left(2x-1\right)-x\right]$$
$$0\le\left(2x-1\right)\left(9x-5\right)$$
$$\frac{1}{2}>x\ge\frac{5}{9}\quad as\quad x\ne\frac{1}{2}$$

**c.** **On the same axes, sketch the curve** $y=x^2$ **and** $y=\left|x\right|$ — **2**

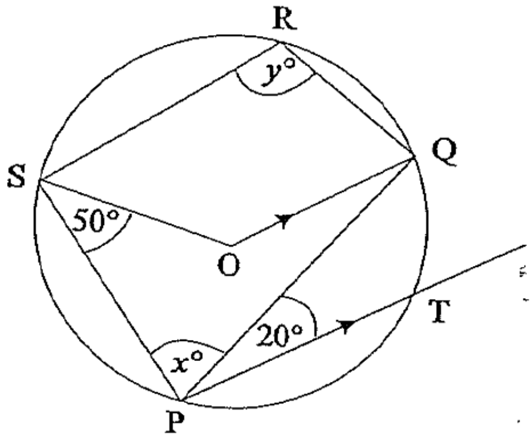
$$0<x<1$$
$$-1<x<0$$

**d.** **The diagram shows the line interval** $AB$ **trisected at** $P$ **and** $Q$**. The coordinates of** $A$ **are** $\left(-6,-2\right)$ **and** $Q$ **are** $\left(2,2\right)$**. Find the coordinates of** $B$**.** — **2**




Horizontal distance=4
vertical distance =2
Therefore $B(6,4)$

**e.** **In the diagram, $O$ is the centre of the circle and $PT \parallel OQ$. Find the values of $x$ and $y$, giving reasons.** **4**




$x+y=180$ opposite angles of a cyclic quadrilateral sum to 180)
$\angle QPT = \angle 0QP = 20$ (alternate angles in parallel lines)
$OP = OQ$ (radii of circle)
$\Delta OPQ$ is isosceles (from above)
$\therefore \angle OPQ = 20$ (base angles of isosceles triangles are equal)
In $\Delta SOP$
$SO = OP$ (radii of circle)
$\Delta SOP$ is isosceles (from above)
$\therefore \angle OPS = 50$ (base angles of isosceles triangles are equal)
$\angle SPQ = \angle SPO + \angle QPO = 50 + 20 = 70$
$x = 70, \quad y = 180 - 70 = 110$

Question 2 13 marks *marks*

a. Given that $f\left(x\right)=\dfrac{a^{x}+a^{-x}}{2}$, show that $f\left(2x\right)=2\left[f\left(x\right)\right]^{2}-1$ *3*

$$f\left(2x\right)=\frac{a^{2x}+a^{-2x}}{2}$$

$$\left[f\left(x\right)\right]^{2}=\left[\frac{a^{x}+a^{-x}}{2}\right]^{2}=\frac{a^{2x}+2\times a^{x}\times a^{-x}+a^{-2x}}{4}=\frac{a^{2x}+a^{-2x}+2}{4}$$

$$2\left[f\left(x\right)\right]^{2}=\frac{a^{2x}+a^{-2x}}{2}+1$$

$$2\left[f\left(x\right)\right]^{2}-1=\frac{a^{2x}+a^{-2x}}{2}+1-1$$

$$=f\left(2x\right)$$

b. Differentiate $2-5x+2x^{2}$ using first principles. *2*

$$f\left(x\right)=2-5x+2x^{2}$$

$$f\left(x+h\right)=2-5\left(x+h\right)+2\left(x+h\right)^{2}=2-5x-5h+2x^{2}+4xh+2h^{2}$$

$$f'\left(x\right)=\lim_{h\to0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

$$f'\left(x\right)=\lim_{h\to0}\frac{2-5x-5h+2x^{2}+4xh+2h^{2}-\left(2-5x+2x^{2}\right)}{h}$$

$$f'\left(x\right)=\lim_{h\to0}\frac{-5h+4xh+2h^{2}}{h}$$

$$f'\left(x\right)=\lim_{h\to0}\frac{h\left(-5+4x+2h^{2}\right)}{h}$$

$$f'\left(x\right)=-5+4x$$

c. Differentiate each of the following

i. $\dfrac{1}{2}x^{4}-3x+7$ *4*

$$2x^{3}-3$$

ii. $\dfrac{2x}{x^2+4}$

$$\frac{\left(x^2+4\right)2-2x\left(2x\right)}{\left(x^2+4\right)^2}$$
$$=\frac{-2x^2+8}{\left(x^2+4\right)^2}$$

4

iii. $x^2\left(5x^3-\dfrac{1}{2x^2}\right)^3$

$$x^2\left(5x^3-\frac{1}{2}x^{-2}\right)^3$$
$$=2x\left(5x^3-\frac{1}{2}x^{-2}\right)^3+x^2\left[3\left(5x^3-\frac{1}{2}x^{-2}\right)^2\left(15x^2+x^{-3}\right)\right]$$
$$=2x\left(5x^3-\frac{1}{2}x^{-2}\right)^3+3x^2\left(5x^3-\frac{1}{2}x^{-2}\right)^2\left(15x^2+x^{-3}\right)$$
$$=x\left(5x^3-\frac{1}{2}x^{-2}\right)^2\left[2\left(5x^3-\frac{1}{2}x^{-2}\right)+3x\left(15x^2+x^{-3}\right)\right]$$
$$=x\left(5x^3-\frac{1}{2}x^{-2}\right)^2\left(10x^3-2x^{-2}+45x^3+3x^{-2}\right)$$
$$=x\left(5x^3-\frac{1}{2}x^{-2}\right)^2\left(55x^3+x^{-2}\right)$$

d. Find the equation of the tangent to the curve $y=2x\sqrt{x+1}$ at the point where $x=3$.

$y = 2x(x+1)^{\frac{1}{2}}$

$y' = 2(x+1)^{\frac{1}{2}} + 2x \times \frac{1}{2}(x+1)^{-\frac{1}{2}} = 2\sqrt{x+1} + \frac{x}{\sqrt{x+1}}$

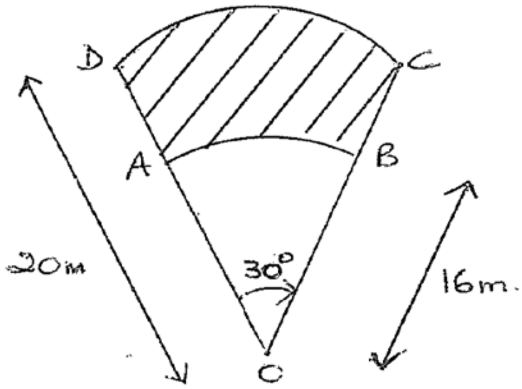
$x = 3: \; y' = 2\sqrt{4} + \frac{3}{\sqrt{4}}$

$if \; \sqrt{4} = 2 \;\; y' = \frac{11}{2}$

$if \; \sqrt{4} = -2 \;\; y' = -\frac{11}{2}$

| | |
|---|---|
| $x = 3, \; y = 2(3)\sqrt{4} = 12$<br>$y - 12 = \frac{11}{2}(x-3)$<br>$11x - 2y - 9 = 0$ | $x = 3, \; y = 2(3)\sqrt{4} = -12$<br>$y + 12 = -\frac{11}{2}(x-3)$<br>$11x - 2y - 9 = 0$ |

| **Question 3** **11 marks** | ***marks*** |
|---|---|
| **a.** If $\tan B = -0.2$ and $0° < B < 270°$, find the value of $\cos ec B$ in surd form. | ***2*** |
| Answer must be in the second quadrant<br>$\sin B = \frac{1}{26}$<br>$\text{cosec}B = \frac{1}{\sin B}$<br>$= \sqrt{26}$ | |
| **b.** The diagram below shows the field laid out for the shotput finals at Homebush. In order to qualify for the finals, the shotput must land in the shaded area $ABCD$. | ***4*** |




**Find the exact perimeter and area of the shaded area $ABCD$.**

$$l_1 = 16\times\frac{\pi}{6} = \frac{8}{3}\pi$$
$$l_2 = 20\times\frac{\pi}{6} = \frac{10}{3}\pi$$
$$P = \frac{8}{3}\pi + \frac{10}{3}\pi + 4 + 4 = 6\pi + 8$$

3

$$A = \frac{1}{2}(20)^2\frac{\pi}{6} - \frac{1}{2}(16)^2\frac{\pi}{6}$$
$$= \frac{1}{2}\frac{\pi}{6}(400-256)$$
$$= 144\frac{\pi}{12} = 12\pi$$

2

**c. Graph the curve** $y=\sqrt{2}\sin x+\cos x$ **for** $0\le x\le 2\pi$

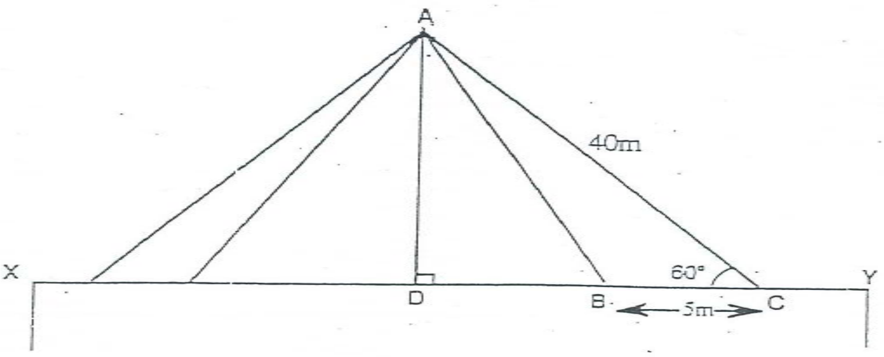
**d. The lines** $3x-y+2=0$ **and** $mx-y-1=0$ **intersect at** $45°$**. Find the possible value(s) of** $m$ **.**

$m_1 = 3,\ m_2 = m$

$$\tan\theta = \left|\frac{m-3}{1+3m}\right|$$

Question 4 13 marks *marks*

a. A horizontal bridge joins points $X$ and $Y$. Cables were used to support the bridge as shown in the diagram. The distance between the cables $AB$ and $AC$ was $5m$ while $AC$ was $40m$ and $\angle ACB = 60°$ *4*

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Question 5 13 marks *marks*

a. *2*

i. Expand $\cos(\alpha + \beta)$

ii. Show that $\cos 2\alpha = 1 - 2\sin^2 \alpha$

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c. $ABCD$ is a cyclic quadrilateral.
Show that $\tan A + \tan B + \tan C + \tan D = 0$ *2*

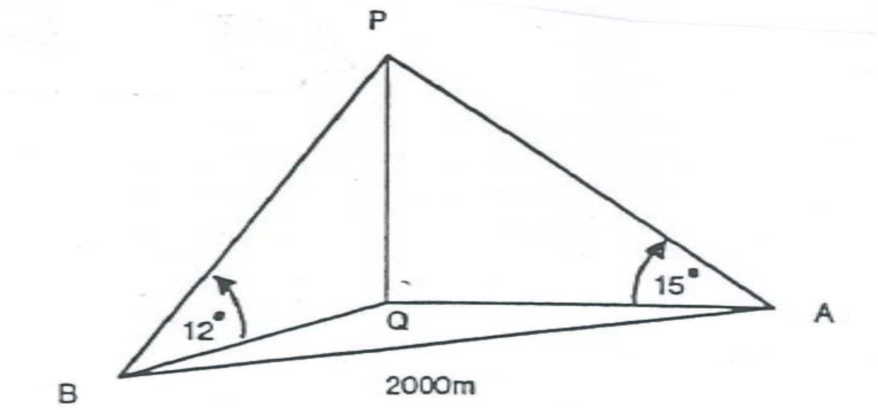
d.

i. Show that $\frac{\sin^3 \theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$ *3*

ii. Hence, or otherwise, solve $\frac{\sin^3 \theta}{\cos\theta} = 1 - \sin\theta\cos\theta$ for $0 \le \theta \le 360$

e.

*4*

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Put all this information into a diagram.

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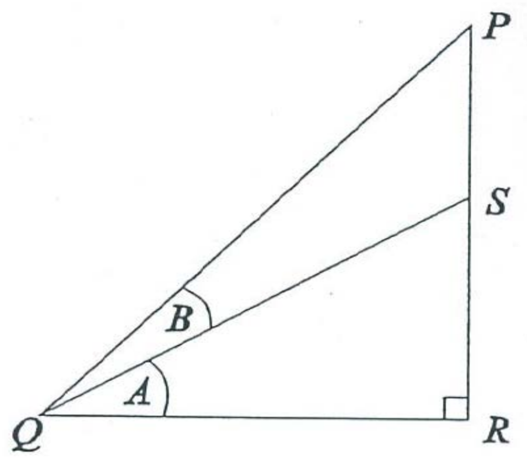
In the diagram $\angle A > \angle B > 0$ and $\dfrac{\cos(A+B)}{\cos(A-B)} = \dfrac{4}{5}$ *8*

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