GIRRAWEEN HIGH SCHOOL

YEAR 11 YEARLY EXAMINATION

2020

MATHEMATICS EXTENSION 1

Time allowed: Two hours

(Plus 10 minutes reading time)

Instructions:

- Write using black or blue pen.
- A reference sheet is provided.
- Calculators approved by NESA may be used.
- For Section I, indicate your answers by filling in the appropriate circle on the answer sheet provided.
- For questions in section II, show relevant mathematical reasoning and/or calculations. Each question is to be answered on a new page in your answer booklet. You may ask for extra booklets if you need them.
- All the graphs are to be drawn on the number plane provided.
- Marks may be deducted for careless or badly arranged work.

Section I Multiple Choice Questions 1 to 5 (5 marks)

Fill in the appropriate circle on your multiple choice answer sheet:

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1. The solution of $x^2 - 5x + 6 \le 0$ is

A. $2 < x < 3$

B. $2 \le x \le 3$

C. $2 < x \le 3$

D. $2 \le x < 3$

2. Simplify: $\dfrac{(n+1)!}{(n-1)!}$

A. $\dfrac{n+1}{n-1}$

B. $n^2 + 1$

C. $n^2 - 1$

D. $n^2 + n$

3. The polynomial $P(x) = 8x^3 + ax^2 - 4x + 1$ has a factor of $2x + 1$. What is the value of a?

A. 8

B. 0

C. 3

D. -8

4. The solution of $|2x - 4| < 6$ is

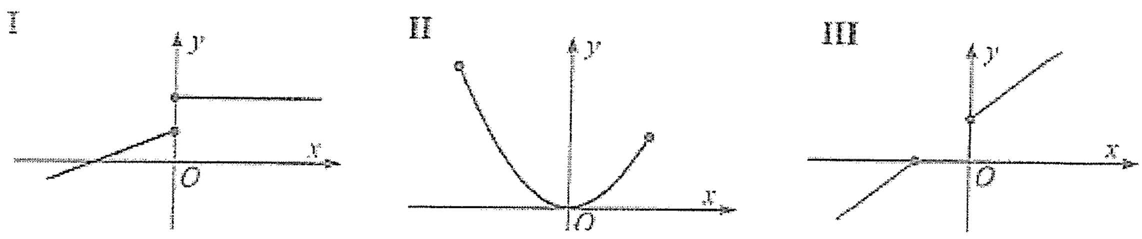
A. $-1 < x < 5$

B. $-5 < x < 1$

C. $-9 < x < 4$

D. $-4 < x < 9$

5. Which of the following graphs represent functions whose inverses are also functions?



A. II and III

B. I and III

C. III only

D. None of I, II and III

END OF MULTIPLE CHOICE QUESTIONS

Section II

- **Show all necessary working in your examination booklet.**
- **Sketch the graphs on the number planes provided.**

Question 6 (24 marks)

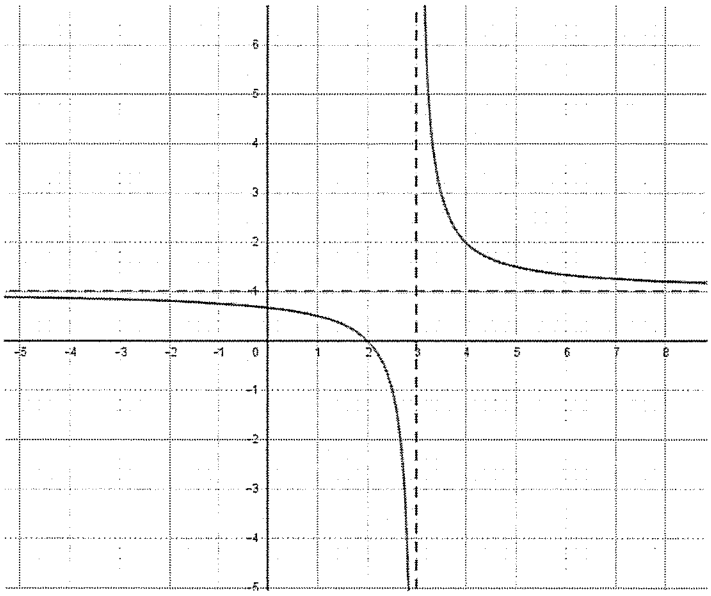
(a) Solve:

(i) $\dfrac{x+2}{x-3} > 2$ 3

(ii) $\dfrac{x^2-25}{x} \geq 0$ 3

(b) A curve is represented by the following parametric equations $x = 2\sin\theta + 3\cos\theta$, $y = 2\sin\theta - 3\cos\theta$. Find the cartesian equation of the curve. 3

(c) The diagram shows the graph of the function $f(x) = \dfrac{1}{x-3} + 1$.



Use the graph of $y = f(x)$ to draw graphs of the following transformations on the number planes provided. Clearly show all intercepts, asymptotes and final graph.

(i) $y = |f(x)|$ 3

(ii) $y = f(|x|)$ 4

(iii) $|y| = f(x)$ 4

(iv) $y^2 = f(x)$ 4

Question 7 (26 marks)

(a) **(i)** Find the inverse f^{-1} of $f(x)=\sqrt{4-x}$. 2

(ii) Show that $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$ 4

(b) **(i)** Write the inverse relation of $(x-3)^2+(y+1)^2=4$ 1

(ii) Graph both relations on the same number plane, showing the reflection line. 3

(iii) Write the domain and range of both relations. 4

(c) **(i)** Find the $x-$intercepts and vertex of $f(x)=x^2-4x$. 2

(ii) Write the domain and range of $f(x)=x^2-4x, x\le 2$. 2

(iii) Write the domain and range of $f^{-1}(x)$. 2

(iv) Find the inverse of $f(x)=x^2-4x, x\le 2$. 3

(v) Sketch $f(x)=x^2-4x, x\le 2$ and its inverse on the same number plane clearly showing the important features and the reflection line. 3

Examination continues on next page.

Question 8 (28 marks)

(a) If α, β and γ are the roots of $2x^3 - 5x^2 - 3x + 1 = 0$, find the value of:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(iii) $\alpha\beta\gamma$ **1**

(iv) $6\alpha + 6\beta + 6\gamma - 5$ **2**

(v) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma}$ **2**

(vi) $(\alpha - 2)(\beta - 2)(\gamma - 2)$ **2**

(b) The polynomial $P(x) = 2x^4 + 5x^3 + 3x^2 - x - 1 = 0$ has a root of multiplicity three.

(i) Find the triple zero using derivatives of $P(x)$. 4

(ii) Find the remaining simple zero. 2

(iii) Factorise $P(x)$. 2

(iv) Sketch the graph of $y = P(x)$ showing all intercepts with the axes. 4

(c) When the polynomial $P(x)$ is divided by $(x+1)(x-3)$, the quotient is $Q(x)$ and the remainder is $-2x + 4$.

(i) Find the remainder when $P(x)$ is divided by $x + 1$. 2

(ii) What is the remainder when $P(x)$ is divided by $x - 3$. 2

(d) Solve the equation $3x^3 - 17x^2 - 8x + 12 = 0$ given that the product of two of the roots is 4. 3

Question 9 (22 marks)

(a) Write as a single fraction: $\dfrac{8}{(n+2))!} - \dfrac{1}{n!}$ 2

(b) A car number plate in Australia usually has three letters followed by three digits. How many different number plates are possible

(i) if there are no restrictions 2

(ii) if there is no repetition 1

(iii) staring with R and ending with 2 (repetition allowed) 1

(iv) starting with R and ending with 2 (repetition not allowed) 1

(v) if the letters are consonants and digits are 3,6,7,9 (repetition allowed) 1

(vi) if letters are consonants and digits are 3,6,7,9 (repetition not allowed) 1

(c) A tea party is arranged for 16 students along two sides of a long table with 8 chairs on each side. Four students wish to sit on one particular side and two on the other side. In how many ways can they be seated? 3

(d) **(i)** In how many ways can a committee of 4 girls and 5 boys be appointed from a group consisting of 10 girls and 9 boys? 2

(ii) What will be the number of ways if the girl Daisy refuses to serve in a committee in which the boy Noah is a member? 2

(e) How many arrangements of the letters of the word COOLANGATTA are there with

(i) no restrictions . 2

(ii) all T's together. 2

(iii) no A's together. 2

Question 10 (20 marks)

(a) A committee of 4 people is to be chosen from a group of 6 men and 5 women.

At least one man and one woman must be on the committee.

(i) Find the number of ways of forming the committee. 3

(ii) What is the probability that a committee chosen at random will consist of a majority of women? 2

(b) In how many ways can 7 boys and 7 girls be arranged in a circle if

(i) there are no restrictions. 1

(ii) boys and girls alternate. 2

(iii) two particular boys want to be together. 2

(iv) a particular boy wants to be opposite to a particular girl. 1

(v) two particular girls do not want to sit next to each other. 2

(c) The numbers 1,2,3,4,5,6,7,8 are arranged in a circle. What is the probability that at least three odd numbers are together? 3

(d) A course has seven elective topics, and students must complete exactly three of them in order to pass the course. If 200 students passed the course, find the least number of students to be chosen so that they all completed the same set of electives. 2

(e) How many of the numbers must be chosen from the set $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

to guarantee that two of the chosen numbers add to 11? 2

END OF QUESTION PAPER

Year 11 Extension 1 Yearly 2020 solutions

Multiple choice (5 marks)

1B 2D 3D 4A 5C

Question 6 (24 marks)

(a)(i) $\dfrac{x+2}{x-3} > 2$

Multiply by $(x-3)^2$, $x \neq 3$

$(x-3)(x+2) > 2(x-3)^2$

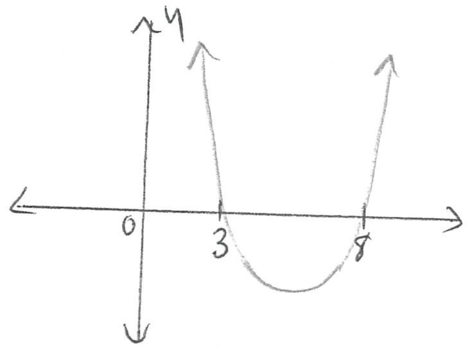
$0 > 2(x-3)^2 - (x-3)(x+2)$

$2(x-3)^2 - (x-3)(x+2) < 0$

$(x-3)\left(2(x-3) - (x+2)\right) < 0$

$(x-3)(2x-6-x-2) < 0$

$(x-3)(x-8) < 0$



3

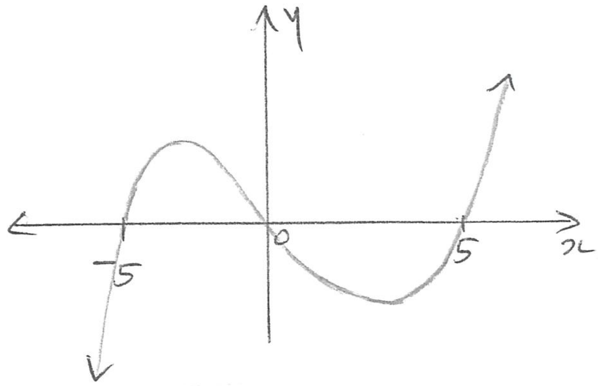
From the graph $(x-3)(x-8)<0$

when $\underline{\underline{3 < x < 8}}$

(ii) $\dfrac{x^2-25}{x} \geq 0$

$\dfrac{x^2(x^2-25)}{x} \geq 0, \; x \neq 0$

$x(x+5)(x-5) \geq 0$



From the graph $x(x+5)(x-5) \geq 0$

when $\underline{\underline{-5 \leq x < 0 \text{ or } x \geq 5}}$ 3

b) $x = 2\sin\theta + 3\cos\theta$

$y = 2\sin\theta - 3\cos\theta$

$x+y = 4\sin\theta$

$\sin\theta = \dfrac{x+y}{4}$

$x-y = 6\cos\theta$

$\cos\theta = \dfrac{x-y}{6}$

$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{(x+y)^2}{16} + \dfrac{(x-y)^2}{36} = 1$ 3

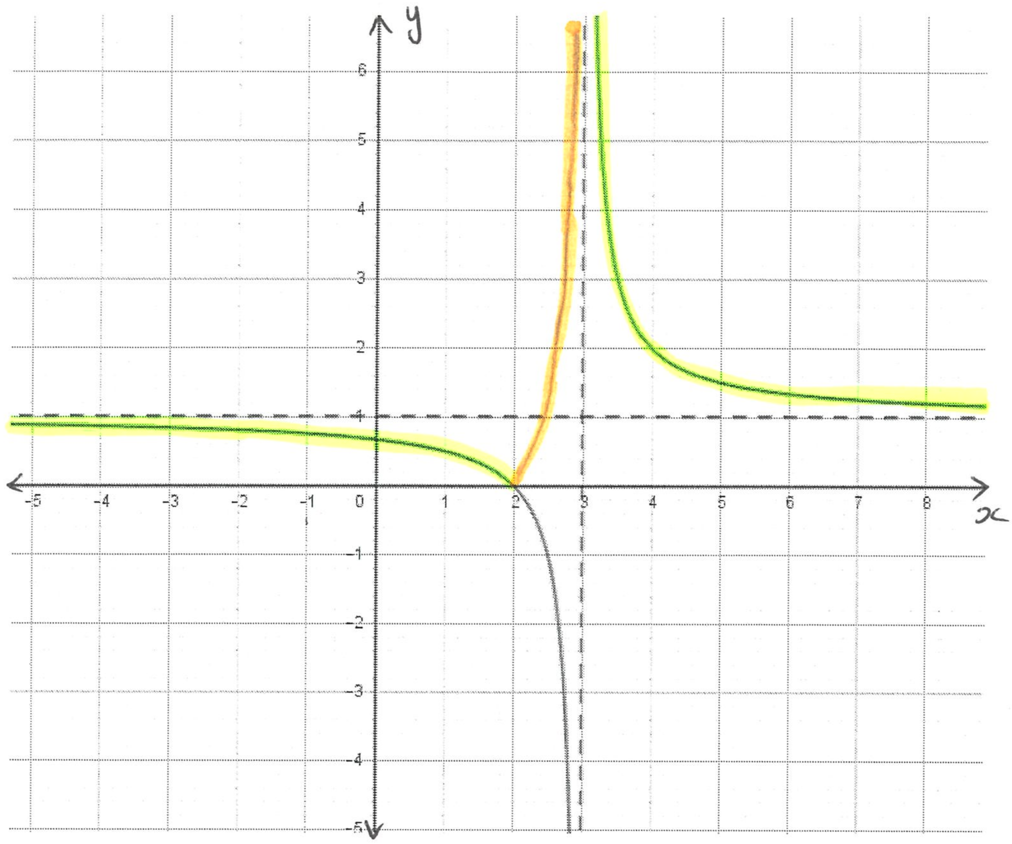
$\dfrac{9(x+y)^2 + 4(x-y)^2}{144} = 1$

$9(x+y)^2 + 4(x-y)^2 = 144$

$9x^2 + 18xy + 9y^2 + 4x^2 - 8xy + 4y^2 = 144$

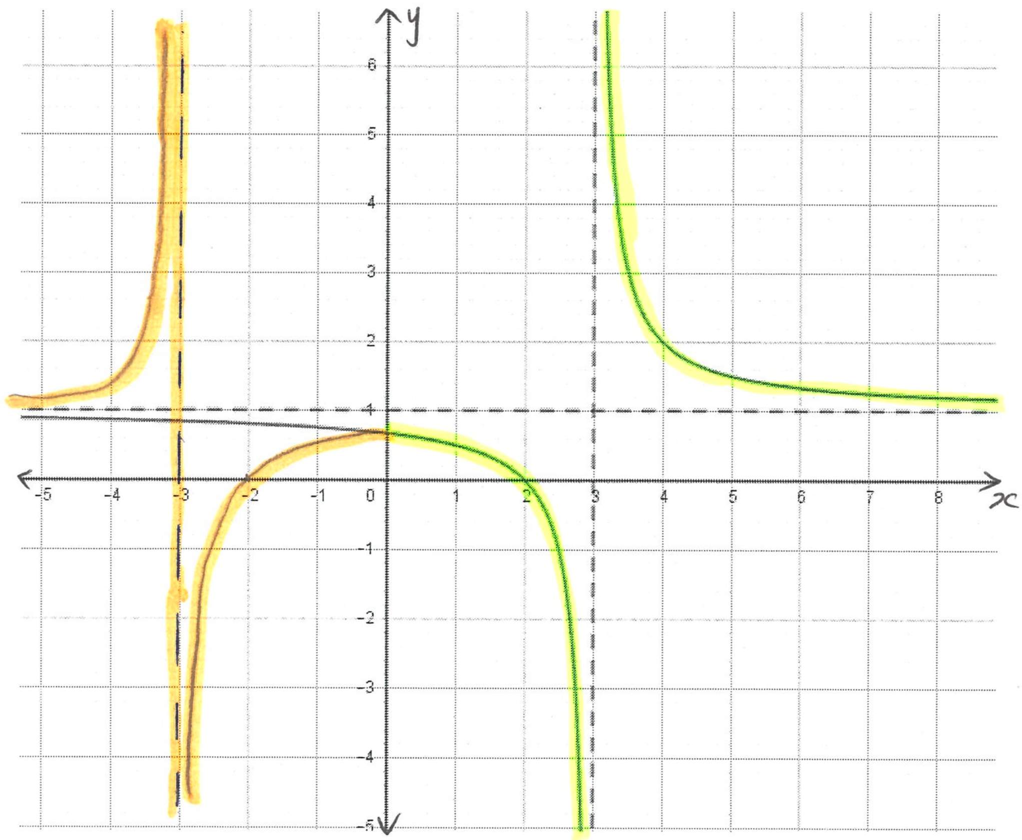
$\underline{\underline{13x^2 + 13y^2 + 10xy = 144}}$

Question 6(c) (i) $y = |f(x)|$



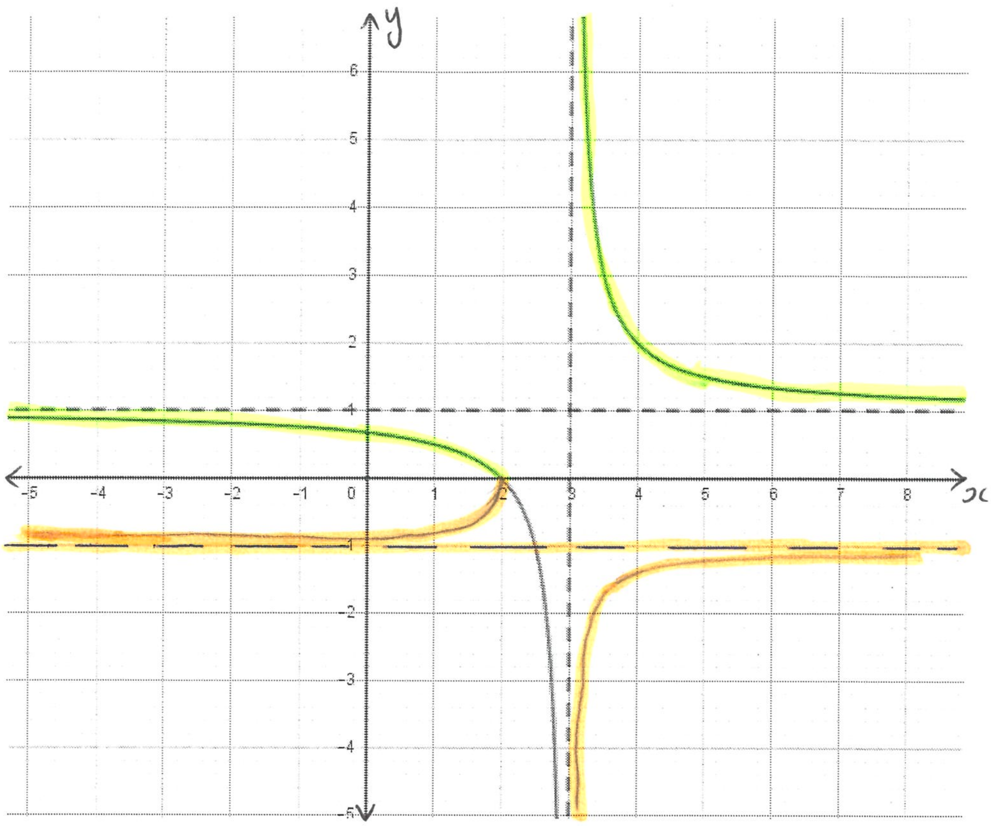
3

(ii) $y = f(|x|)$



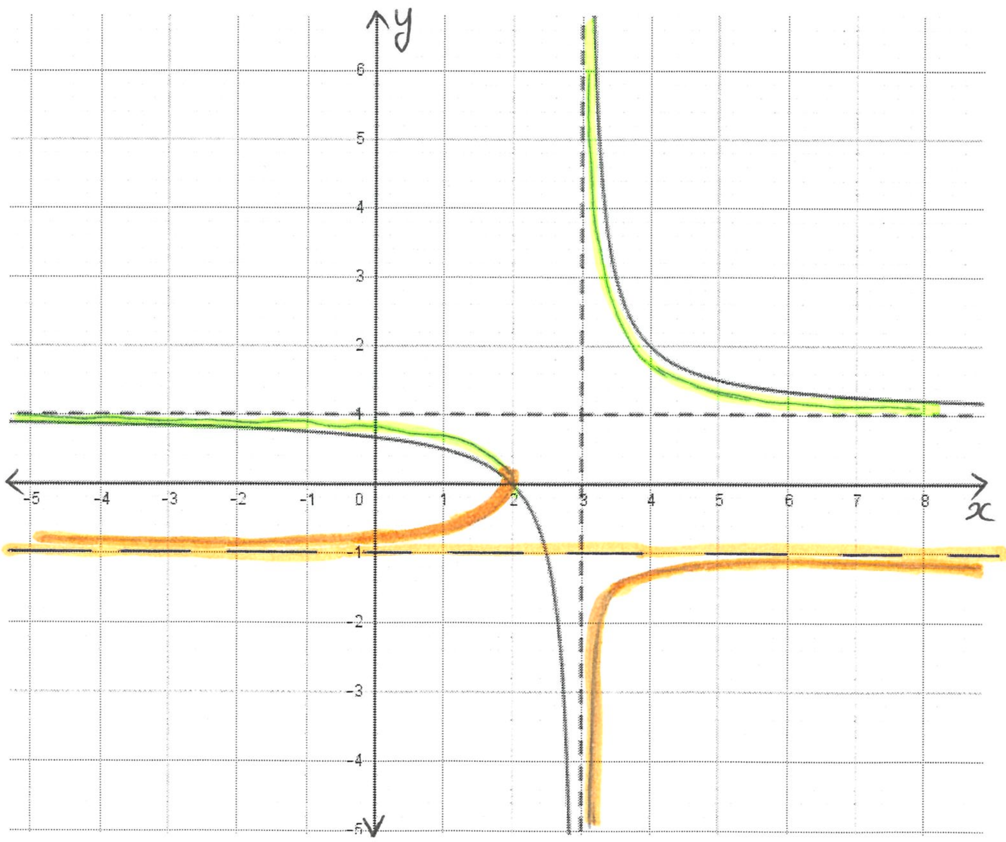
4

(iii) $|y| = f(x)$



4

(iv) $y^2 = f(x)$



4

Question 7 (26 marks)

(a) (i) $y = \sqrt{4-x}$

$x = \sqrt{4-y}$

$x^2 = 4-y$

$y = 4-x^2$ (2)

$f^{-1}(x) = 4-x^2$

(ii) $f(f^{-1}(x))$

$= f(4-x^2)$

$= \sqrt{4-(4-x^2)}$

$= \sqrt{4-4+x^2}$ (2)

$= \sqrt{x^2} = x$

$f^{-1}(f(x))$

$f^{-1}(\sqrt{4-x})$ (2)

$= 4-(4-x)$

$= 4-4+x = x$

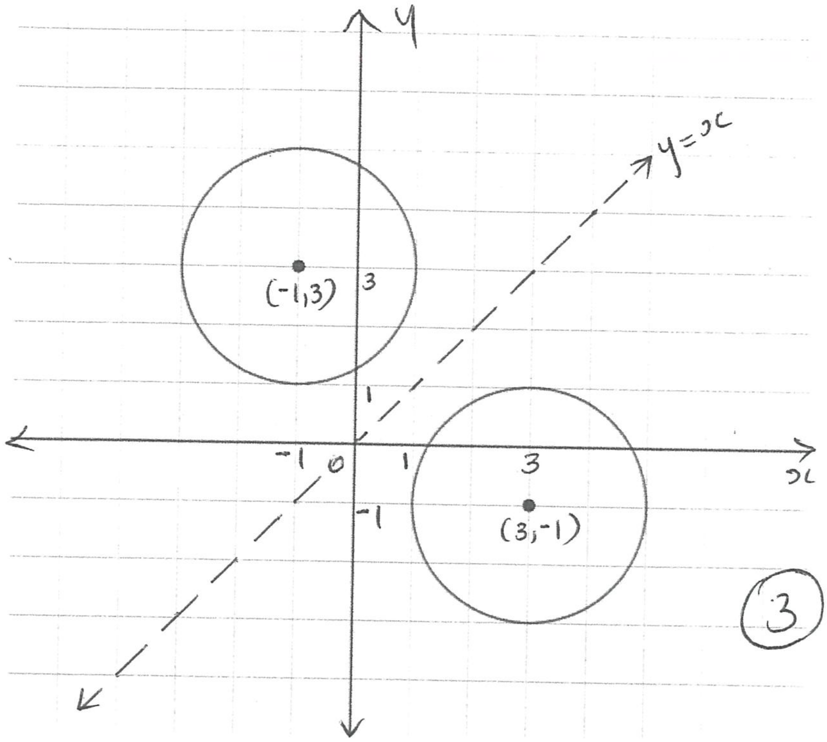
b(i) $(x-3)^2+(y+1)^2=4$

Interchanging x and y

$(y-3)^2+(x+1)^2=4$

(1)

(ii)



(3)

(iii) $(x-3)^2+(y+1)^2=4$

$D: 1 \le x \le 5, \quad R: -3 \le y \le 1$ (4)

$(x+1)^2+(y-3)^2=4$

$D: -3 \le x \le 1, \quad R: 1 \le y \le 5$

(c)(i) $f(x) = x^2-4x$

$= x(x-4)$

x intercepts $x=0, 4$

axis of symmetry $x=2$ (2)

when $x=2$, $f(x) = 4-8=-4$

Vertex $= (2,-4)$

(ii) $D_f: x \le 2, \quad R_f: y \ge -4$ (2)

(iii) $D_{f^{-1}}: x \ge -4, \quad R_{f^{-1}}: y \le 2$ (2)

(iv) $y = x^2 - 4x$

$x = y^2 - 4y$

$x + 4 = y^2 - 4y + 4$

$x + 4 = (y - 2)^2$

$y - 2 = \pm\sqrt{x+4}$

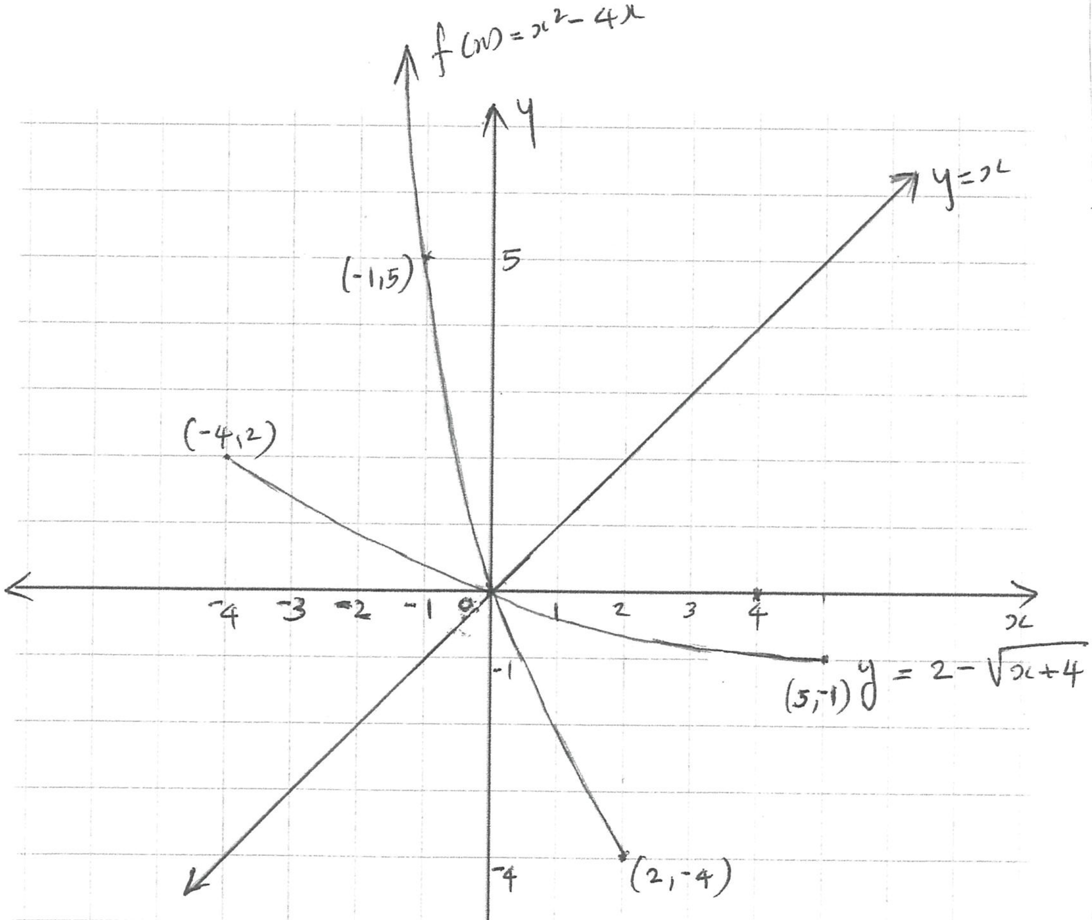
$y = 2 \pm \sqrt{x+4}$

$y = 2 - \sqrt{x+4}$ (3)

$(\because R_{f^{-1}} \ y \leq 2)$

$f^{-1}(x) = 2 - \sqrt{x+4}$

(v)



(3)

Question 8 (28 marks)

(a) $\alpha+\beta+\gamma = \frac{5}{2}$ 1

(ii) $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{-3}{2}$ 1

(iii) $\alpha\beta\gamma = \frac{-1}{2}$ 1

(iv) $6\alpha+6\beta+6\gamma-5$

$= 6(\alpha+\beta+\gamma)-5$

$= 6\times\frac{5}{2}-5$ 2

$= 10$

(v) $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$ 2

$= \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$

$= \frac{5}{2} \div \frac{-1}{2} = -5$

(vi) $(\alpha-2)(\beta-2)(\gamma-2)$

$=(\gamma-2)(\alpha\beta-2\alpha-2\beta+4)$

$= \alpha\beta\gamma-2\alpha\gamma-2\beta\gamma+4\gamma-2\alpha\beta+4\alpha+4\beta-8$ 2

$= \alpha\beta\gamma - 2(\alpha\gamma+\alpha\beta+\beta\gamma)+4(\alpha+\beta+\gamma)-8$

$= \frac{-1}{2} - 2\times\frac{-3}{2} + 4\times\frac{5}{2} - 8$

$= 4\frac{1}{2}$

(b). $p(x) = 2x^4 + 5x^3 + 3x^2 - x - 1$

(i) $p'(x) = 8x^3 + 15x^2 + 6x - 1$

$p''(x) = 24x^2 + 30x + 6$

Triple root of $p(x)$ is a simple zero of $p''(x)$

$p''(x) = 0$

$24x^2 + 30x + 6 = 0$

$6(4x^2 + 5x + 1) = 0$

$4x^2 + 5x + 1 = 0$

$(x+1)(4x+1) = 0$ (4)

$x = -1$ or $x = -\frac{1}{4}$

$p(-1) = 2 - 5 + 3 + 1 - 1 = 0$

$\therefore x = -1$ is the triple zero of $p(x)$

(ii) Let α be the 4^{th} zero.

$\alpha - 3 = \frac{-5}{2}$ (2)

$\alpha = 3 - \frac{5}{2} = \frac{1}{2}$

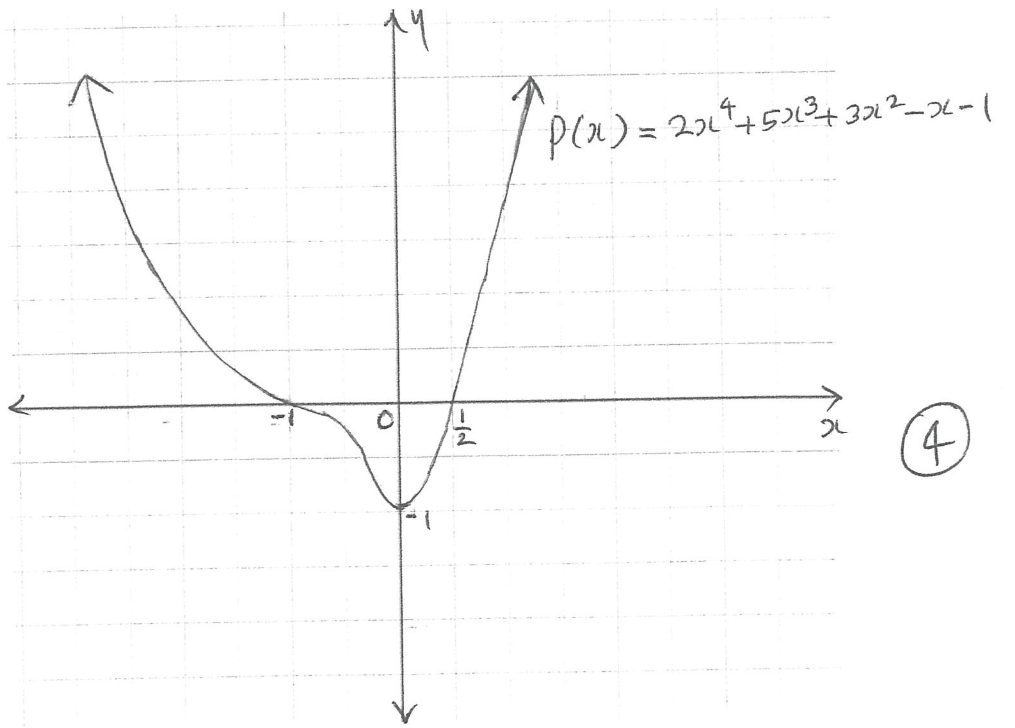
(iii) $p(x) = 2\left(x^4 + \frac{5}{2}x^3 + \frac{3}{2}x^2 - \frac{x}{2} - \frac{1}{2}\right)$

$= 2(x+1)^3\left(x - \frac{1}{2}\right)$

$= 2(x+1)^3\left(\frac{2x-1}{2}\right)$

$= (x+1)^3(2x-1)$ (2)

(iv)



(4)

(c)(i) $P(x) = (x+1)(x-3)Q(x) + 4-2x$

$P(-1) = 0 + 4 - 2\times -1 = 6$ (2)

(ii) $P(3) = 4 - 2\times 3 = -2$ (2)

(d) $3x^3 - 17x^2 - 8x + 12 = 0$

Let α, β and $\frac{4}{\alpha}$ be the roots.

$\alpha + \beta + \frac{4}{\alpha} = \frac{17}{3}$ — $\boxed{1}$

$\alpha\beta \times \frac{4}{\alpha} = -4$ — $\boxed{2}$

$4\beta = -4$

$\beta = -1$

Substitute $\beta = -1$ in $\boxed{1}$

$\alpha + \frac{4}{\alpha} - 1 = \frac{17}{3}$

$\alpha + \frac{4}{\alpha} = \frac{20}{3}$

$\alpha^2 + 4 = \frac{20\alpha}{3}$

$3\alpha^2 + 12 = 20\alpha$

$3\alpha^2 - 20\alpha + 12 = 0$

$(\alpha - 6)(3\alpha - 2) = 0$ (3)

$\alpha = 6$ or $\alpha = \frac{2}{3}$

when $\alpha = 6$, $\frac{4}{\alpha} = \frac{2}{3}$

when $\alpha = \frac{2}{3}$, $\frac{4}{\alpha} = 6$

The roots are

$6, \frac{2}{3}$ and -1

Question 9 (22 marks)

(a) $\frac{8}{(n+2)!} - \frac{1}{n!}$

$= \frac{8 - (n+1)(n+2)}{(n+2)!}$ (2)

$= \frac{8 - (n^2+3n+2)}{(n+2)!}$

$= \frac{6 - n^2 - 3n}{(n+2)!}$

(b)(i) $26^3 \times 10^3 = 17576000$ (2)

(ii) $26 \times 25 \times 24 \times 10 \times 9 \times 8$

$= 11232000$ (1)

(iii) $26^2 \times 10^2 = 67600$ (1)

(iv) $25 \times 24 \times 9 \times 8 = 43200$ (1)

(v) $21^3 \times 4^3 = 592704$ (1)

(vi) $21 \times 20 \times 19 \times 4 \times 3 \times 2 = 191520$ (1)

(c) $8 \times 7 \times 6 \times 5 \times 8 \times 7 \times 10!$

$= 3.4 \times 10^{11}$ (3)

(d)(i) ${}^{10}C_4 \times {}^{9}C_5 = 26460$ (2)

(ii) Number of committees in which both Noah and Daisy are present

$= {}^{9}C_3 \times {}^{8}C_4 = 5880$ (2)

Number of committees not containing both Noah and Daisy

$= 26400 - 5880$ (2)

$= \underline{\underline{20580}}$

(e) (i) COOLANGATTA

(i) $\frac{11!}{2! \times 3! \times 2!} = 1663200$ (2)

(ii) $\frac{10!}{3! \times 2!} = 302400$ (2)

(iii) $\frac{8!}{2! \times 2!} \times \frac{9 \times 8 \times 7}{3!}$

$= 846720$ (2)

Question 10 (20 marks)

(a) (i) 1M 3W

${}^6C_1 \times {}^5C_3 = 60$

2M 2W

${}^6C_2 \times {}^5C_2 = 150$

3M 1W (3)

${}^6C_3 \times {}^5C_1 = 100$

Total $= 310$

(ii) $\frac{60}{310} = \frac{6}{31}$ (2)

(b) (i) $13! = 6227020800$ (1)

(ii) $6! \times 7! = 3628800$ (2)

(iii) $12! \times 2! = 958003200$ (2)

(iv) $12! = 479001600$ (1)

(v) $13! - 12! \times 2 = 5269017600$ (2)

(c) 3 odd numbers together

$= 5! \times 3! \times 4$

4 odd numbers together $= 4! \times 4!$

p(atleast 3 odd numbers together)

$= \frac{({}^4C_3 \times 3! \times {}^4P_2 \times 3!) + (4! \times 4!)}{7!}$ (3)

$= \frac{16}{35}$

(d) $\frac{200}{{}^7C_3} = 5\frac{5}{7}$ (2)

At least six students is to be chosen

(e) The pairs which give a sum of 11 are $\{1,10\}$ $\{2,9\}$ $\{3,8\}$ $\{4,7\}$ $\{5,6\}$

If 6 integers are selected from A, at least two must be from the same subset. (2)