FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS EXTENSION
2019
YEAR 11 Yearly Examination
ANSWERS COVER SHEET

Student Number: ______________________________

QUESTION	MARK	ME11-1	ME11-2	ME11-3	ME11-4	ME11-5	ME11-6	ME11-7
1-5	/5	✓	✓					✓
6	/20	✓	✓					✓
7	/13	✓	✓					✓
8	/25		✓					✓
9	/28					✓		✓
10	/14	✓	✓			✓		✓
TOTAL								
	/105	/52	/77			/42		/105

Year 11 Mathematics Extension 1 outcomes

A student:

ME11-1 uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2 manipulates algebraic expressions and graphical functions to solve problems

ME11-3 applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

ME11-4 applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change

ME11-5 uses concepts of permutations and combinations to solve problems involving counting or ordering

ME11-6 uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts

ME11-7 communicates making comprehensive use of mathematical language, notation, diagrams and graphs

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION

YEAR 11

YEARLY EXAMINATION

- **Instructions**
  - Reading Time – 10 minutes
  - Working time – 2 hours
  - Write using black pen
  - Calculators approved by NESA may be used
  - A reference sheet is provided
  - For Section I, use the multiple-choice answer sheet
  - For questions in section II, show relevant mathematical reasoning and /or calculations
  - Marks may be deducted for careless or badly arranged work

Section I (5 marks)
Answer Questions 1 – 5

Use the multiple-choice answer sheet on page 5 for questions 1 – 5

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1. Which of the following is the inverse function of $f(x) = 2x^{\frac{1}{5}}$?

   A. $f^{-1}(x) = \dfrac{x^{\frac{1}{5}}}{32}$

   B. $f^{-1}(x) = \dfrac{x^5}{32}$

   C. $f^{-1}(x) = \dfrac{x^5}{16}$

   D. $f^{-1}(x) = \dfrac{x^5}{2}$

2. The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor $x - 2$.
   What is the value of k ?

   A. 2

   B. 12

   C. 20

   D. 36

3. How many arrangements of the word *OLYMPIC* are possible if the *C* and the *L* are to be together in any order?

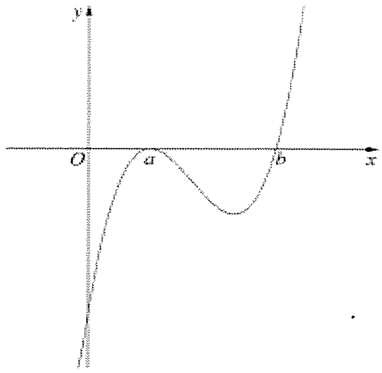
   A. 5!

   B. 6!

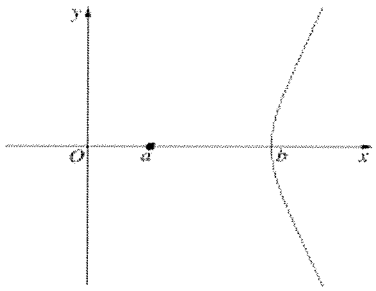
   C. 2 x 5!

   D. 2 x 6!

4. The diagram shows the graph of the function $y = f(x)$.



A second graph is obtained from the graph of $y = f(x)$.



Which equation best represents the second graph?

A. $y^2 = |f(x)|$

B. $y^2 = f(x)$

C. $y = \sqrt{f(x)}$

D. $y = f(\sqrt{x})$

5. A curve is represented by the following parametric equations $x = t + \frac{1}{t}$, $y = t^2 + \frac{1}{t^2}$.

The Cartesian equation of the curve is:

A. $y^2 = x - 2$

B. $y^2 - x^2 = 2$

C. $y = x^2 - 2$

D. $y = x - 2$

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Section II (102 marks)

- Answer questions in the spaces provided.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 6 (20 marks)

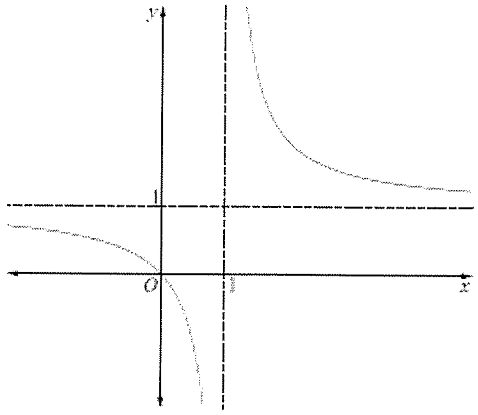
a. Solve

(i) $|3x - 5| \leq 4$ [2]

(ii) $|5x + 4| \geq 6$ [3]

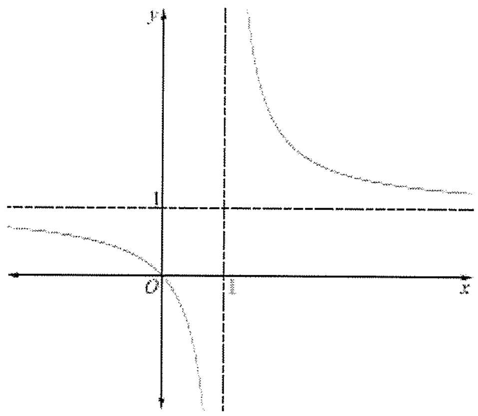
(iii) $\dfrac{x^2 + 5}{x} > 6$ [3]

c. The diagram shows the graph of the function $f(x) = \dfrac{x}{x-1}$.

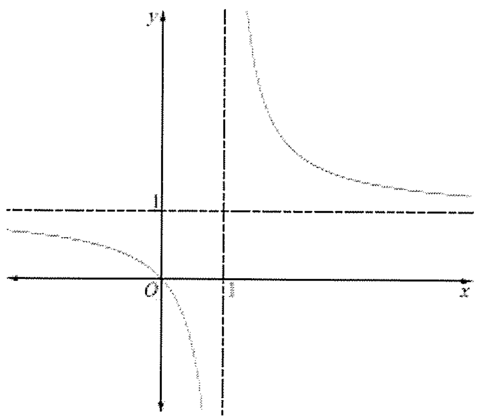


Use the graph of $y = f(x)$ to draw graphs the following transformations on the number planes provided. Clearly show all intercepts and asymptotes.

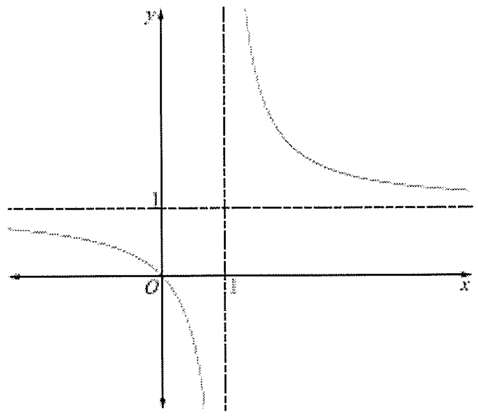
(i) $y = |f(x)|$ [3]



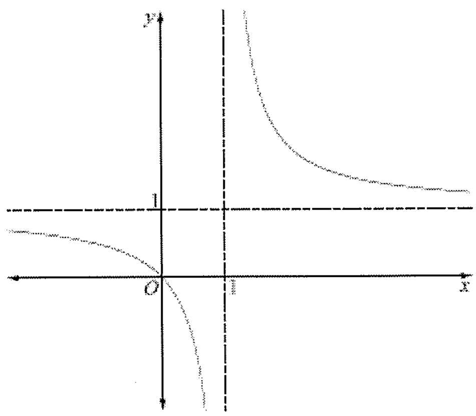
(ii) $y = (f(x))^2$ [3]



(iii) $y = \sqrt{f(x)}$ [3]

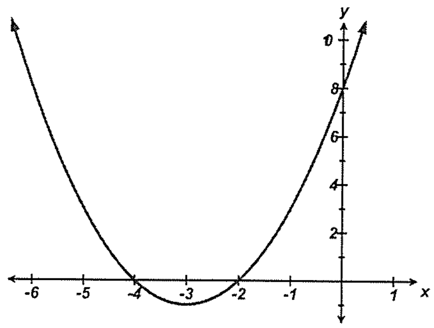


(iv) $y = f(|x|)$ [3]



Question 7 (13 marks)

a. The graph of $f(x) = x^2 + 6x + 8$ is shown below.



(i) Explain why the domain of $f(x)$ must be restricted if $f(x)$ is to have an inverse function. [1]

(ii) Consider $f(x) = x^2 + 6x + 8$ for $x \geq -3$. Find the equation of $f^{-1}(x)$. [3]

(iii) State the domain of $f^{-1}(x)$ [1]

(iv) Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ for $x \geq -3$ on the grid below, clearly showing the important features. [3]

b. The parametric equation of a graph are:

$$x = 1 - \frac{1}{t}$$

$$y = 1 + \frac{1}{t}$$

(i) Find the Cartesian equation of the graph. [3]

(ii) Sketch the graph. [2]

Question 8 (25 marks)

a. If α, β, γ are the roots of $2x^3 - 3x^2 + 6x + 8 = 0$, find the value of:

(i) $\alpha + \beta + \gamma$ [1]

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ [1]

(iii) $\alpha\beta\gamma$ [1]

(iv) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ [2]

(v) $\alpha^2 + \beta^2 + \gamma^2$ [3]

(vi) $\dfrac{\alpha}{\beta\gamma} + \dfrac{\beta}{\alpha\gamma} + \dfrac{\gamma}{\alpha\beta}$ [2]

b. $x^3 + 3x^2 + ax - 1 = 0$ has one root equal to the reciprocal of another.
Find the roots. [3]

c. Consider the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$

(i) Show that $x = 1$ is a zero of $P(x)$. [1]

(ii) Find the other zeros. [3]

d. Factorise $2x^3 - 5x^2 - 4x + 12 = 0$ if it has a zero of multiplicity 2 at $x = 2$. [4]

e. Let $P(x) = (x + 1)(x - 3)Q(x) + ax + b$ where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x - 3$. When $P(x)$ is divided by $x + 1$, the remainder is 8.

(i) Find the values of a and b. [3]

(ii) Find the remainder $P(x)$ is dvided by $(x + 1)(x - 3)$. [1]

Question 9 (28 marks)

a. Simplify

(i) $n! + (n - 1)!$ [2]

(ii) $\frac{n!}{(n-1)!} \cdot \frac{(n-2)!}{(n+1)!}$ [3]

b. Siya has five different coloured blocks. She stacks two, three, four or five blocks on top of one another to form a vertical tower.

(i) How many different towers can she form that are three blocks high? [1]

(ii) How many different towers can she form in total? [2]

c. (i) In how many different ways can the letters of the word ESTEEM be arranged? [1]

(ii) In how many ways can the letters of the word ESTEEM be arranged so that the E's are together? [2]

d. Find the number of ways in which 3 boys and 4 girls line up in a queue so that girls occupy the middle position and exactly one of the two end positions. [3]

e. (i) In how many ways can a committee of 7 be selected from a group of 18 people? [1]

(ii) In how many ways can a committee of 3 men and 4 women be selected if there are 8 men and 10 women in this group? [2]

(iii) In how many ways can a committee of 3 men and 4 women be selected from this group if particular man and a particular woman can't be in the committee together? [3]

f. Six men and six women are to be seated at a round table.
In how many ways can they be seated:
(i) if there are no restrictions? [1]

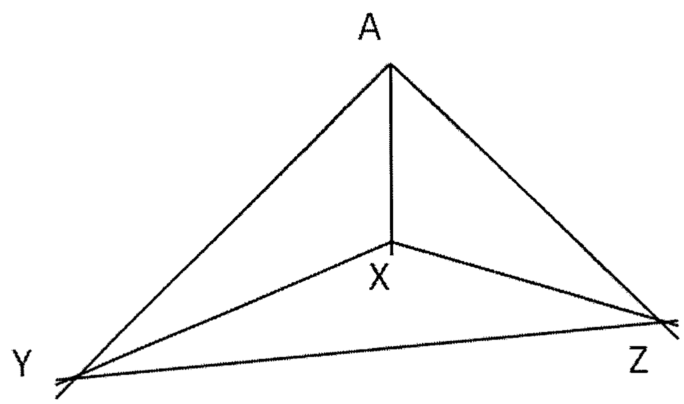
(ii) if men and women alternate? [2]

(iii) if two particular people are not to sit together? [3]

(iv) if three particular people are to sit together? [2]

Question 10 (15 marks)

a.



The tower AX, of height h metres, is built on flat ground. X, Y and Z are points on the same horizontal plane.
From Y and Z, the angles of elevation to the top of the tower are 28° and 40° respectively. Give that YZ = 100m and $\angle$ YXZ = 90°,

(i) show that XY = h cot 28° and find a similar statement for XZ. [2]

(ii) find the height of the tower to the nearest metre. [3]

b. Find the inverse of $y = 3\log_e(2x + 5) - 4$. [2]

c. Solve $x^5 + 2x^4 - 2x^3 - 8x^2 - 7x - 2 = 0$ if it has a root of multiplicity 4. [4]

d. A hostel has four vacant rooms. Each room can accommodate a maximum of four people. In how many ways can six people be accommodated in the four rooms? [3]

End of Examination

Additional Paper (Please clearly indicate the question numbers)

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TOTAL								
	/105	/52	/77			/42		/105

Year 11 Mathematics Extension 1 outcomes

A student:

ME11-1	uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses
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MATHEMATICS EXTENSION

YEAR 11

YEARLY EXAMINATION

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  - Marks may be deducted for careless or badly arranged work

Section I (5 marks)
Answer Questions 1 – 5

Use the multiple-choice answer sheet on page 5 for questions 1 – 5

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1. Which of the following is the inverse function of $f(x) = 2x^{\frac{1}{5}}$?

A. $f^{-1}(x) = \dfrac{x^{\frac{1}{5}}}{32}$

B. $f^{-1}(x) = \dfrac{x^5}{32}$

C. $f^{-1}(x) = \dfrac{x^5}{16}$

D. $f^{-1}(x) = \dfrac{x^5}{2}$

$y = 2x^{1/5}$

$f^{-1}(x) \; ; \; x = 2y^{1/5}$

$y^{1/5} = \dfrac{x}{2}$

$y = \dfrac{x^5}{32}$

2. The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor $x - 2$.
What is the value of k ?

A. 2

B. 12

C. 20

D. 36

$P(2) = 2^3 - 4(2)^2 - 6(2) + k = 0$

$k = 20$

3. How many arrangements of the word *OLYMPIC* are possible if the *C* and the *L* are to be together in any order?

A. 5!

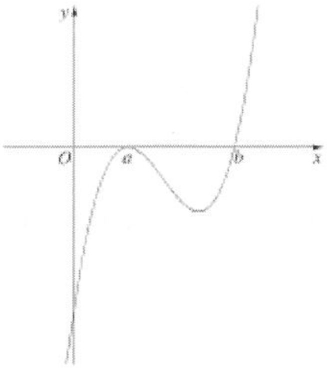
B. 6!

C. 2 x 5!

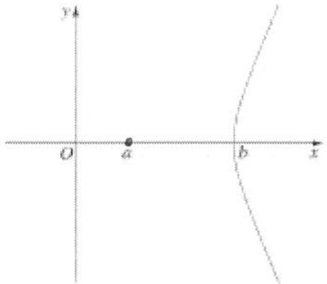
D. 2 x 6!

$\boxed{CL}$ O Y M P I

4. The diagram shows the graph of the function $y = f(x)$.



A second graph is obtained from the graph of $y = f(x)$.



D: $x = a$, $x \geq b$ $\Rightarrow$ not A, D

$y = \pm\sqrt{f(x)}$

$\therefore y^2 = f(x)$

Which equation best represents the second graph?

A. $y^2 = |f(x)|$

B. $y^2 = f(x)$

C. $y = \sqrt{f(x)}$

D. $y = f(\sqrt{x})$

5. A curve is represented by the following parametric equations $x = t + \dfrac{1}{t}$, $y = t^2 + \dfrac{1}{t^2}$.

The Cartesian Equation of the curve is :

A. $y^2 = x - 2$

B. $y^2 - x^2 = 2$

C. $y = x^2 - 2$

D. $y = x - 2$

$x^2 = (t + \frac{1}{t})^2$

$= t^2 + 2 + \frac{1}{t^2}$

$\therefore x^2 - 2 = t^2 + \frac{1}{t^2}$

$= y$

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	●	C	○	D	○
2.	A	○	B	○	C	●	D	○
3.	A	○	B	○	C	○	D	●
4.	A	○	B	●	C	○	D	○
5.	A	○	B	○	C	●	D	○

Section II (100 marks)

- Answer questions in the spaces provided.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 6 (20 marks)

a. Solve

(i) $|3x - 5| \leq 4$ [2]

$-4 \leq 3x - 5 \leq 4$

$1 \leq 3x \leq 9$

$\frac{1}{3} \leq x \leq 3$

(ii) $|5x + 4| \geq 6$ [3]

$5x + 4 \leq -6$ or $5x + 4 \geq 6$

$5x \leq -10$ $5x \geq 2$

$x \leq -2$ $x \geq \frac{2}{5}$

(iii) $\dfrac{x^2 + 5}{x} > 6$ [3]

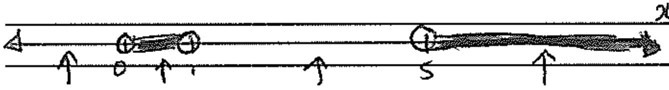
$x \neq 0$ Solve $\frac{x^2+5}{x} = 6$

$x^2 + 5 = 6x$

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1, 5$



Test

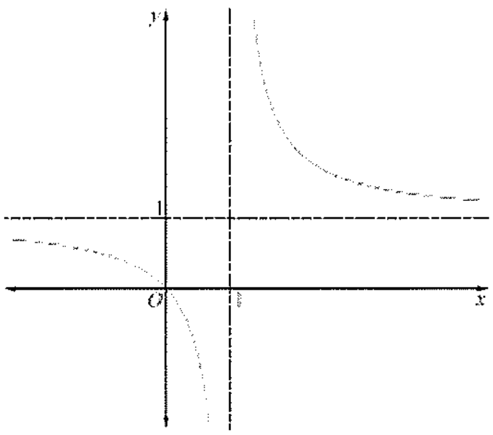
$x = -1$ $x = \frac{1}{2}$ $x = 2$ $x = 6$

$\frac{(-1)^2+5}{-1} > 6$ $\frac{\frac{1}{4}+5}{\frac{1}{2}} > 6$ $\frac{4+5}{2} > 6$ $\frac{36+5}{6} > 1$

× ✓ × ✓

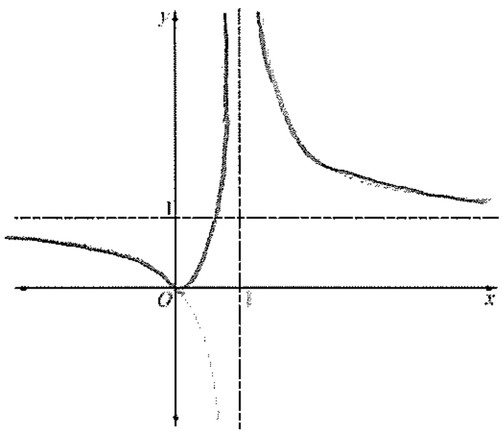
$\therefore$ $0 < x < 1$, $x > 5$

c. The diagram shows the graph of the function $f(x) = \dfrac{x}{x-1}$.

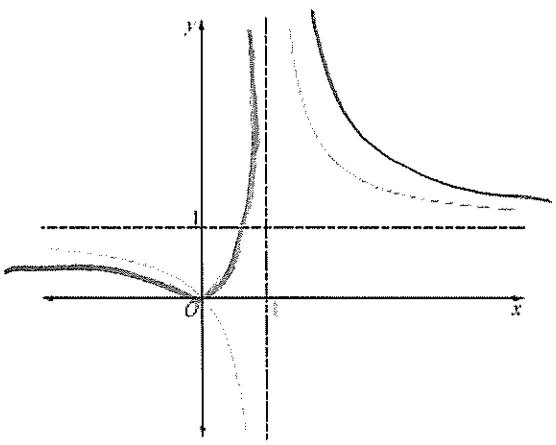


Use the graph of $y = f(x)$ to draw graphs the following transformations on the number planes provided. Clearly show all intercepts and asymptotes.

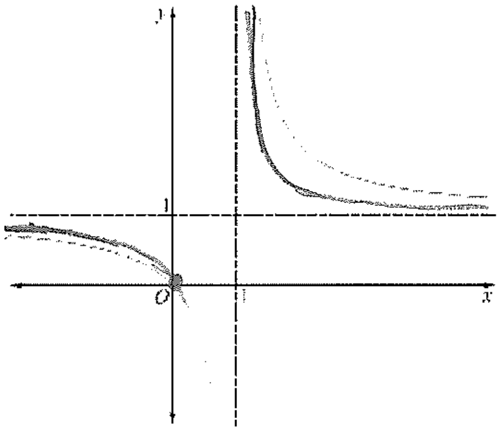
(i) $y = |f(x)|$ [3]



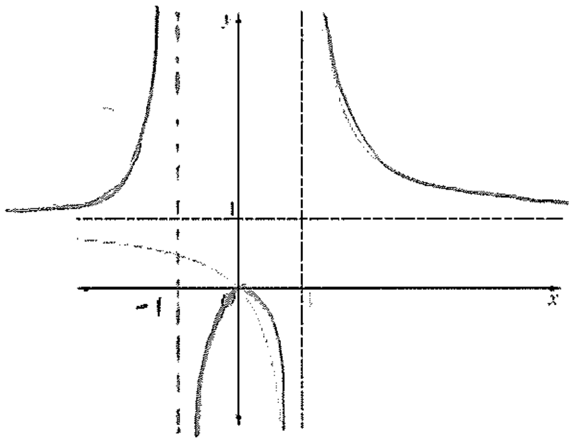
(ii) $y = (f(x))^2$ [3]



(iii) $y = \sqrt{f(x)}$ [3]

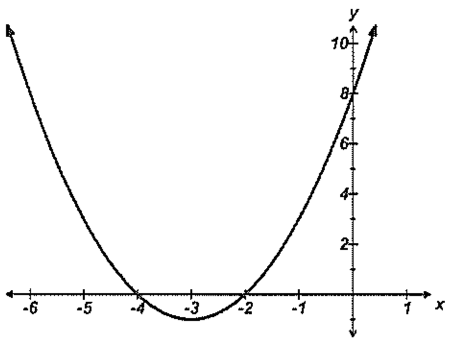


(iv) $y = f(|x|)$ [3]



Question 7 (13 marks)

a. The graph of $f(x) = x^2 + 6x + 8$ is shown below.



(i) Explain why the domain of $f(x)$ must be restricted if $f(x)$ is to have an inverse function. [1]

Horizontal line cuts this graph in more than 1 place

(OR) The function is not a one-to-one function.

(ii) Consider $f(x) = x^2 + 6x + 8$ for $x \geq -3$. Find the equation of $f^{-1}(x)$. [3]

$f(x)$: $y = x^2 + 6x + 8$

$f^{-1}(x)$: $x = y^2 + 6y + 8$

$x = (y+3)^2 - 1$

$x + 1 = (y+3)^2$

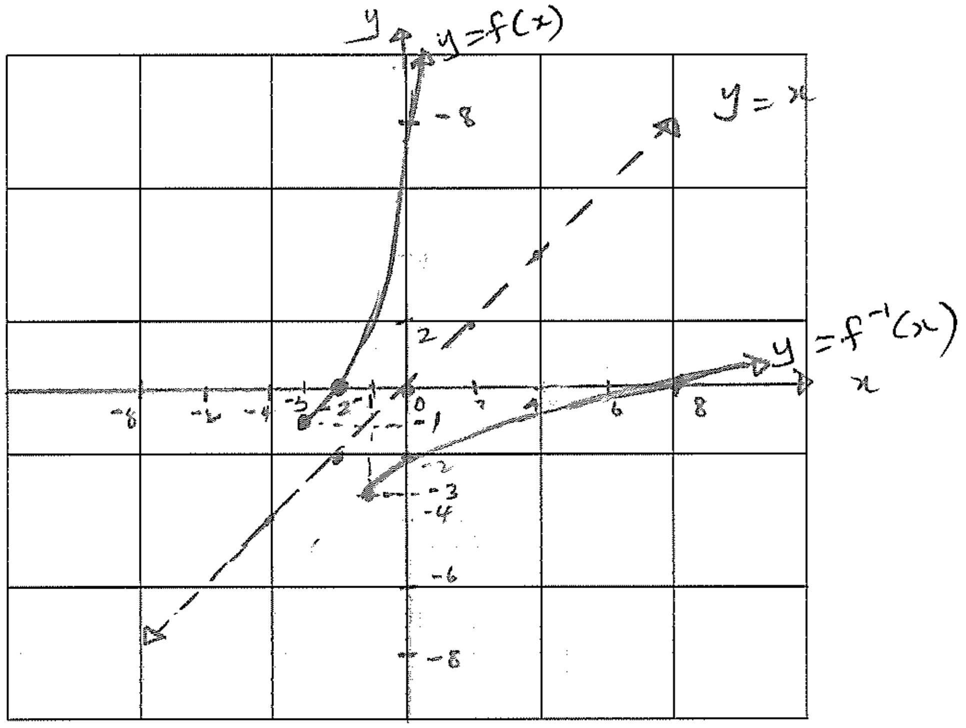
$y + 3 = \pm\sqrt{x+1}$

$\therefore y = -3 + \sqrt{x+1}$, since $R_{f^{-1}(x)}$: $y \geq -3$

(iii) State the domain of $f^{-1}(x)$ [1]

$D_{f^{-1}(x)} = R_{f(x)} = x \geq -1$

(iv) Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ for $x \geq -3$ on the grid below, clearly showing the important features. [3]



b. The parametric equation of a graph are:

$$x = 1 - \frac{1}{t}$$

$$y = 1 + \frac{1}{t}$$

(i) Find the Cartesian equation of the graph. [3]

$x = 1 - \frac{1}{t}$ — 1 ; $y = 1 + \frac{1}{t}$ — 2

From 1 $tx = t - 1$ Substitute $t = \frac{1}{1-x}$ into 2

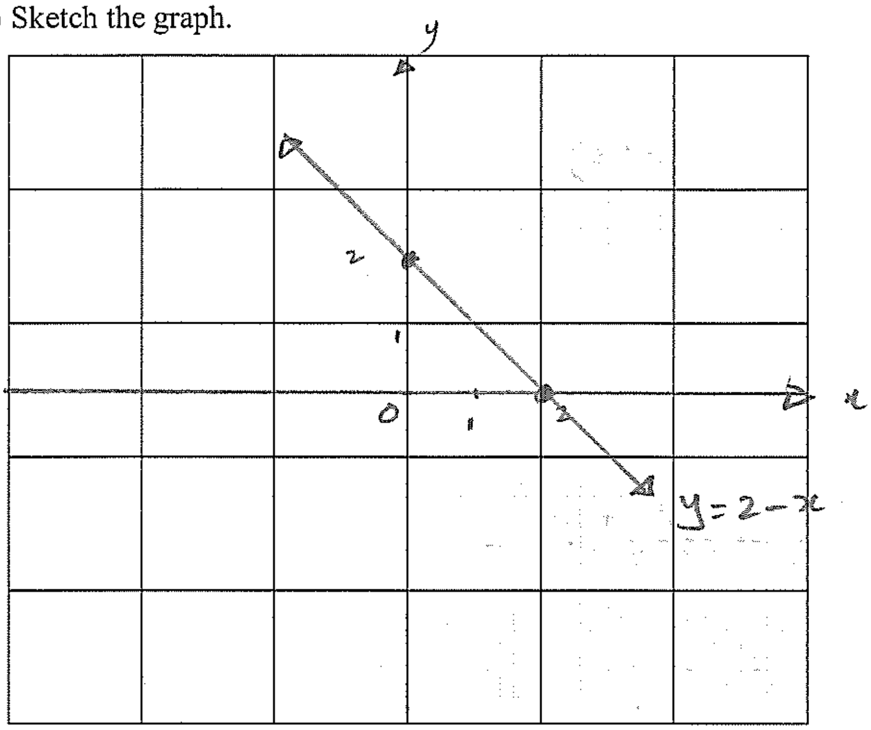
$t - tx = 1$

$t(1-x) = 1$ $y = 1 + \frac{1}{\frac{1}{1-x}}$

$t = \frac{1}{1-x}$ $y = 1 + 1 - x$

$y = 2 - x$

(ii) Sketch the graph. [2]



Question 8 (26 marks)

a. If α, β, γ are the roots of $2x^3 - 3x^2 + 6x + 8 = 0$, find the value of:

(i) $\alpha + \beta + \gamma$ [1]

$= -\frac{b}{a} = \frac{3}{2}$

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ [1]

$= \frac{c}{a} = \frac{6}{2} = 3$

(iii) $\alpha\beta\gamma$ [1]

$= \frac{-d}{a} = \frac{-8}{2} = -4$

(iv) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ [2]

$\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-4} = \frac{-3}{4}$

(v) $\alpha^2 + \beta^2 + \gamma^2$ [3]

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= \left(\frac{3}{2}\right)^2 - 2(3)$

$= \frac{9}{4} - 6 \quad = \frac{-15}{4}$

(vi) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$ [2]

$= \frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$

$= \frac{-15/4}{-4} \quad = \frac{15}{16}$

c. $x^3 + 3x^2 + ax - 1 = 0$ has one root equal to the reciprocal of another. Find the roots. [3]

Roots : $\alpha, \beta, \frac{1}{\beta}$

product of roots : $\alpha = 1$

$\therefore$ roots $= 1, \beta, \frac{1}{\beta}$

Sum of roots : $1 + \beta + \frac{1}{\beta} = -3$

$\beta + \beta^2 + 1 = -3\beta$

$\beta^2 + 4\beta + 1 = 0$

$\beta = \frac{-4 \pm \sqrt{12}}{2} \quad = \frac{-4 \pm 2\sqrt{3}}{2}$

$= -2 \pm \sqrt{3}$

$\therefore$ Roots : $1, \sqrt{3}-2, \frac{1}{\sqrt{3}-2}$

or $1, -\sqrt{3}-2, \frac{1}{\sqrt{3}-2}$

b. Consider the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$

(i) Show that $x = 1$ is a zero of $P(x)$. [1]

$P(1) = 1 - 2 - 5 + 6 = 0$

$\therefore x = 1$ is a zero

(ii) Find the other zeros. [3]

$$x^2 - x - 6$$

$$x-1 \overline{) x^3 - 2x^2 - 5x + 6}$$

$$-\ x^3 - x^2$$

$$-x^2 - 5x$$

$$-\ -x^2 + x$$

$$-6x + 6$$

$$-\ 6x + 6$$

$$\therefore P(x) = (x-1)(x^2 - x - 6)$$

$$= (x-1)(x-3)(x+2)$$

Other zeros : $-2, 3$

c. Factorise $2x^3 - 5x^2 - 4x + 12 = 0$ if it has a zero of multiplicity 2 at $x = 2$. [4]

Roots : $2, 2, \alpha$ $(x-2)^2$ is a factor

Sum of roots : $2 + 2 + \alpha = \frac{5}{2}$

$$\alpha = -\frac{3}{2}$$

$\therefore$ $(2x+3)$ is a factor

$$\therefore 2x^3 - 5x^2 + 4x + 12 = (x-2)^2(2x+3)$$

d. Let $P(x) = (x+1)(x-3)Q(x) + ax + b$ where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x - 3$. When $P(x)$ is divided by $x + 1$, the remainder is 8.

(i) Find the values of a and b . [3]

$(x-3)$ is a factor $\therefore P(3) = 0$

$\therefore 3a + b = 0$ —— (1)

When $\div$ by $(x+1)$, remainder $= 8$ $\therefore P(-1) = 8$

$\therefore -a + b = 8$ —— (2)

(1) − (2) $4a = -8$

$$a = -2$$

substitute $a = -2$ into (1)

$$-6 + b = 0$$

$$b = 6$$

$$\therefore a = -2, b = 6$$

(ii) Find the remainder $P(x)$ is dvided by $(x + 1)(x - 3)$. [1]

$P(x) = (x+1)(x-3)\,Q(x) - 2x + 6$ (from(i))

$\therefore$ Remainder when divided by $(x+1)(x-3)$

$= -2x+6$

Question 9 (28 marks)

a. Simplify

(i) $n! + (n-1)!$ [2]

$= n(n-1)! + (n-1)!$

$= (n-1)!\,(n+1)$

(ii) $\dfrac{n!}{(n-1)!} \cdot \dfrac{(n-2)!}{(n+1)!}$ [3]

$= \dfrac{n(n-1)!\;(n-2)!}{(n-1)!\,(n+1)\,n\,(n-1)(n-2)!}$

$= \dfrac{1}{(n+1)(n-1)}$

b. Siya has five different coloured blocks. She stacks two, three, four or five blocks on top of one another to form a vertical tower.

(i) How many different towers can she form that are three blocks high? [1]

$5 \times 4 \times 3$ (or) ${}^5P_3 = 60$

$= 60$

(ii) How many different towers can she form in total? [2]

${}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$= 20 + 60 + 120 + 120$

$= 320$

c. (i) In how many different ways can the letters of the word ESTEEM be arranged? [1]

6 letters, 3Es

$\dfrac{6!}{3!} = 120$

(ii) In how many ways can the letters of the word ESTEEM be arranged so that the E's are together? [2]

$\boxed{EEE}$ _ _ _

$4! = 24$

d. Find the number of ways in which 3 boys and 4 girls line up in a queue so that girls occupy the middle position and exactly one of the two end positions. [3]

G . . G . . B or B . . G . . G

$2 \times {}^4C_2 \times {}^3C_1 \times 4! \times 2!$

$= 1728$

e. (i) In how many ways can a committee of 7 be selected from a group of 18 people? [1]

${}^{18}C_7 = 31824$

(ii) In how many ways can a committee of 3 men and 4 women be selected if there are 8 men and 10 women in this group? [2]

${}^8C_3 \times {}^{10}C_4 = 11760$

(iii) In how many ways can a committee of 3 men and 4 women be selected from this group if particular man and a particular woman can't be in the committee together? [3]

Total ways − ways both chosen

$11760 - {}^7C_2 \times {}^9C_3 = 9996$

e. Six men and six women are to be seated at a round table.
In how many ways can they be seated:
(i) if there are no restrictions? [1]

$11! = 39916800$

(ii) if men and women alternate? [2]

$5! \times 6!$

$= 86400$

(iii) if two particular people are not to sit together? 11 grops [3].

2 sit together $= 10! \times 2! = 7257600$

$\therefore$ Not sit together $= 11! - 10! \, 2!$

$= 10!\,(11-2) = 9 \times 10! = 32659200$

(iv) if three particular people are to sit together? [2]

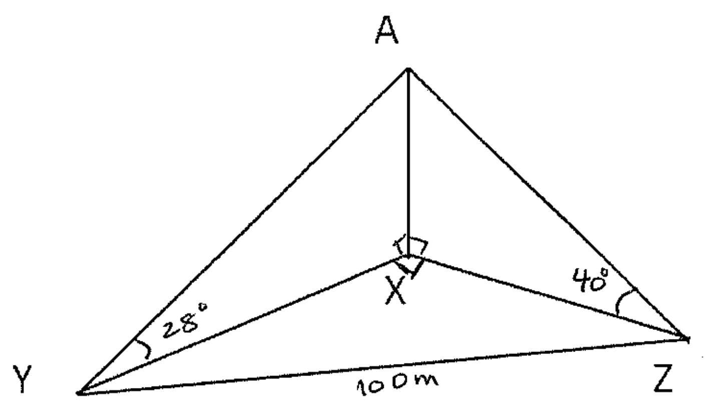
10 grops

$9!\,3!$

$= 2177280$

Question 10 (15 marks)

a.



The tower AX, of height h metres, is built on flat ground. X, Y and Z are points on the same horizontal plane.
From Y and Z, the angles of elevation to the top of the tower are 28° and 40° respectively. Give that YZ = 100m and $\angle$ YXZ = 90°,

(i) show that XY = h cot 28° and find a similar statement for XZ. [2]

In ΔAXY

$\tan 28° = \dfrac{h}{XY}$

$XY = \dfrac{h}{\tan 28°}$

$XY = h \cot 28°$

Similarly, using ΔAXZ

$XZ = h \cot 40°$

(ii) find the height of the tower to the nearest metre. [3]

Using pythagoras' theorem in ΔXYZ

$YZ^2 = XY^2 + XZ^2$

$(100)^2 = (h\cot 28°)^2 + (h\cot 40°)^2$

$= h^2\cot^2 28° + h^2\cot^2 40°$

$(100)^2 = h^2(\cot^2 28° + \cot^2 40°)$

$h = \dfrac{100}{\sqrt{\cot^2 28° + \cot^2 40°}}$

$= 44.913\ldots$

height of tower = 45 metres.

b. Find the inverse of $y = 3\log_e(2x+5) - 4$ $= f(x)$ [2]

$f^{-1}(x)$: $x = 3\log_e(2y+5) - 4$

$\dfrac{x+4}{3} = \log_e(2y+5)$

$e^{\frac{x+4}{3}} = 2y+5 \Rightarrow y = \frac{1}{2}\left(e^{\frac{x+4}{3}} - 5\right)$

c. Solve $x^5 + 2x^4 - 2x^3 - 8x^2 - 7x - 2 = 0$ if it has a root of multiplicity 4. [4]

$P(x) = x^5 + 2x^4 - 2x^3 - 8x^2 - 7x - 2$ (root of multiplicity)

$P'(x) = 5x^4 + 8x^3 - 6x^2 - 16x - 7$ (same root of multiplicity 3)

$P''(x) = 20x^3 + 24x^2 - 12x - 16$ (- - - - - × 2)

$P'''(x) = 60x^2 + 48x - 12$ (- - - - - - × 1)

Roots of $60x^2 + 48x - 12 = 0$

$5x^2 + 4x - 1 = 0$

$(5x-1)(x+1) = 0$

$x = \frac{1}{5}, -1$

$P(-1) = (-1)^5 + 2(-1)^4 - 2(-1)^3 - 8(-1)^2 - 7(-1) - 2$

$= 0$

$\therefore x = -1$ is the zero of multiplicity 4

$\therefore (x+1)^4$ is a factor

$\therefore P(x) = (x+1)^4(x-2)$ By observation

$\therefore x = -1, -1, -1, -1, 2$

d. A hostel has four vacant rooms. Each room can accommodate a maximum of four people. In how many ways can six people be accommodated in the four rooms? [3]

Total number of ways (no maximum) $= 4^6 = 4096$

Number of ways with max 4 = Total − (ways to have 5 + ways to have 6)

No. of ways to have 5 in one room $= {}^6C_5 \times {}^4C_1 \times {}^3C_1 = 72$

No. of ways to have 6 in one room $= 4$

∴ No. of ways with 4 maximum in a room.

$= 4096 - (4 + 72)$

$= \underline{4020}$

End of Examination