Name: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2017

ASSESSMENT TASK 3 – PRELIMINARY

Mathematics Extension 1

Time allowed: 90 minutes
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
P4	chooses and applies appropriate arithmetic, algebraic, graphical and trigonometric techniques	2,3,4
P6 P7	relates the derivative of a function to the slope of its graph determines the derivative of a function through routine application of the rules of differentiation	1, 5
PE6	makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations	5

General Instructions:

- Questions 1-5 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-5, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Total Marks 65

Attempt Questions 1-5

Question	Out of	Marks
Q1	/13	
Q2	/13	
Q3	/13	
Q4	/13	
Q5	/13	
Total	/65	
	Percent:	

Question 1 (13 marks) Use a SEPARATE writing booklet.

a) P (8, 8) is a point on the parabola $x^2 = 8y$. Find the equation of the tangent at P. **2**

b) Differentiate the following functions.

i) $f(x) = \frac{x^3 - 3x^2 + 1}{x^2}$ **1**

ii) $f(x) = \left(x + \frac{1}{x}\right)^2$ **2**

iii) $f(x) = \frac{3x^2 - 1}{4x - 2}$ **2**

iv) $f(x) = 2x^3\,(3 + x^2)^2$ **2**

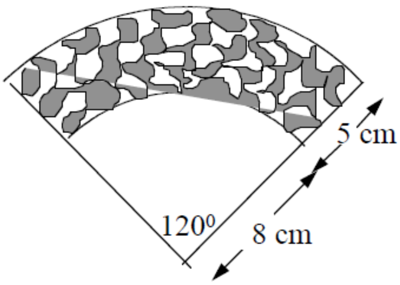
c) Find $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ **1**

d) Find from first principles the derivative of $y = x^2 + 3x$ **3**

Question 2 (13 marks) Use a SEPARATE writing booklet.

a) Convert 210^0 into radians and leave your answer in terms of π **1**

b) A fan in the shape of a sector of a circle is as follows



i) Find the perimeter of the shaded region, to the nearest cm? **2**

ii) What area of material is used in the shaded part of the fan, to the nearest cm^2? **2**

c) i) Show that $x = \frac{\pi}{3}$ is a solution of $sinx = \frac{1}{2}\, tanx$ **1**

ii) On the same set of axes, sketch the graphs of the function $y = sinx$ and $y = \frac{1}{2}tanx$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ **2**

iii) Using your graphs, or otherwise solve $sinx \leq \frac{1}{2}tanx$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ **2**

d) i) Show that $cosec\, 2\theta - cot2\theta = tan\theta$ **2**

ii) Hence find the exact value of $tan\frac{\pi}{8}$ **1**

Question 3 (13 marks) Use a SEPARATE writing booklet.

a) i) Express $cos\theta + \sqrt{3}\,\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ **2**

ii) Sketch $y = R\cos(\theta - \alpha)$, $0 \leq \theta \leq 2\pi$. Showing all essential features. **2**

iii) Solve the equation $cos\theta + \sqrt{3}\,\sin\theta = \sqrt{2}$ for $0 \leq x \leq 2\pi$. **2**

b) Find the general solution to $\cos 2\theta + cos\theta + 1 = 0$, leaving your answer in exact form in radians. **3**

c) Using the t results solve $1 + \cos x = 2\sin^2 x$ for $0 \leq x \leq 2\pi$. **4**

Question 4 (13 marks) Use a SEPARATE writing booklet.

a) Find the equation of the locus of a point P, which moves so it is always equidistant from the points A (3,5) and B (1,1). **3**

b) Let A and B be the points (4,1) and (-2,2) respectively.

Find the equation of the locus of a point P, which moves so PA is perpendicular to PB.

Describe the locus geometrically. **3**

c) For the parabola $y = \frac{1}{20}\,(x^2 - 8x - 4)$, find the **3**

i) Vertex

ii) Focus

d) i) Find the equation of the parabola that has directrix $y = -4$ and cuts the x −axis at $x = -2$ and $x = 6$. **3**

ii) Hence, find the coordinates of the focus. **1**

Question 5 (13 marks) Use a SEPARATE writing booklet.

a) Consider the function $f(x) = x^4 - 4x^3$.

i) Show that $f'(x) = 4x^2(x-3)$. **1**

ii) Find the coordinates of the stationary point(s) of the curve $y = f(x)$, and determine their nature. **3**

iii) Sketch the graph of the curve $y = f(x)$, showing the stationary point(s). **2**

iv) Find the values of x for which the graph of $y = f(x)$ is concave down. **2**

b) i) Show that for all values of m, the line $y = mx - 3m^2$ touches the parabola $x^2 = 12y$. **2**

ii) Find the values of m for which this line passes through the point (5, 2). **2**

iii) Hence determine the equation of the tangent to the parabola $x^2 = 12y$, from the point (5, 2) when the value of m is an integer. **1**

End of paper

Comments

Question 1

a) $x^2 = 8y$

$y = \frac{x^2}{8}$

$f'(x) = \frac{x}{4}$

$f'(8) = 2$ ✓

The equation of the tangent at $(8, 8)$

$y - y_1 = m(x - x_1)$

$y - 8 = 2(x - 8)$

$y = 2x - 8$ ✓

b)

i) $f(x) = \frac{x^3 - 3x^2 + 1}{x^2}$

$= x - 3 + x^{-2}$

$f'(x) = 1 - \frac{2}{x^2}$ ✓

ii $f(x) = \left(x + \frac{1}{x}\right)^2$

$f'(x) = 2\left(x + \frac{1}{x}\right)\left(1 - \frac{1}{x^2}\right)$ ✓ ✓

iii) $f(x) = \frac{3x^2 - 1}{4x - 1}$

let $u = 3x^2 - 1$

$u' = 6x$

$V = 4x - 1$

$V' = 4$

$f'(x) = \frac{Vu' - uV'}{V^2}$

$= \frac{(4x-1)(6x) - (3x^2-1)(4)}{(4x-1)^2}$ ✓

$= \frac{12x^2 - 6x + 4}{(4x-1)^2}$ ✓

iv $f(x) = 2x^3(3+x^2)^2$

let $u = (3+x^2)^2$

$u' = 2(3+x^2)(2x)$

$V = 2x^3$

$V' = 6x^2$

① correct derivatives of u and v

① correct answer using the product rule formula.

$f'(x) = uv' + vu'$

$= (3+x^2)^2(6x^2) + 2x^3(4x)(3+x^2)$

$= (3+x^2)[(3+x^2)(6x^2) + 8x^3]$

$= (3+x^2)(6x^4 + 8x^3 + 18x^2)$

c $\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$= 4$ ✓

d) $f(x) = x^2 + 3x$

$f(x+h) = (x+h)^2 + 3(x+h)$

$= x^2 + 2xh + h^2 + 3x + 3h$ ✓

$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2xh + 3h + h^2}{h}$ ✓

$= \lim_{h \to 0} \frac{\cancel{h}(2x+3+h)}{\cancel{h}}$

$= 2x + 3$ ✓

②

Question 2

a) $210^\circ \times \frac{\pi}{180} = \frac{7\pi}{6}$ ✓

b i) $P = 10 + r\theta + R\theta$ $\left(120^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}\right)$

$= 10 + \left(\frac{2\pi}{3} \times 8 + \frac{2\pi}{3} \times 13\right)$

$= 54$ cm ✓

ii) $A = \frac{1}{2}\theta(R^2 - r^2)$

$= \frac{\pi}{3}(105)$ ✓

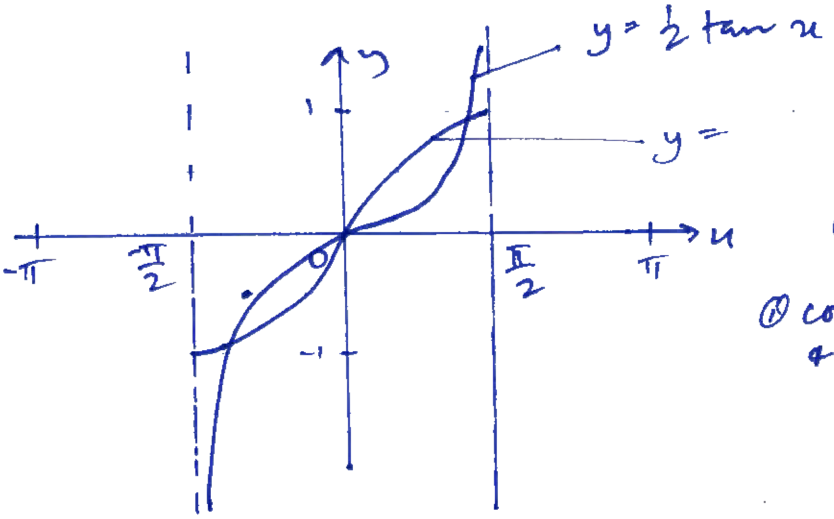
$= 110 \text{ cm}^2$ ✓

c i) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\frac{1}{2}\tan\frac{\pi}{3} = \frac{1}{2} \times \sqrt{3}$

$= \frac{\sqrt{3}}{2}$ ✓

$\therefore$ LHS = RHS

ii)



① correct domain

① correct shapes & range

iii) $-\frac{\pi}{6} \le x \le 0$ ✓

$\frac{\pi}{6} \le x < \frac{\pi}{2}$ ✓

S	A
T	C

OR $\sin x \le \frac{1}{2}\tan x$

$\cos x \le \frac{1}{2}$

$x \le \frac{\pi}{6}$

$\therefore -\frac{\pi}{6} \le x \le 0$

3

e) i) $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$

$$LHS = \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$ ✓ ① ratio ① working

$$= \tan\theta$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

ii) $\tan\frac{\pi}{8} = \operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}$

$$= \sqrt{2} - 1$$ ✓

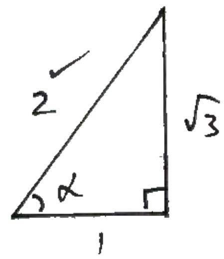
4

Question 3

a i) $\cos\theta + \sqrt{3}\sin\theta = R\cos(\theta - \alpha)$

$\tan\alpha = \sqrt{3}$

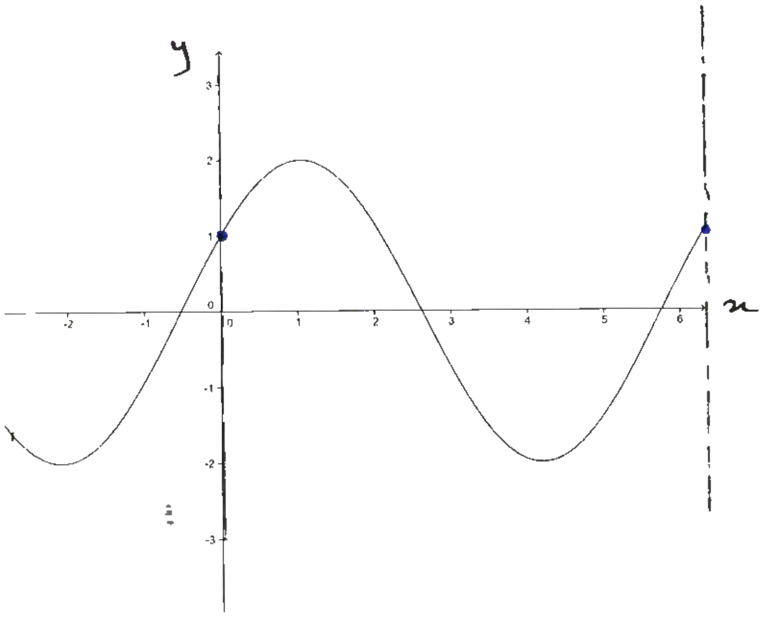
$\alpha = \frac{\pi}{3}$ ✓



$\therefore R\cos(\theta - \alpha) = 2\cos\left(\theta - \frac{\pi}{3}\right)$

ii) sketch:

$0 \leq \theta \leq 2\pi$



iii) $2\cos\left(\theta - \frac{\pi}{3}\right) = \sqrt{2}$

$\cos\left(\theta - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

$\theta - \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}$ ✓

$\theta = \frac{7\pi}{12}, \frac{25\pi}{12}$ ✓

OR $\theta = \pm\frac{\pi}{4} + \frac{\pi}{3}$

$= \frac{7\pi}{12}, \frac{\pi}{12}$

5

3 b $\cos 2\theta + \cos\theta + 1 = 0$

$2\cos^2\theta - 1 + \cos\theta + 1 = 0$

$2\cos^2\theta + \cos\theta = 0$

$\cos\theta\,(2\cos\theta + 1) = 0$

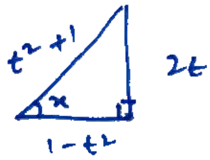
$\cos\theta = 0$ $\qquad\qquad$ $\cos\theta = -\frac{1}{2}$ ✓ for values

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $\qquad\qquad$ $\theta = \frac{2\pi}{3}$

$\theta = 2\pi n \pm \frac{\pi}{2}$ ✓ $\qquad$, $\theta = 2\pi n \pm \frac{2\pi}{3}$ ✓

where n is an integer

c) $2\sin^2 x - \cos x - 1 = 0$



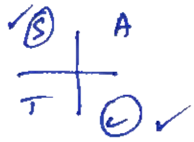
$2\left(\frac{2t}{t^2+1}\right)^2 - \left(\frac{1-t^2}{t^2+1}\right)^2 - 1 = 0$ ✓

$8t^2 - [(1-t^2)(1+t^2)] - (t^2+1)^2 = 0$

$8t^2 - (1-t^4) - (t^4+2t^2+1) = 0$

$6t^2 - 2 = 0$

$t = \pm\frac{1}{\sqrt{3}}$ ✓



$\tan x = \frac{1}{\sqrt{3}}$ $\qquad\qquad$ $\tan x = -\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$ ✓ $\qquad$ $x = \left(\pi - \frac{\pi}{6}\right), \left(2\pi - \frac{\pi}{6}\right)$

$= \frac{5\pi}{6}, \frac{11\pi}{6}$ ✓

OR $\qquad$ $= -\frac{\pi}{6}, -\frac{7\pi}{6}$

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2017 Extension 1, Preliminary Task3 Solutions

Question 4

a) $PA^2 = PB^2$

$(x-3)^2 + (y-5)^2 = (x-1)^2 + (y-1)^2$ ✓

$x^2 - 6x + 9 + y^2 - 10y + 25$

$= x^2 - 2x + 1 + y^2 - 2y + 1$ ✓

$-4x - 8y + 32 = 0$

$x + 2y + 8 = 0$ ✓

$\therefore$ The locus of point P is a straight line

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b) $m_{PA} = \dfrac{y-1}{x-4}$, $m_{PB} = \dfrac{y-2}{x+2}$

for both gradients.

Since $PA \perp PB$

$m_{PA} \times m_{PB} = -1$

$\dfrac{y-1}{x-4} = -\dfrac{(x+2)}{y-2}$ ✓

$(y-1)(y-2) = -(x+2)(x-4)$

$y^2 - 3y + 2 = -(x^2 - 2x - 8)$

$y^2 - 3y + 2 = -x^2 + 2x + 8$

$x^2 - 2x + y^2 - 3y - 6 = 0$ ✓

$x^2 - 2x + (-1)^2 + y - 3y \quad = 6 + 1 + \frac{9}{4}$

$(x-1)^2 + (y-\frac{3}{2})^2 = \left(\frac{\sqrt{37}}{2}\right)^2$

The locus is a circle with centre $(1, \frac{3}{2})$ and radius is $\frac{\sqrt{37}}{2}$ ✓

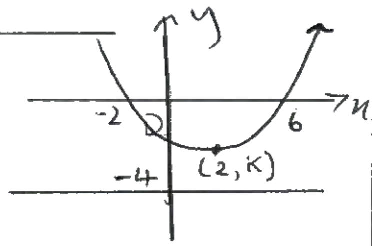
c) $y = \frac{1}{20}(x^2 - 8x - 4)$

$$x^2 - 8x - 4 = 20y$$
$$x^2 - 8x + (-4)^2 = 20y + 20$$

$$(x-4)^2 = 20(y+1)$$ ✓
$$4a = 20$$
$$a = 5$$

i) Vertex $(4, -1)$ ✓

ii) Focus $(4, 4)$ ✓



d) $(x-2)^2 = 4a(y-K)$
when $x = 6$, $y = 0$
$$16 = -4aK \quad — ①$$

$$K - a = -4$$
$$K = a - 4 \quad — ②$$

Sub ② into ①
$$16 = -4a(a-4)$$
$$16 = -4a^2 + 16a$$
$$4a^2 - 16a + 16 = 0$$
$$a^2 - 4a + 4 = 0$$
$$(a-2)^2 = 0$$
$$a = 2$$ ✓

from ②
$$K = 2 - 4$$
$$= -2$$ ✓

i) The equation of the Parabola:
$$\therefore (x-2)^2 = 8(y+2)$$ ✓

ii) vertex : $(2, -2)$
Focus : $(2, 0)$ ✓

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Question 5

a) $f(x) = x^4 - 4x^3$

i) $f'(x) = 4x^3 - 12x^2$ ✓
$= 4x^2(x-3)$

ii) for stationary points : $\frac{dy}{dx} = 0$

$4x^2(x-3) = 0$
$x = 0, 3$ ✓

to determine their nature :

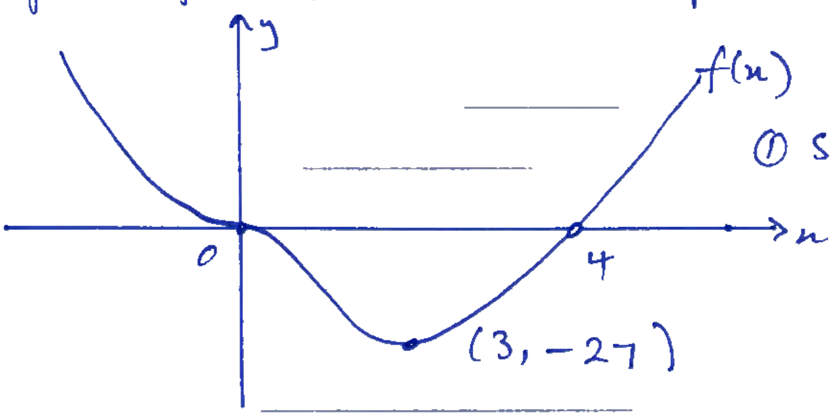
$f''(x) = 12x^2 - 24x$
$f''(0) = 0$ (a possible p.o.I)
$f''(3) = 36$
since $f''(3) > 0$, $\therefore$ min when $x = 3$ ✓

x	-1	0	1	3	4
$f'(x)$	-16	0	-8	0	64
	$-$	0	$-$	0	$+$

* the table of values must include the $f'(x)$ values.
+ and − signs are not accepted in the HSC.

$\therefore$ from $f'(x)$, $x = 0$ is a p.o.I ✓

iii)



① Shape + x intercepts
① minimum point.

iv) $f(x)$ is concave downward where $f''(x) < 0$

$0 \le x \le 2$ ✓

$12x^2 - 24x < 0$ ✓

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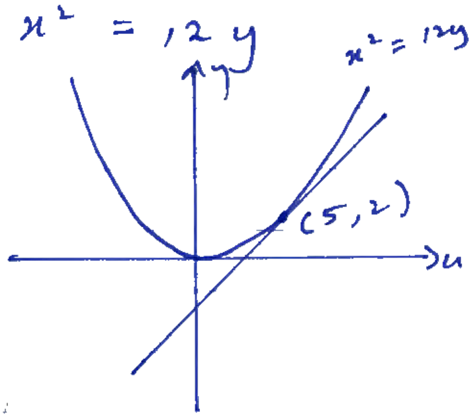
b) $y = mx - 3m^2$, $x^2 = 12y$



i) $x^2 = 12(mx - 3m^2)$

$x^2 = 12mx - 36m^2$

for y to be a tangent

$\Delta = 0$

$b^2 - 4ac = 0$

$\Delta = (12m)^2 - 4(1)(36m^2)$

$= 144m^2 - 144m^2$

$= 0$ ✓

ii) when passes through (5, 2)

$y = mx - 3m^2$

$2 = 5m - 3m^2$ ✓

$3m \quad 1$
$m \quad -2$

$3m^2 - 5m + 2 = 0$

$(3m+1)(m-2) = 0$

$\therefore m = 2, -\frac{1}{3}$ ✓

iii) The equation of the tangent is

$y = mx - 3m^2$ (for $m = 2$)

$y = 2x - 12$ ✓

END