Name: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2018

ASSESSMENT TASK 3 – PRELIMINARY

Mathematics Extension 1

Time allowed: 90 minutes
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
P4	chooses and applies appropriate arithmetic, algebraic, graphical and trigonometric techniques	1, 3, 5
P6 PE5 PE6	Understands the concept of a function and the relationship between a function and its graph, determines the derivatives which require the application of more than one rule of differentiation, relates the derivative of a function to the slope of its graph	4, 2, 5
PE6	makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations	5

General Instructions:

- Questions 1-5 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-5, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Total Marks 58

Attempt Questions 1-5

Question	Out of	Marks
Q1	/10	
Q2	/12	
Q3	/11	
Q4	/12	
Q5	/13	
Total	/58	
	Percent:	

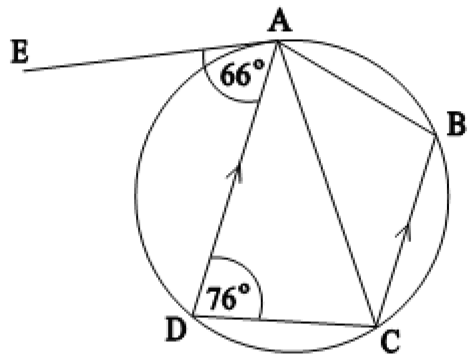
Question 1 (10 marks) Use a SEPARATE writing booklet.

a) Given that $R(x, y)$ divides the interval joining $P(-\ 4, 5)$ and $Q\,(-\ 1, 9)$ externally in the ratio $3\!:\!5$, find the coordinates of R. 2

b) i) Sketch $y = |x| - x$ 2

ii) Hence or otherwise, solve $|x| - x > 2$ 1

c) In the circle shown in the diagram, $ABCD$ is a cyclic quadrilateral in which AD is parallel to BC and $\angle ADC\ = 76^0$. AE is tangent to the circle such that $\angle EAD = 66^0$.
Copy diagram into your solution booklet.



i) Find the value of $\angle DAC$. Give reasons. 2

ii) Show that the triangle ABC is isosceles. 3

End of question 1

Question 2 (12 marks) Use a SEPARATE writing booklet.

a) i) Find the equation of the locus of a point $P(x, y)$ moving so that PA is perpendicular to PB, where $A = (3,4)$ and $B = (-1,2)$. 2

ii) Show that the locus is a circle and state its centre and radius. 3

b) i) State the focal length and the equation of the directrix of a parabola with focus $(3,1)$ and vertex $(5,1)$. 2

ii) State the equation of the parabola. 1

iii) Find the coordinates of the endpoints of the latus rectum. 2

c) The point $P(k,6)$ lies on the parabola $x^2 = 16y$. Find the distance from P to the focus of the parabola. ***Show working out/ reasoning*** 2

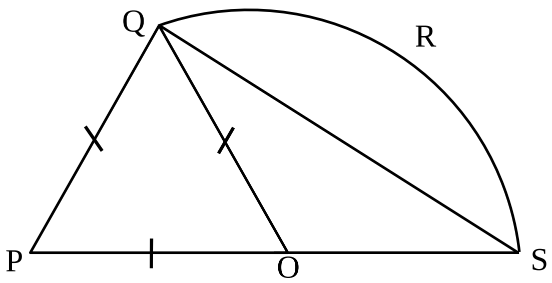
End of question 2

Question 3 (11 marks) Use a SEPARATE writing booklet.

a) Find the exact value of 300^0 in radians. 1

b) Solve $2\sin 2\alpha - 1 = 0$ for $0 \le \alpha \le 2\pi$. 3

c) OPQ is an equilateral triangle of side $12cm$. PS is a straight line. QRS is an arc of a circle with centre O.



Giving your answer in exact form find:

i. the perimeter of the region $PQRSOP$ 3

ii. the area of the segment SQR (i.e. the segment bounded by the chord QS and the arc QRS) 1

d) Express $2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right)$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Show all working out. 3

End of question 3

Question 4 (12 marks) Use a SEPARATE writing booklet.

a) Differentiate the following:

i) $y = 4x^2 - 5x$ 1

ii) $y = \dfrac{1}{2x-4}$ 1

b) Consider the function $f(x) = \dfrac{2x^2}{x^2+1}$

i) Find the equation of any horizontal asymptotes. **Show all working out** 2

ii) Using the first derivative, find the coordinates of any turning point(s) and determine their nature. 4

iii) List all the steps for identifying the x coordinate of any points of inflexion. **Do not perform any calculations.** 1

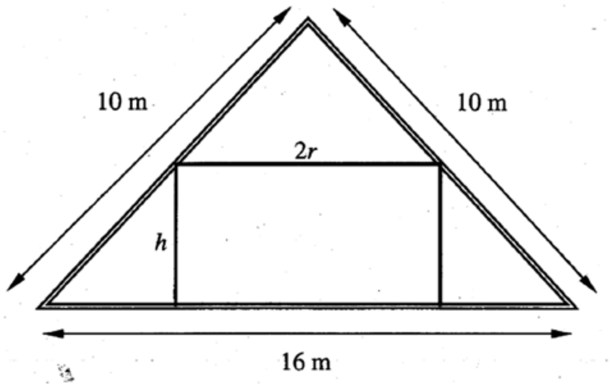
iv) Sketch the graph of this function, showing all intercepts and asymptotes. 3

End of question 4

Question 5 (13 marks) Use a SEPARATE writing booklet.

a) Draw a sketch of a function that has $f'(x) > 0$ for $x < -2$ and $x > 2$, $f'(x) = 0$ at $x = -2$ and at $x = 2$, and $f'(x) < 0$ for $-2 < x < 2$ 2

b) In some rural areas, hot water tanks are installed in the roofs of large homesteads. The diagram below shows the cross-section of a cylindrical tank in such a homestead's roof. The cylindrical tank, with diameter $2r$ metres and height h metres, fits exactly into the roof. The cross-section of the roof is an isosceles triangle with dimensions shown.



i) Show that the height of the roof is 6 metres. 1

ii) Hence, show that $h = 6 - \frac{3}{4}r$ 2

iii) Show that the volume of the cylindrical tank can be expressed by 1

$$V = 3\pi r^2\left(2 - \frac{r}{4}\right)$$

iv) Find the value of r which gives the hot water tank its greatest volume. 3

c) Find all the values of θ for which $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$ 4

End of Examination

2018

HIGHER SCHOOL CERTIFICATE PRELIMINARY COURSE

ASSESSMENT TASK 3

Mathematics Extension 1

Solutions

Question 1 (10 marks)

a) Given that $R(x, y)$ divides the interval joining $P(-4, 5)$ and $Q(-1, 9)$ externally in the ratio $3:5$, find the coordinates of R. 2

Solution :

Divided externally $\Rightarrow$ the ratio is $3:-5$

$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{-5(-4)+3(-1)}{-5+3} \qquad = \frac{-5(5)+3(9)}{-5+3}$$

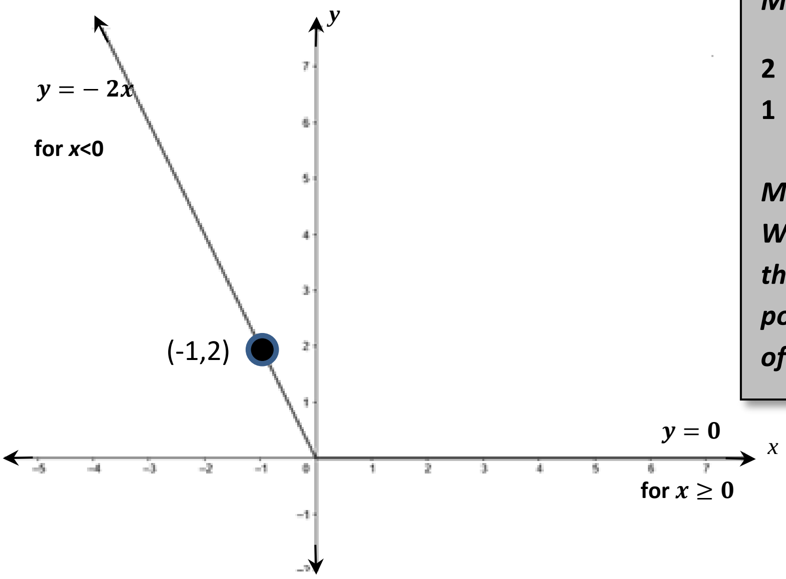
$$= -\frac{17}{2} \qquad = -1$$

$$\therefore R\left(-\frac{17}{2}, -1\right)$$

> ***Marking guidelines***
>
> **2** **Both x and y are correct**
> **1** **1 of either x or y correct**
>
> ***Marker's comments***
> ***Well done. A few students made arithmetic errors.***

b) i) Sketch $y = |x| - x$ 2

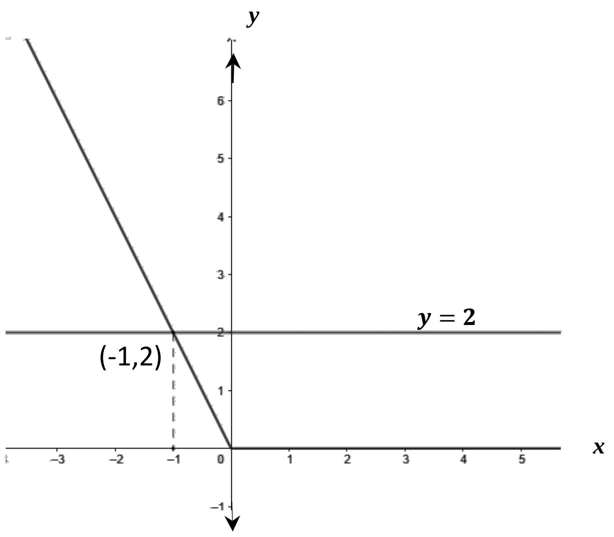
Solution :



> ***Marking guidelines***
>
> **2** **Correct sketch**
> **1** **Either ray correctly drawn**
>
> ***Marker's comments***
> ***Well done. Some students left out scales on the axes or didn't give the co-ordinates of a point on the ray y=-2x for x<0. At least one of these should be shown.***

ii) Hence or otherwise, solve $|x| - x > 2$ 1

Solution : $\boldsymbol{x < -1}$

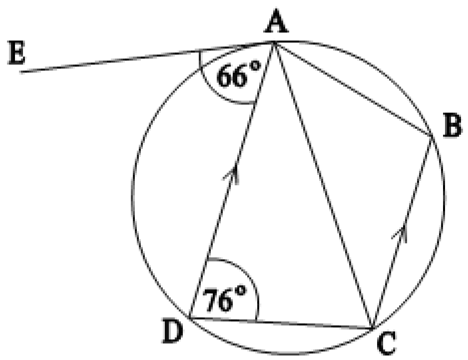


Marking guidelines

1 Correct solution

Marker's comments
Well done.

c) In the circle shown in the diagram, $ABCD$ is a cyclic quadrilateral in which AD is parallel to BC and $\angle ADC = 76^0$. AE is tangent to the circle such that $\angle EAD = 66^0$.
Copy diagram into your solution booklet.



i) Find the value of $\angle DAC$. Give reasons. 2

Solution :

$\angle ACD = 66^0$ (angle between tangent and chord is equal to angle in the alternate segment)

$\angle DAC + 76^0 + 66^0 = 180^0$ (angle sum of a triangle is 180^0)

Hence, $\angle DAC = 38^0$

Marking guidelines

2 Correct value of $\angle ACD$ with correctly stated reasons.
1 Either $\angle ACD$ or $\angle DAC$ correct

Marker's comments
Many students wrote a poorly worded reason for the "The angle between the tangent and chord, at point of contact, equals the angle in the alternate segment.

ii) Show that the triangle ABC is isosceles. 3

Solution :

$\angle ACB = 38^0$ (alternate angles on parallel lines)

$\angle ABC = 180^0 - 76^0 = 104^0$ (opposite angles of cyclic quadrilateral are supplementary)

$\therefore \angle CAB + 104^0 + 38^0 = 180^0$ (angle sum of triangle ABC is 180^0)

$\therefore \angle CAB = 38^0$

$\therefore \angle CAB = \angle ACB = 38^0$

$\therefore \triangle ABC$ is isosceles (two angles equal)

Marking guidelines

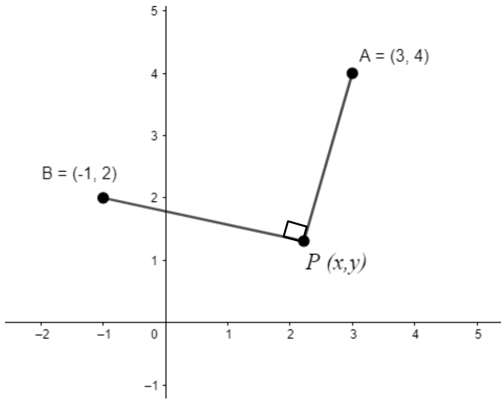
3 Correct proof
2 Correctly finding 2 angles with correctly stated reasons.
1 Correctly finding an angle with a correctly stated reason.

Marker's comments
Well done

Question 2 (12 marks)

a) i) Find the equation of the locus of a point $P(x, y)$ moving so that PA is perpendicular to PB, where $A = (3,4)$ and $B = (-1,2)$. 2

Solution :



$$\text{Grad. } PA = -\frac{1}{\text{Grad. } PB}$$

$$m_{PA} = \frac{y-4}{x-3} \qquad m_{PB} = \frac{y-2}{x+1}$$

Hence,

$$\frac{y-4}{x-3} = -\left(\frac{x+1}{y-2}\right)$$

$$(y-4)(y-2) = -(x+1)(x-3)$$

$$y^2 - 6y + 8 = -\left(x^2 - 2x - 3\right)$$

$$\therefore x^2 - 2x + y^2 - 6y + 5 = 0$$

Marking guidelines

2 Correct solution

1 $\dfrac{y-4}{x-3} = -\left(\dfrac{x+1}{y-2}\right)$

Marker's comments

- ***Marks awarded for alternate methods – using of Pythagoras Theorem or Circle Geometry Theorems***
- ***Some students recognised it was a circle with diameter AB and found the midpoint to determine the centre of the circle.***
- ***A few students incorrectly used the distance formula or equation of a line***
- ***A few students were unable to correctly recall and apply the gradient formula***

ii) Show that the locus is a circle and state its centre and radius. 3

Solution :

$$x^2 - 2x + y^2 - 6y + 5 = 0$$

$$x^2 - 2x + \left(\frac{-2}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 = -5 + \left(\frac{-2}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$$

$$(x-1)^2 + (y-3)^2 = 5$$

Therefore it is a circle since it's in the form $(x-h)^2 + (y-k)^2 = r^2$. Centre $(1,3)$ and radius $\sqrt{5}$.

Marking guidelines

3 Correct solution

2 Completing square and writing in the form $(x-1)^2 + (y-3)^2 = 5$

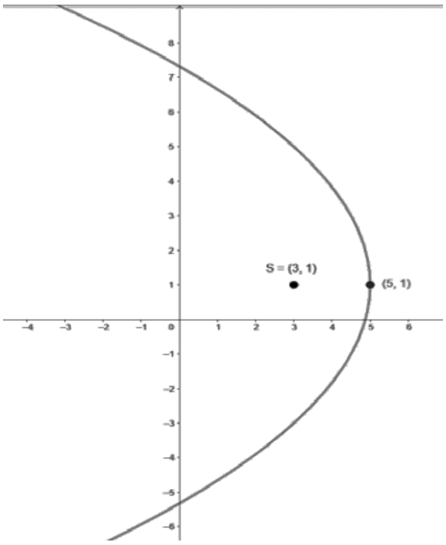
1 Completing square, correct radius or correct centre

Marker's comments

- ***A few students did not attempt the question***
- ***This question uncovered weak algebraic skills some students were unable to complete the square correctly.***

b) i) State the focal length and the equation of the directrix of a parabola with focus $(3,1)$ and vertex $(5,1)$. 2

Solution :



Focal Length: $a = 5-3$

$= 2$

Equation of directrix: $x = 5+2$

$x = 7$

Marking guidelines

2 Correct focal length and equation of directrix

1 Correct focal length or equation of directrix

Marker's comments

- ***Some students did not answer the question fully and left out either the focal length or directrix equation***
- ***Some had incorrect orientation of the parabola***
- ***A few incorrectly gave the equation of the directrix as a point***
- ***Students who used the a diagram tended to arrive at the correct answer***

ii) State the equation of the parabola. 1

Solution :

Parabola is in the form $(y-k)^2 = -4a(x-h)$

Hence equation is $(y-1)^2 = -8(x-5)$

Marking guidelines

1 Correct solution

Marker's comments

- ***Not answered very well***
- ***Many students didn't realise it was a sideways parabola which led to problems when attempting to answer part (iii)***
- ***A few swapped the x and y coordinates in the equation***
- ***Those that used the correct coordinates for the vertex forgot the negative in front of the 8.***

iii) Find the coordinates of the endpoints of the latus rectum. 2

Solution :

Latus rectum passes through the focus.

Hence end points are $(3, y_1)$ and $(3, y_2)$

When $x = 3$

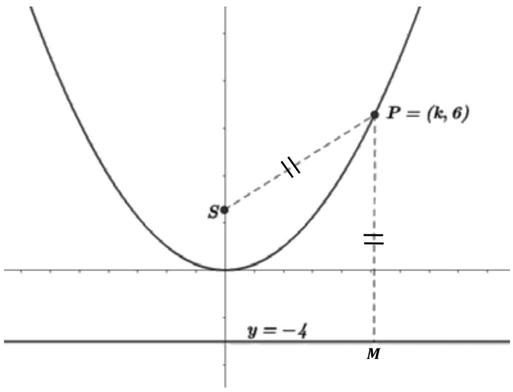
$$(y-1)^2 = -8(3-5)$$
$$(y-1)^2 = 16$$
$$y-1 = \pm 4$$
$$\therefore y = 5 \text{ or } y = -3$$

$\therefore$ Coordinates of endpoints are $(3,5)$ and $(3,-3)$

> *Marking guidelines*
>
> **2** **Correct solution**
>
> **1** $\boldsymbol{x = 3}$
>
> *Marker's comments*
>
> - ***Not answered very well – it was apparent that many students didn't fully understand the concept***
> - ***Weak algebraic skills were apparent with few students able to solve the quadratic once $x = 3$ was substituted.***
> - ***Because of the error in part (ii) some students were trying to find the square root of a negative number***

c) The point $P(k,6)$ lies on the parabola $x^2 = 16y$. Find the distance from P to the focus of the parabola. ***Show working out/ reasoning*** 2

Solution :



> *Marking guidelines*
>
> **2** **Correct solution**
>
> **1** $\boldsymbol{a = 4}$ **or equation of directrix** $\boldsymbol{y = -4}$
>
> *Marker's comments*
>
> - ***Not answered very well***
> - ***Almost all students did not realise they could use the geometric definition of the parabola (distance from a point to the directrix is equal to the distance from the point to the focus). However, marks were awarded if the correct answer was achieved using the distance formula.***
> - ***When labelling coordinates of the focus, a few were confusing x and y***

$x^2 = 16y$, hence focal length, $a = 4$.

Therefore, equation of directrix is $y = -4$.

Perpendicular distance from $P(k, 6)$ to the directrix

$$= 6 + 4$$
$$= 10$$

Since distance from any point $P(x, y)$ on a parabola to the directrix is equal to distance from $P(x, y)$ to the focus then distance from $P(k, 6)$ to the focus $= 10$

Question 3 (11 marks)

a) Find the exact value of 300^0 in radians. 1

Solution :

$$1^0 = \frac{\pi}{180}$$
$$300^0 = \frac{300\pi}{180}$$
$$= \frac{5\pi}{3}\text{radians}$$

Marking guidelines

1 Correct solution

Marker's comments

Generally well done

b) Solve $2\sin 2\alpha - 1 = 0$ for $0 \le \alpha \le 2\pi$. 3

Solution :

$2\sin 2\alpha - 1 = 0 \qquad 0 \le 2\alpha \le 4\pi$

$\sin 2\alpha = \frac{1}{2} \qquad$ related angle $= \frac{\pi}{6}$

Hence,

$$2\alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$
$$\therefore \alpha = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

Marking guidelines

3 Correct solution

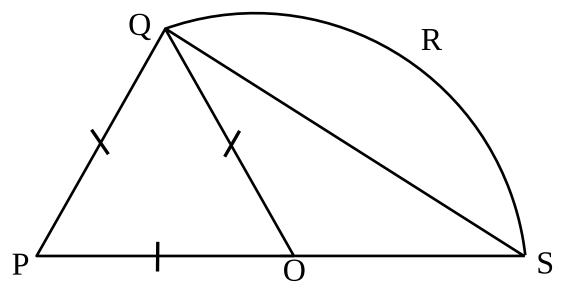
2 Related angle and 2 correct solutions

1 Related angle or change of domain

Marker's comments

Many students didn't explicitly state the change of domain. Most of those students only gave 2 out of 4 correct solutions.

c) OPQ is an equilateral triangle of side $12cm$. PS is a straight line. QRS is an arc of a circle with centre O.



Giving your answer in exact form find:

i. the perimeter of the region $PQRSOP$ 3

Solution :

$$\angle QOP = \frac{\pi}{3}$$

$$\angle QOS = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Length of arc QRS $= r\theta$

$$= 12 \times \frac{2\pi}{3}$$

$$= 8\pi$$

Perimeter

$$P = 3 \times 12 + 8\pi$$

$$\therefore P = 36 + 8\pi \ cm$$

Marking guidelines

3 Correct solution

2 $\angle QOS$ and length of $arcQRS$

1 $\angle QOS$ or length of $arcQRS$

Marker's comments

Mostly well done, but quite a few students didn't leave in exact form.

ii. the area of the segment SQR 1

Solution :

$$A = \frac{1}{2} r^2 \left(\theta - \sin\theta\right)$$

$$= \frac{1}{2} \times 12^2 \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$$

$$= 72 \times \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

$$\therefore A = 12\left(4\pi - 3\sqrt{3}\right) cm^2$$

Marking guidelines

1 Correct solution

Marker's comments

Many students didn't refer to the reference sheet which could have saved some time.

Quite a few students didn't leave in exact form.

d) Express $2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right)$ in the form $\text{R}\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. 3

Solution :

$$2\cos\theta + 2\left(\cos\theta + \frac{\pi}{3}\right)$$

$$= 2\cos\theta + 2\left(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right)$$

$$= 2\cos\theta + 2\left(\frac{\cos\theta}{2} - \frac{\sqrt{3}\sin\theta}{2}\right)$$

$$= 2\cos\theta + \cos\theta - \sqrt{3}\sin\theta$$

$$= 3\cos\theta - \sqrt{3}\sin\theta$$

Hence,

$$3\cos\theta - \sqrt{3}\sin\theta = R\cos(\theta + \alpha)$$

$$3\cos\theta - \sqrt{3}\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$$R\cos\alpha = 3 \ \ldots\ldots \ (1)$$

$$R\sin\alpha = \sqrt{3} \ \ldots\ldots (2)$$

$(1)^2 + (2)^2$

$$R^2(\cos^2\alpha + \sin^2\alpha) = 9 + 3$$

$$\therefore R = 2\sqrt{3}$$

$$2\sqrt{3}\cos\alpha = 3 \qquad 2\sqrt{3}\sin\alpha = \sqrt{3}$$

$$\cos\alpha = \frac{\sqrt{3}}{2} \qquad \sin\alpha = \frac{1}{2}$$

$$\therefore \alpha = \frac{\pi}{6}$$

$$\therefore 2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right) = 2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right)$$

Marking guidelines

3	**Reduction to $3\cos\theta - \sqrt{3}\sin\theta$, correct R and correct α**
2	**1 mark deducted for 1 error in reduction, R or α**
1	**2 marks deducted for 2 errors**

Marker's comments

Many students failed to change the expression to the form $a\cos\theta - b\sin\theta$ before trying to determine R and α

Those students who successfully simplified the expression were mostly able to complete a full solution.

Question 4 (12 marks)

a) Differentiate the following: 1

i) $y = 4x^2 - 5x$

Solution :

$$\frac{dy}{dx} = 8x - 5$$

> ***Marking guidelines***
>
> **1 Correct answer**
>
> ***Marker's comments***
> ***Well done.***

ii) $y = \dfrac{1}{2x-4}$ 1

Solution :

$$y = (2x-4)^{-1}$$

$$\frac{dy}{dx} = -1 \times 2(2x-4)^{-2}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{(2x-4)^2}$$

> ***Marking guidelines***
>
> **1 Correct answer**
>
> ***Marker's comments***
> ***Generally well done but there were some errors with indices.***

b) Consider the function $f(x) = \dfrac{2x^2}{x^2+1}$

i) Find the equation of any horizontal asymptotes. **Show all working out** 2

Solution :

$$\lim_{x\to\infty} \frac{2x^2}{x^2+1} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{2}{1+\frac{1}{x^2}}, \quad \lim_{x\to\infty} \frac{1}{x^2} = 0$$

Hence,

$$\lim_{x\to\infty} \frac{2}{1+\frac{1}{x^2}} \Rightarrow \frac{2}{1+0}$$

$\therefore$ Horizontal asymptote is $y = 2$

> ***Marking guidelines***
>
> **2 Division by x^2, $\lim_{x\to\infty} \frac{1}{x^2} = 0$ and $y = 2$**
>
> **1 Division by x^2 or $\lim_{x\to\infty} \frac{1}{x^2} = 0$ or $y = 2$**
>
> ***Marker's comments***
> ***Students had a reasonable idea of concept but couldn't set it out properly.***
> ***Some students did not answer the question!***
> ***i.e. concluding statement.***
> ***HA and VA are NOT acceptable abbreviations.***
> ***Note: cannot divide by ∞***

ii) Using the first derivative, find the coordinates of any turning point(s) and determine their nature. 4

Solution :

$$f(x) = \frac{2x^2}{x^2+1}$$

$$f'(x) = \frac{\mathrm{vu}' - \mathrm{uv}'}{\mathrm{v}^2} \qquad \mathrm{u} = 2x^2 \qquad \mathrm{v} = x^2+1$$

$$\mathrm{u}' = 4x \qquad \mathrm{v}' = 2x$$

$$f'(x) = \frac{4x\left(x^2+1\right) - 2x^2(2x)}{(x^2+1)^2}$$

$$\therefore f'(x) = \frac{4x}{(x^2+1)^2}$$

Stationary Point at $f'(x) = 0$

$$\frac{4x}{(x^2+1)^2} = 0$$

$$4x = 0$$

$$x = 0$$

$\therefore$ Coordinates of S.P $= (0,0)$

x	-1	0	1
$f'(x)$	-1	0	1

Therefore, there is a minimum turning point at $(0,0)$

Marking guidelines

4 **1st derivative using quotient rule**
S.P. at $(0,0)$
Sign table
Minimum/ concave up at $(0,0)$

3 **1 error**

2 **2 errors**

1 **3 errors**

Marker's comments

Some students confused first derivative with fist principles which wasted time and often led to errors.

Students MUST state for stationary points $f'(x) = 0$

Many students ignored the instruction and used the second derivative to determine the nature.

iii) List all the steps for identifying the x coordinate of any points of inflexion. **Do not perform any calculations.** 1

Solution :

1. Find second derivative
2. Set second derivative to 0 and solve for x
3. Setup sign table, find values of $f''(x)$ on either side of x from step 2
4. If there is a change in sign of $f''(x)$ on either side of x then there is a change in concavity and a point of inflexion exists.

Marking guidelines

1 **ALL steps listed**

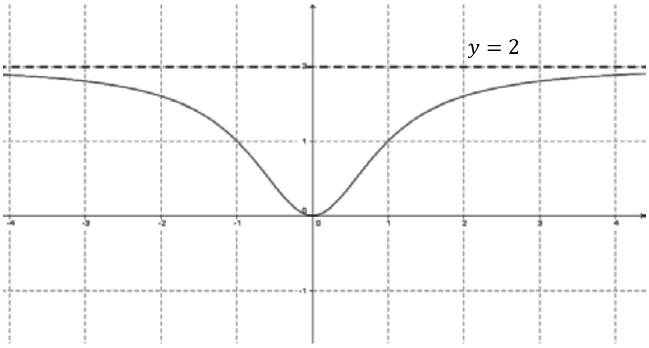
Marker's comments

Poorly done. Some students ignored instructions and performed calculations- no marks.

There seems to be a lot of confusion between inflexion points and a horizontal point of inflexion!

iv) Sketch the graph of this function, showing all intercepts and asymptotes. 3

Solution :



Marking guidelines

3 Correct curve to asymptote relationship.
Turning point at (0,0)
Shape/symmetry of curve

2 1 feature of curve incorrect

1 2 features of curve incorrect

Marker's comments

A sketch requires a ruler to be used, axes to be labelled with reasonable relative scale and should be a suitable size.

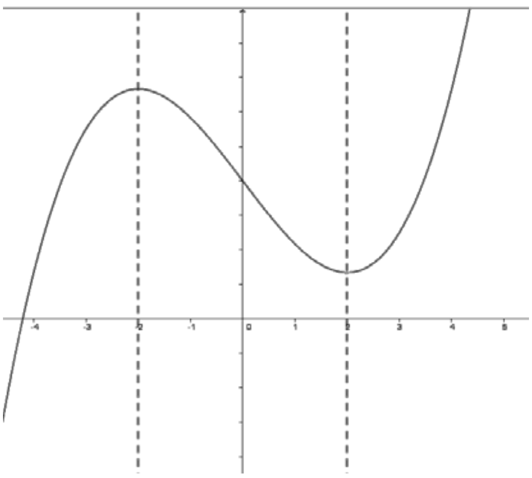
Many had a 'V' instead of a minimum turning point.

More care required with shape and the curve as it approaches the asymptote.

Question 5 (13 marks)

a) Draw a sketch of a function that has $f'(x) > 0$ for $x < -2$ and $x > 2$, $f'(x) = 0$ at $x = -2$ and at $x = 2$, and $f'(x) < 0$ for $-2 < x < 2$ 2

Solution :



Turning points at $x = 2$ and $x = -2$

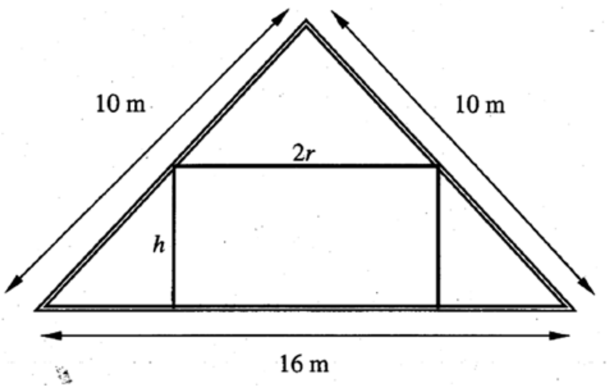
Marking guidelines

2	**Correct shape and turning points at $x = 2$ and $x = -2$**
1	**Correct shape or turning points**

Marker's comments

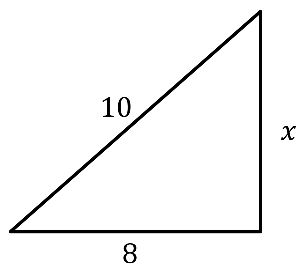
Students should label their axes and graph.

b) In some rural areas, hot water tanks are installed in the roofs of large homesteads. The diagram below shows the cross-section of a cylindrical tank in such a homestead's roof. The cylindrical tank, with diameter $2r$ metres and height h metres, fits exactly into the roof. The cross-section of the roof is an isosceles triangle with dimensions shown.



i) Show that the height of the roof is 6 metres. 1

Solution :



$$x^2 = 10^2 - 8^2$$
$$= 36$$
$$\therefore x = 6m \text{ as req'd}$$

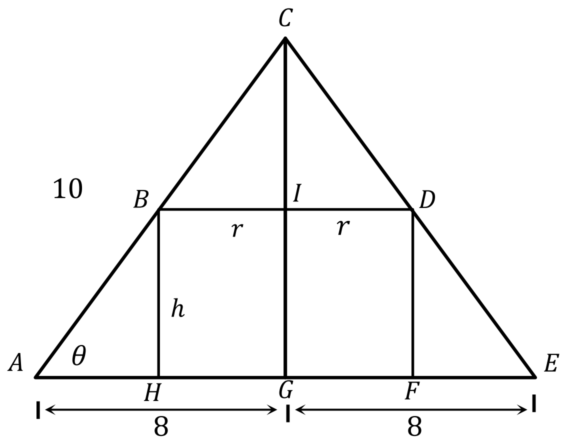
Marking guidelines

1 Correct answer

Marker's comments

Generally well done.

ii) Hence, show that $h = 6 - \dfrac{3}{4}r$ 2



Let $\angle CAE = \theta$

In ΔACG $\tan\theta = \dfrac{6}{8} = \dfrac{3}{4}$

$\Delta ACG \,|||\, \Delta ABH$ hence,

$\tan\theta = \dfrac{h}{8-r}$ in ΔABH

Therefore,

$$\frac{h}{8-r} = \frac{3}{4}$$

$$h = \frac{3}{4}(8-r) = \frac{3}{4}\times 8 - \frac{3}{4}r$$

$$\therefore h = 6 - \frac{3}{4}r \quad \text{as req'd}$$

Marking guidelines

2 Identifying similar triangles, equating ratios and finding *h* in required form

1 Identifying similar triangles or equating ratios

Marker's comments

Poorly answered. Students should think similarity when seeing 'triangles within triangles.'

Some clever students equated areas to find *h*.

iii) Show that the volume of the cylindrical tank can be expressed by 1

$$V = 3\pi r^2\left(2-\frac{r}{4}\right)$$

Solution :

$V = \pi r^2 h$

$= \pi r^2\left(6-\frac{3}{4}r\right)$ from part (ii)

$= 6\pi r^2 - \frac{3}{4}\pi r^3$

$= 3\pi r^2\left(2-\frac{r}{4}\right)$ as req'd

Marking guidelines

1 Correct answer

Marker's comments

Generally well answered.

iv) Find the value of r which gives the hot water tank its greatest volume. 3

Solution :

$V = 6\pi r^2 - \frac{3}{4}\pi r^3$

$\frac{dV}{dr} = 12\pi r - \frac{9\pi r^2}{4}$

Sationary point at $\frac{dV}{dr} = 0$

$12\pi r - \frac{9\pi r^2}{4} = 0$

$r\left(12\pi - \frac{9\pi r}{4}\right) = 0$

$r = 0$ or $12\pi - \frac{9\pi r}{4} = 0$

$\therefore r = \frac{16}{3} = 5\frac{1}{3}m$

$\frac{d^2V}{dr^2} = 12\pi - \frac{9\pi r}{2}$

When $r = \frac{16}{3}$

$\frac{d^2V}{dr^2} = 12\pi - \frac{9\pi \times 16}{6}$

$= -12\pi$

$< 0 \quad \therefore$ maximum

$\therefore$ tank radius of $5\frac{1}{3}m$ will give maximum volume

Marking guidelines

3 Correct with proof that *r* is a maximum using the test for max.

2 Correctly solved $\frac{dV}{dr} = 0$

1 Correctly found $\frac{dV}{dr}$

Marker's comments

Many students should practise differentiating.

Clearer setting-out would help minimise careless errors.

Students should be aware that they must demonstrate their value is a maximum by using either $\frac{d^2V}{dr^2}$ or $\frac{dV}{dr}$ sign table.

Students should check the reasonableness of their answers. For example many students had $r = 16$ giving the maximum value; however looking at the diagram $r < 8$.

c) Find all the values of θ for which $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$ 4

Solution :

$$8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$$
$$8\left(1-\cos^2\theta\right) - 2\cos^2\theta = 2\cos^2\theta - 1$$
$$8 - 8\cos^2\theta - 2\cos^2\theta = 2\cos^2\theta - 1$$
$$12\cos^2\theta - 9 = 0$$
$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

Related acute angle $\alpha = \frac{\pi}{6}$

General solution:

$$\theta = 2n\pi \pm \frac{\pi}{6} \quad \text{and}$$
$$\theta = 2n\pi \pm \frac{5\pi}{6}$$

This can be simplified to

$\theta = n\pi \pm \frac{\pi}{6}$, where n is an integer.

Alternate solution:

$$8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$$
$$8\sin^2\theta - 2\left(1-\sin^2\theta\right) = 1 - 2\sin^2\theta$$
$$8\sin^2\theta - 2 + 2\sin^2\theta = 1 - 2\sin^2\theta$$
$$12\sin^2\theta - 3 = 0$$
$$\sin^2\theta = \frac{1}{4}$$
$$\sin\theta = \pm\frac{1}{2}$$

Related acute angle $\alpha = \frac{\pi}{6}$

General solution:

$$\theta = n\pi + (-1)^n\frac{\pi}{6} \quad \text{and} \quad \theta = n\pi - (-1)^n\frac{\pi}{6}$$

$\theta = n\pi \pm \frac{\pi}{6}$, where n is an integer.

Marking guidelines

4 **Finding the simplified expression for θ or Finding two general solutions.**

3 **Partially correct general solution** or **General solutions using degrees.**

2 **Solving for $\cos\theta$ or $\sin\theta$ or $\tan\theta$**

e.g. $\cos\theta = \pm\frac{\sqrt{3}}{2}$

1 **Simplifying the equation in terms of one ratio.**

Marker's comments

Very poorly answered.

Many students did not know the trigonometric identities and consequently were unable to simplify the equation.

When solving for $\cos\theta$ many students did not take the $\pm$ of the root.

Many students incorrectly answered in degrees (often mixing their answer with radians).

Many students restricted their domain to $0 < \theta < 2\pi$ and do not realise trig equation have infinite solutions.

Many students missed out on the second solution i.e. $\theta = 2n\pi \pm \frac{5\pi}{6}$.

Many students did not explain their pattern by saying "where *n* is an integer".