GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION

YEAR 11

TASK 3

- **Instructions**

  - Time Allowed – 1 hour.
  - Write using black pen.
  - Calculators approved by NESA may be used
  - This task is open book but NOT open technology (ICT).
  - Show all relevant mathematical reasoning and/or calculations.
  - Marks may be deducted for careless or badly arranged work.

You must have your
- camera on (no virtual backgrounds)
- microphone off
- speakers on
- phone on the desk facing upwards and on silent and be visible to the teachers

Show all relevant mathematical reasoning and/or calculations.

Question 1 (15 marks)

a. Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$.

(i) Find the remainder when P(x) is divided by (x + 2). [1]

(ii) Use the factor theorem to show that $(x + 3)$ is a factor of $P(x)$. [1]

(iii) Factorise $P(x)$ completely. [2]

b. If α β and γ are the roots of $2x^3 + 3x^2 - 5x + 1 = 0$ find:

(i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ [2]

(ii) $\alpha^2 + \beta^2 + \gamma^2$ [2]

(iii) $(\alpha\beta)^2\gamma + \alpha(\beta\gamma)^2 + (\alpha\gamma)^2\beta$ [2]

c. The polynomial $P(x) = ax^4 + bx^3 + cx^2 + e$

has a remainder -3 when divided by $x - 1$.

The polynomial has a double root at $x = -1$.

Find the value of $4a + 2c$. [2]

d. Let $P(x) = (x + 1)(x - 3)Q(x) + a(x + 1) + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x + 1)$ the remainder is -11.

When $P(x)$ is divided by $(x - 3)$ the remainder is 1.

(i) Find the value of b [1]

(ii) What is the remainder when $P(x)$ is divided by $(x + 1)(x - 3)$? [2]

Question 2 (15 marks)

a. In how many ways can the letters of the word **DEDUCED** be arranged? [2]

b. How many numbers greater than 3 000 can be formed from the digits 1, 3, 5, 7, 9 if no repetition is permitted? [2]

c. In how many ways can a committee of 3 women and 4 men be chosen from 8 women and 7 men if two particular women cannot serve on the committee together? [3]

d. Mr and Mrs Antony (the hosts) and 6 guests sit around a circular dining table. In how many ways can they be arranged if the two hosts are separated? [2]

e. At a fund-raising dinner, each rectangular table has 9 seats, 5 facing the stage and 4 with their backs to the stage. In how many ways can nine people be seated at a particular table if:

(i) Angelo and Agam must sit on the same side? [2]

(ii) Angelo and Agam must sit on opposite sides? [2]

f. A box contains 2 red, 3 blue, 9 green and 9 yellow balls. What is the minimum number of balls that must be drawn to be certain that there will be 4 balls of the same colour? [2]

Question 3 (15 marks)

a. Simplify $\dfrac{5^n + 5^{n+2}}{5^{n+1}}$ [2]

b. Solve for x: $2\log_7 5 = \log_7 2x - \log_7 3$ [2]

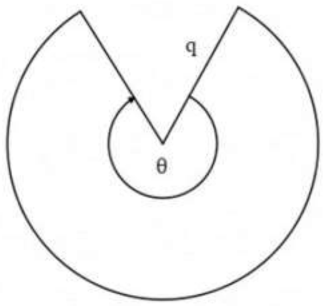
c. A function is described by the parametric equations

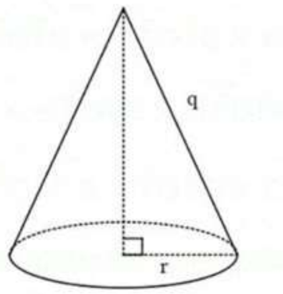
$x = t + 1; y = t^2 - 1$

Find the cartesian equation of this function. [2]

d. A sector with angle θ with fixed radius q is folded to form a cone with radius r, as shown below.

Let the area of the sector be A and the area of the base of the cone be B.





(i) Show that $r = \dfrac{q\theta}{2\pi}$. [1]

(ii) Find the exact value of θ such that $3A^2 = 4B^2$. [2]

e. Solve:

(i) $2\cos(2\theta - 30°) = \sqrt{3}$; $0° \leq \theta \leq 360°$ [3]

(ii) $2\log_2(x + 5) - \log_2(x + 9) = 1$ [3]

End of Examination

2021 GHS Year 11 Mathematics Ext Task 3

SOLUTIONS

Question 1

a) $P(x) = 2x^3 + 3x^2 - 29x - 60$

i) $P(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$

$= -6$

$\therefore$ Remainder is -6.

ii) $P(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$

$= 0$

$\therefore (x+3)$ is a factor of $P(x)$.

iii)

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 20 \\
\hline
x+3 & 2x^3 + 3x^2 - 29x - 60 \\
 & -\underline{2x^3 + 6x^2} \\
 & -3x^2 - 29x \\
 & -\underline{-3x^2 - 9x} \\
 & -20x - 60 \\
 & -\underline{-20x - 60} \\
 & \,
\end{array}
$$

$\therefore 2x^3 + 3x^2 - 29x - 60 = (x+3)(2x^2 - 3x - 20)$

$\therefore P(x) = (x+3)(2x+5)(x-4)$

b) $2x^3 + 3x^2 - 5x + 1 = 0$

i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

$= \dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \dfrac{-5/2}{-1/2} = 5$

ii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= \left(\frac{-3}{2}\right)^2 - 2\left(\frac{-5}{2}\right)$

$= \frac{29}{4} = 7\frac{1}{4}$

iii) $(\alpha\beta)^2\gamma + \alpha(\beta\gamma)^2 + (\alpha\gamma)^2\beta$

$= \alpha\beta\gamma\,(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= \left(-\frac{1}{2}\right)\left(\frac{-5}{2}\right)$

$= \frac{5}{4}$

c) $P(x) = ax^4 + bx^3 + cx^2 + e$

$P'(x) = 4ax^3 + 3bx^2 + 2cx$

$P(1) = a+b+c+e = -3$ —— (1)

$P(-1) = a-b+c+e = 0$ —— (2)

$P'(-1) = -4a+3b-2c = 0$

$\therefore\ 4a+2c = 3b$ —— (3)

(1) − (2)

$2b = -3$

$b = \frac{-3}{2}$

substitute $b = \frac{-3}{2}$ into (3)

$4a+2c = 3\left(\frac{-3}{2}\right)$

$\therefore\ 4a+2c = \frac{-9}{2}$

d) $P(x) = (x+1)(x-3)\,Q(x) + a(x+1) + b$

i) $P(-1) = (0)(-4)\,Q(x) + a(0) + b = -11$

$\therefore\ b = -11$

ii) $P(3) = (0)\,Q(x) + 4a + b = 1$

$4a + b = 1$

$4a - 11 = 1$

$4a = 12$

$a = 3$

when $P(x)$ is divided by $(x+1)(x-3)$

$R(x) = a(x+1) + b$

$= 3(x+1) - 11$

$= 3x + 3 - 11$

$= 3x - 8$

Question 2

a) DEDUCED

7 letters
3 Ds
2 Es.

$= \dfrac{7!}{3!\,2!}$

$= 420$ ways.

b) 1, 3, 5, 7, 9 ; Numbers > 3000

5 digit numbers $= 5! = 120$

4 digit numbers $= 4 \times 4!$

$= 96$

Total $= 120 + 96$

$= 216$ numbers.

c) only 1 of the 2 particular woman chosen

$$^{6}C_{2} \times 2 \times {}^{7}C_{4}$$

$$= 1050$$

both of the women <u>not</u> chosen.

$$^{6}C_{3} \times {}^{7}C_{4}$$

$$= 700$$

Total ways $= 1750$

d) Hosts separated = Total − hosts together.

$$= 7! - (1 \times 2 \times 6!)$$

$$= 3600$$

(OR) $1 \times 5 \times 6! = 3600$

e) i) same side

Both facing stage : $^{5}P_{2} \times 7!$

Both not facing : $^{4}P_{3} \times 7!$

Both on the same side $= 161\ 280$

ii) on opposite sides :

$$= 2 \left[{}^{5}P_{1} \times {}^{4}P_{1} \times 7! \right]$$

$$= 201\ 600$$

f) $2R, \ 3B, \ 9G, \ 9Y$

Minimum number drawn to guarantee 4 balls of same colour.

$$2+3+3+3+1$$
$$= 12$$

Question 3

a) $$\frac{5^n + 5^{n+2}}{5^{n+1}}$$

$$= \frac{5^n(1+5^2)}{5^n \times 5^1}$$

$$= \frac{26}{5}$$

b) $$2\log_7 5 = \log_7 2x - \log_7 3$$

$$\log_7(5^2) = \log_7\left(\frac{2x}{3}\right)$$

$$25 = \frac{2x}{3}$$

$$x = \frac{75}{2}$$

c) $x = t+1$ ① ; $y = t^2 - 1$ ②

$\Rightarrow t = x - 1$

substitute $t = x - 1$ into ②

$$y = (x-1)^2 - 1$$

$$= x^2 - 2x + 1 - 1$$

$$y = x^2 - 2x$$

d) i)

the arc length of the sector will become the circumference of the base of the cone.

$$\therefore q\theta = 2\pi r$$

$$r = \frac{q\theta}{2\pi}$$

ii)

$$3A^2 = 4B^2$$

$$A = \frac{1}{2}q^2\theta \quad ; \quad B = \pi r^2$$

$$r = \frac{q\theta}{2\pi} \Rightarrow B = \pi\left(\frac{q\theta}{2\pi}\right)^2$$

$$= \frac{q^2\theta^2}{4\pi}$$

$$3A^2 = 3\left(\frac{1}{2}q^2\theta\right)^2$$

$$= \frac{3q^4\theta^2}{4}$$

$$4B^2 = 4\left(\frac{q^2\theta^2}{4\pi}\right)^2$$

$$= \frac{q^4\theta^4}{4\pi^2}$$

Solving $\quad 3A^2 = 4B^2$

$$\frac{3q^4\theta^2}{4} = \frac{q^4\theta^4}{4\pi^2}$$

$$3\pi^2 = \theta^2$$

$$\theta = \sqrt{3}\,\pi \quad ; \quad \theta > 0$$

e) $\quad 2\cos(2\theta - 30^\circ) = \sqrt{3} \quad ; \quad 0^\circ \leq \theta \leq 360^\circ$

$$\cos(2\theta - 30^\circ) = \frac{\sqrt{3}}{2}$$

$$0^\circ \leq 2\theta \leq 720^\circ$$

$$-30^\circ \leq 2\theta - 30^\circ \leq 690^\circ$$

$$2\theta - 30^\circ = -30^\circ, 30^\circ, 330^\circ, 390^\circ, 690^\circ$$

$$2\theta = 0^\circ, 60^\circ, 360^\circ, 420^\circ, 720^\circ$$

$$\theta = 0^\circ, 30^\circ, 180^\circ, 210^\circ, 360^\circ$$

f) $2\log_2(x+5) - \log_2(x+9) = 1$

$\log_2(x+5)^2 - \log_2(x+9) = 1$

$\log_2\left(\frac{(x+5)^2}{x+9}\right) = 1$

$$\frac{(x+5)^2}{x+9} = 2$$

$(x+5)^2 = 2(x+9)$

$x^2 + 10x + 25 = 2x + 18$

$x^2 + 8x + 7 = 0$

$(x+7)(x+1) = 0$

$x = -7, -1.$

$\therefore\ x = -1$; since $x > -5$