GIRRAWEEN HIGH SCHOOL
MATHEMATICS
Extension
2023
Year 11 Yearly Examination

- **Reading time – 10 minutes**
- **Working time – 2 hours**

Instructions:

- There are 11 questions in this paper. All questions are compulsory.
- Use blue or black pen.
- Write all your answers in the Answer Booklets provided.
- For Questions 1 – 5, fill in the circle corresponding to the correct answer in your answer booklet.
- For Questions 6 – 11, start each question on a new page.
- Write on both sides of the paper.
- Show all necessary working.
- Board-approved calculators may be used.
- Mathematics reference sheets are provided.
- Marks may be deducted for careless or badly arranged work.

Section I

5 marks
Attempt questions 1 - 5

Use the multiple-choice answer sheet provided for questions 1- 5.

1. Which of the following is a factor of $6x^3 - 5x^2 - 34x - 15$?

   A. $x + 1$

   B. $x - 3$

   C. $x - 2$

   D. $x + 5$

2. What is the Cartesian equation of the curve with parametric equations $x = 20t$ and $y = 40t - 5t^2$?

   A. $y = 2x - \dfrac{x^2}{400}$

   B. $y = 2x - \dfrac{x^2}{200}$

   C. $y = 2x - \dfrac{x^2}{160}$

   D. $y = 2x - \dfrac{x^2}{80}$

3. Which of the following is the inverse function of $f(x) = 3 - \dfrac{1}{2x + 6}$?

   A. $f^{-1}(x) = -3 - \dfrac{1}{2x - 6}$

   B. $f^{-1}(x) = 3 - \dfrac{1}{6 - 2x}$

   C. $f^{-1}(x) = 6 - \dfrac{2}{x + 4}$

   D. $f^{-1}(x) = 6 - \dfrac{1}{6 - 2x}$

4. In how many ways can the letters of the word TROOPS be arranged if the two Os are to be separated?

   A. 60

   B. 240

   C. 300

   D. 600

5. What is the seventh term in the expansion of $(x + 3y)^{11}$?

   A. ${}^{11}C_7(3)^7x^7y^6$

   B. ${}^{11}C_7(3)^7x^6y^7$

   C. ${}^{11}C_6(3)^6x^6y^5$

   D. ${}^{11}C_6(3)^6x^5y^6$

Section II

105 marks

Attempt questions 6 – 11.

Start each question on a new page in the answer booklet provided.

Your responses should include relevant mathematical reasoning and/or calculations.

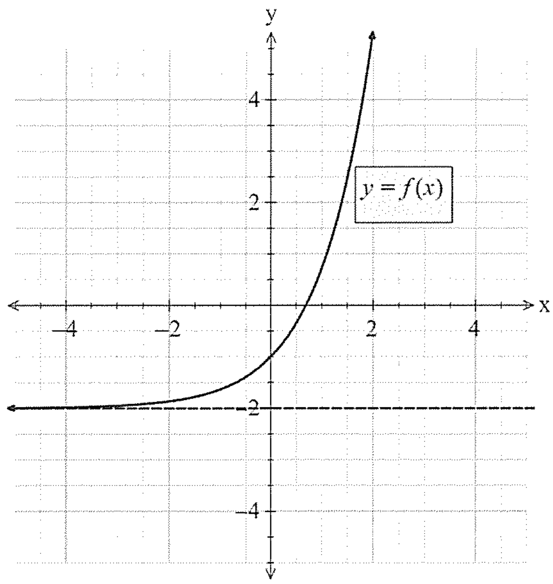
Question 6 (20 marks)

a. Solve:

(i) $\dfrac{2x + 1}{x - 1} \geq 3$ [3]

(ii) $\dfrac{x^2 + 6}{x} < 5$ [3]

b. The diagram below shows the graph of $f(x) = e^x - 2$ and it's horizontal asymptote $y = -2$.



On separate diagrams, on the templates provided in the Answer Booklet, sketch the following graphs, clearly showing the intercepts with the axes and the equations of any asymptotes.

(i) $y = \dfrac{1}{f(x)}$ [2]

(ii) $y^2 = |f(x)|$ [2]

c. A curve is represented by the parametric equations:

$x = \cos\theta + \sin\theta$, $y = \cos\theta - \sin\theta$.

Find the Cartesian equation of the curve. [2]

d. (i) Explain why $f(x) = x^2 - 3x$ does not have an inverse function. [1]

(ii) Find the largest possible domain of $f(x)$ that includes $x = 0$ for the inverse function to exist. [1]

(iii) Find the expression for $f^{-1}(x)$. [2]

(iv) State the domain and range of $f^{-1}(x)$. [2]

(v) Hence, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same number plane (provided in the Answer Booklet). [2]
Clearly label the endpoints and the intercepts with the axes for each graph.

Question 7 (15 marks)

a. In how many ways can the letters of the word GEOMETRY be arranged in a straight line if the vowels must occupy the 2nd, 4th and 6th positions. [2]

b. There are 6 women and 10 men available to serve in a committee.

(i) How many different committees of 8 people can be chosen? [1]

(ii) How many different committees of 8 people with equal numbers of men and women can be chosen? [2]

(iii) How many different committees of 8 people with equal numbers of men and women can be chosen if Mary and Peter must be in the committee? [2]

c. A group of 10 people arrive to have dinner at a restaurant. The only seating available to them are two circular tables, one that seats six people and another that seats four. Assuming that the seating arrangement is random, find:

(i) The number of ways in which Omee and Adi will be seated at the same table. [2]

(ii) The probability that Omee and Adi will be seated at the same table. [2]

d. A soccer team's record for the 2023 season was 16 wins and 4 losses.
None of their games ended in a draw. Prove that the team must have won at least 4 games in a row somewhere during the season. [2]

e. The members of a club voted for a new president.
There were 15 candidates for the position of president and 3543 members voted.
Each member voted for one candidate only.
One candidate received more votes than anyone else so became the new president.
What is the smallest number of votes the new president could have received?
Explain your answer. [2]

Question 8 (18 marks)

a. Express $P(x) = x^4 - 3x^3 - 2x^2 - 5x$ in the form

$P(x) = x(x-4)Q(x) + R(x)$ [2]

b. Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$

(i) Find the remainder when $P(x)$ is divided by $(x+2)$. [1]

(ii) Show that $(x+3)$ is a factor of $P(x)$. [1]

(iii) Factorise $P(x)$ completely. [3]

c. The polynomial $P(x) = x^4 - x^3 - 7x^2 + 13x - 6$ has only integer roots.

(i) Prove that $x = 1$ is a root of multiplicity 2 of $P(x)$. [2]

(ii) Factorise $P(x)$ completely. [2]

(iii) Sketch the graph of $y = P(x)$ on the number plane provided in the Answer Booklet. [2]

(iv) Hence, solve $P(x) \geq 0$. [2]

d. The polynomial $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ has a double root at $x = 3$.

Find the values of a and b, where a and b are real numbers. [3]

Question 9 (17 marks)

a. The polynomial $P(x) = x^3 - 10x^2 - 39x + 180$ has roots α, β and γ. Find:

(i) $\alpha + \beta + \gamma$ [1]

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ [1]

(iii) $\alpha\beta\gamma$ [1]

(iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ [2]

(v) $\alpha^2 + \beta^2 + \gamma^2$ [2]

(vi) $(\alpha\beta)^{-2} + (\beta\gamma)^{-2} + (\alpha\gamma)^{-2}$ [2]

b. The polynomial $P(x) = x^4 + 4x^3 - 3x^2 + 5x - 7$ has roots α, β, γ and δ.

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$. [3]

c. Let $P(x) = x^3 + 3x^2 + ax + b$, where a and b are integers.

When $P(x)$ is divided by $(x+1)$, the remainder is 4. [2]

When $P(x)$ is divided by $(x-1)$, the remainder is -2.

What is the remainder when $P(x)$ is divided by $(x^2 - 1)$?

d. The equation $x^3 + 3ax + b = 0$ has two equal roots.

Prove that $b^2 + 4a^3 = 0$. [3]

Question 10 (17 marks)

a. Write the expansion of $(2x - y)^5$. [2]

b. Expand $\left(\frac{x}{2} - 3\right)^4$. [3]

c. Find the value of the constant a such that $8\,750x^4$ is a term in the binomial expansion of $(a + 2x)^7$. [3]

d. Find the values of a and b if $\left(2 - \sqrt{3}\right)^5 = a + b\sqrt{3}$. [3]

e. What is the coefficient of x^5 in the expansion of $(1 - 3x + 2x^3)(1 - 2x)^6$? [3]

f. (i) Find the general term in the expansion of $\left(x + \frac{1}{x}\right)^8$. [1]

(ii) Find the coefficient of x^2 in the expansion of $\left(x + \frac{1}{x}\right)^8$. [2]

Question 11 is on the next page.

Question 11 (18 marks)

a. Find the value of n if ${}^{n}\mathrm{C}_2 + {}^{n+1}\mathrm{C}_1 = 22$ [3]

b. By considering $(1 + x)^{11} = (1 + x)^3(1 + x)^8$, show that

${}^{11}\mathrm{C}_5 = {}^{8}\mathrm{C}_5 + {}^{3}\mathrm{C}_1\,{}^{8}\mathrm{C}_4 + {}^{3}\mathrm{C}_2\,{}^{8}\mathrm{C}_3 + {}^{8}\mathrm{C}_2.$ [3]

c. In the expansion of $(5x + 2)^{20}$, the coefficients of x^k and x^{k+1} are equal.

Find the value of k. [3]

d. By considering the binomial expansion

$(1 + x)^n = 1 + {}^{n}\mathrm{C}_1\, x + {}^{n}\mathrm{C}_2\, x^2 + \ldots + {}^{n}\mathrm{C}_n\, x^n$, show that

${}^{n}\mathrm{C}_1 + 2\,{}^{n}\mathrm{C}_2 + \ldots + n\,{}^{n}\mathrm{C}_n = n\,2^{n-1}$ [2]

e. By considering the binomial expansion

$(1 + x)^{2n} = {}^{2n}\mathrm{C}_0 + {}^{2n}\mathrm{C}_1 x + {}^{2n}\mathrm{C}_2 x^2 + \ldots + {}^{2n}\mathrm{C}_n x^n + \ldots + {}^{2n}\mathrm{C}_{2n}\, x^{2n}$,

show that:

$$2^{2n} = 2\sum_{r=0}^{n} {}^{2n}\mathrm{C}_r - {}^{2n}\mathrm{C}_n$$

[3]

f. (i) By using binomial expansion,

show that $(q + p)^n - (q - p)^n = 2\,{}^{n}\mathrm{C}_1\, q^{n-1} p + 2\,{}^{n}\mathrm{C}_3\, q^{n-3} p^3 + \ldots$ [2]

(ii) Find the last term in the expansion when n is odd. [1]

(iii) Find the last term in the expansion when n is even. [1]

End of Examination

GMS 2023 Year 11 Extension Yearly Examination Solutions

Section I

1. B 2. D 3. A 4. B 5. D

1. $P(3) = 6(3)^3 - 5(3)^2 - 34(3) - 15$

$= 0$

$\therefore (x-3)$ is a factor $\boxed{B}$

2. $x = 20t \Rightarrow t = \frac{x}{20}$

$y = 40\left(\frac{x}{20}\right) - 5\left(\frac{x}{20}\right)^2$

$= 2x - \frac{x^2}{80}$ $\boxed{D}$

3. f^{-1}: $x = 3 - \frac{1}{2y+6}$

$2y + 6 = \frac{1}{3-x}$

$2y = \frac{1}{3-x} - 6$

$y = -3 + \frac{1}{6-2x} = -3 - \frac{1}{2x-6}$ $\boxed{A}$

4. n(Os separated) = n(Total) − n (Os together)

$= \frac{6!}{2!} - 5!$

$= 240$ $\boxed{B}$

5. $T_7 \Rightarrow r = 6$

${}^{11}C_6 \, x^5 (3y)^6$ $\boxed{D}$

Section II

Question 6 (21 marks)

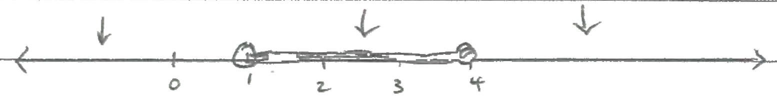
a) $\dfrac{2x+1}{x-1} \geq 3$

$x \neq 1$

Solve $\dfrac{2x+1}{x-1} = 3$

$2x+1 = 3x-3$

$x = 4$



Test $x=0$: $\dfrac{1}{-1} \geq 3$ ✗

Test $x = 2$: $\dfrac{5}{1} \geq 1$ ✓

Test $x = 5$: $\dfrac{11}{4} \geq 3$ ✗

$\therefore$ Solution: $1 < x \leq 4$

/3

6 a) and 6 (b) Done reasonably well by majority of students.

• A small number of students need to learn/revise this skill as they had no idea how to solve these inequalities.

b) $\dfrac{x^2+6}{x} < 5$

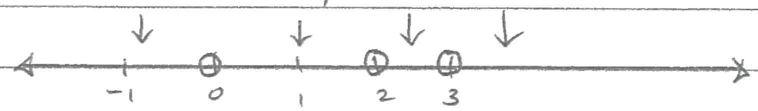
$x \neq 0$

Solve $\dfrac{x^2+6}{x} = 5$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$



Test $x = -1$: $\dfrac{7}{-1} < 5$ ✓

Test $x = 1$: $\dfrac{7}{1} < 5$ ✗

Test $x = 2\frac{1}{2}$: $\dfrac{\frac{25}{4}+6}{\frac{5}{2}} < 5$ ✓

Test $x = 4$: $\dfrac{22}{4} < 5$ ✗

* A number of students did not show full working out (mainly not showing tests for points)

$\therefore$ Solution: $x < 0$, $2 < x < 3$

/3

Question 6

a, b ← alternative method.

a) $\dfrac{2x+1}{x-1} \geq 3$

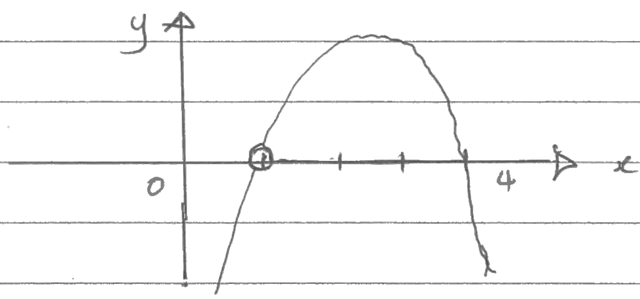
Multiplying both sides by $(x-1)^2$:

$(2x+1)(x-1) \geq 3(x-1)^2$; $x \neq 1$

$(2x+1)(x-1) - 3(x-1)^2 \geq 0$

$(x-1)[2x-1-3(x-1)] \geq 0$

$(x-1)(4-x) \geq 0$



Some students who used this method showed very little understanding of the process.

Solution : $1 < x \leq 4$ (from graph)

b) $\dfrac{x^2+6}{x} < 5$

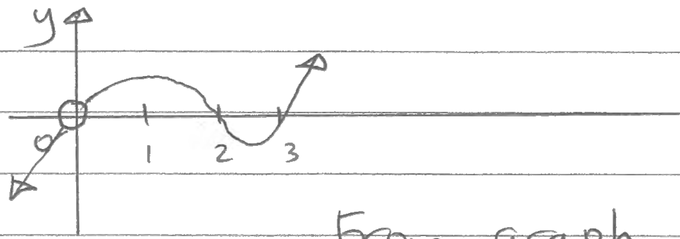
Multiplying both sides by x^2

$x(x^2+6) < 5x^2$; $x \neq 0$

$x(x^2+6) - 5x^2 < 0$

$x(x^2-5x+6) < 0$

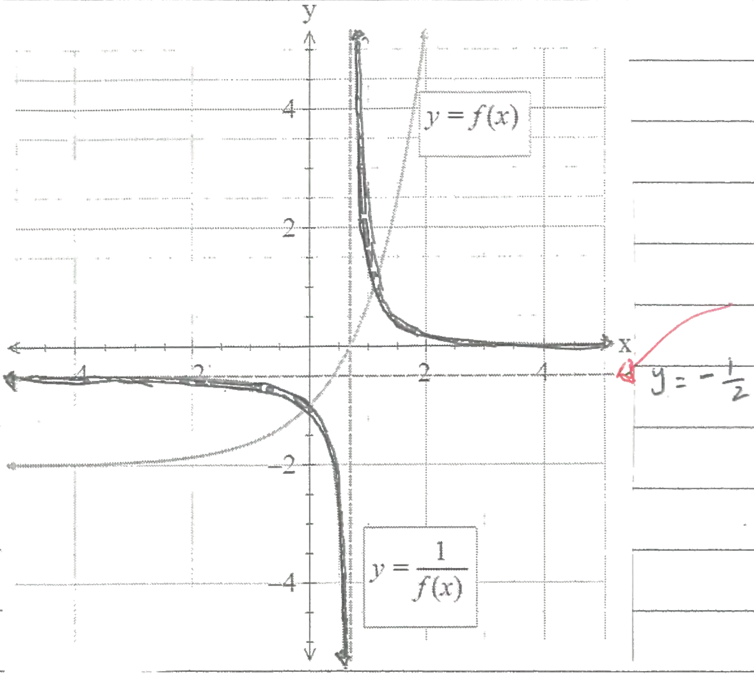
$x(x-2)(x-3) < 0$



From graph :

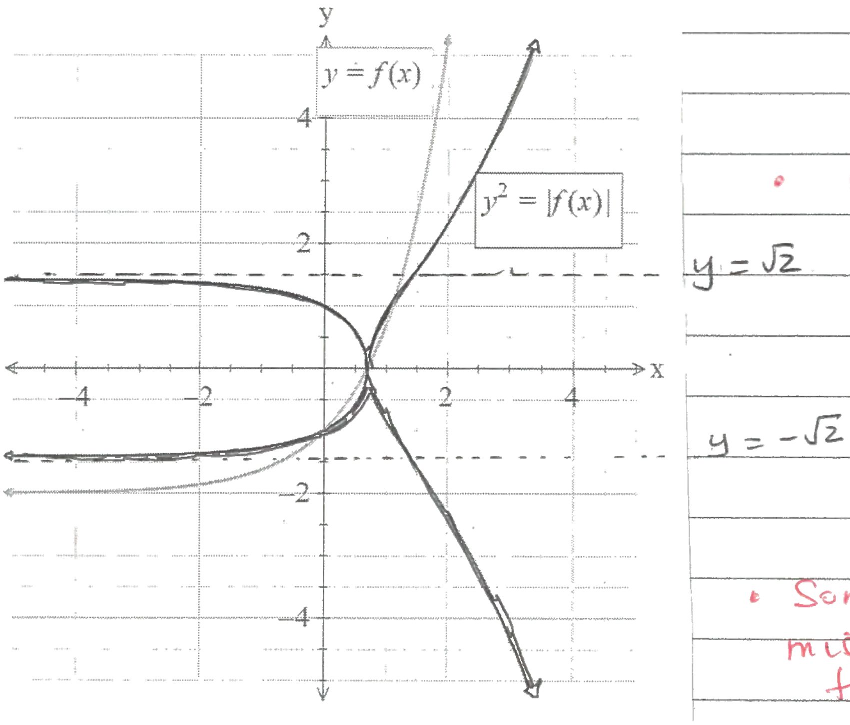
Solution : $x < 0$, $2 < x < 3$

6 b) i)



A significant number of students did not show this horizontal asymptote.

b (ii)



- A large number of students did not identify the horizontal asymptotes.
- Some students missing parts of the graph.

c) $x = \cos\theta + \sin\theta$ — 1 ; $y = \cos\theta - \sin\theta$ — 2

squaring and adding 1 and 2

$$x^2 + y^2 = (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$$

$$= \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta$$

$$= 2\sin^2\theta + 2\cos^2\theta$$

$$= 2(\sin^2\theta + \cos^2\theta)$$

$$\therefore \quad x^2 + y^2 = 2$$

Some students left the answer in terms of θ.

6 d) i) It is not a one-to-one function

ie. for at least one y-value there is more than one x-value.

just mentioning horizontal line test is not sufficient.

ii) $D_f : x \leq \frac{3}{2}$ $\quad\quad R_f : y \geq \frac{-9}{4}$

iii) $f^{-1}(x) : x = y^2 - 3y$

$$y^2 - 3y + \frac{9}{4} = x + \frac{9}{4}$$

$$\left(y - \frac{3}{2}\right)^2 = x + \frac{9}{4}$$

$$y - \frac{3}{2} = \pm\sqrt{x + \frac{9}{4}}$$

$$y = \frac{3}{2} \pm \sqrt{x + 9/4}$$

since it included $(0, 0)$

$$0 = \frac{3}{2} - \sqrt{0 + \frac{9}{4}}$$

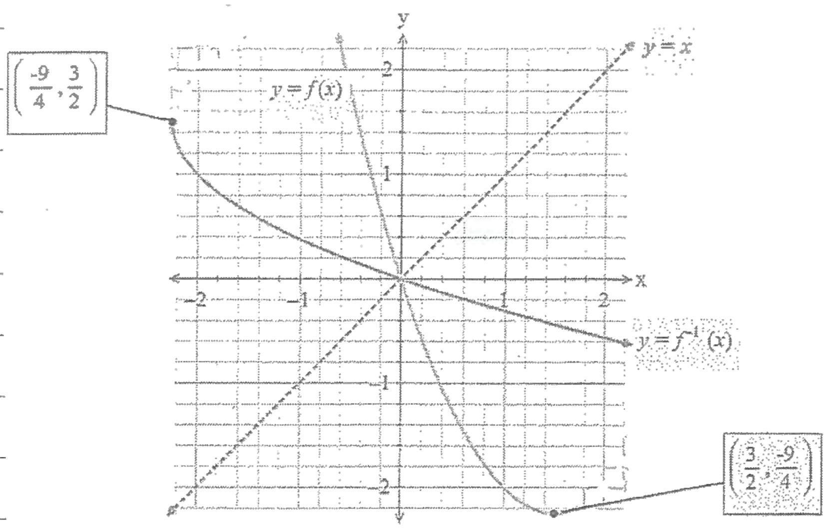
$$\therefore y = \frac{3}{2} - \sqrt{x + \frac{9}{4}}$$

Quite well done.

iv) $D_{f^{-1}(x)} : x \geq \frac{-9}{4}$

$R_{f^{-1}(x)} : y \leq \frac{3}{2}$

v)



Some students had trouble with this

4

Question 7 (19 marks)

a) GEOMETRY (8 letters; 3V, 5C)

_ V _ V _ V _ _

V: EOE

C: GMTRY

Some students missed 'E' repeated twice and hence lost a mark

No. of ways of arranging vowels

$= \frac{3!}{2!} = 3$

No. of ways of arranging consonants

$5!$

Total ways $= 3 \times 5!$

$= 360$ ways.

b) 6W, 10M

Mostly done well

i) ${}^{16}C_8 = 12870$

ii) ${}^{6}C_4 \times {}^{10}C_4 = 3150$

iii) ${}^{5}C_3 \times {}^{9}C_3 = 840$

Very few students wrote ${}^{6}C_4 + {}^{10}C_4$ & ${}^{5}C_3 + {}^{9}C_3$ and lost a mark each.

c) i) - Omee and Adi seated at 6-seater table

$= {}^{8}C_4 \times 5! \times 3!$

ways to arrange 4 people around seats

ways of selecting 4 others to join them

ways to arrange 6 people around seats

Most students lost marks in this Question.

$= 50400$

- Omee and Adi seated at 4-seater table

$= {}^{8}C_2 \times 5! \times 3!$

$= 20160$

Students are advised to revise circular arrangements.

$\therefore$ Total number of ways $= 70560$

7 c (ii) Total ways (without restrictions)

$= {}^{10}C_6 \times 5! \times 3!$

$= 151\ 200$

If students did 7c i, correct then they mostly got this part right.

$P(\text{Omee and Adi at the same table}) = \frac{70\ 560}{151\ 200}$

$= \frac{7}{15}$

d) 16 wins, 4 losses

_ L _ L _ L _ L _

- there are 5 spots to put 16 wins

Very well done!

$\frac{16}{5} = 3.2$

$\therefore$ there must be at least 4 wins in at least one category

By the pigeonhole principle, they must have won 4 games in a row somewhere during the season.

e) $\frac{3543}{15} = 236\frac{3}{15}$

$\therefore$ 3 candidates with 237 votes and the rest 236.

$\Rightarrow$ no clear winner.

$\therefore$ 238 votes is the smallest number for a majority.

lost a mark if students did not realise that 238 votes is the smallest number of votes.

Question 8 (18 marks)

a) $P(x) = x^4 - 3x^3 - 2x^2 - 5x$

$$\begin{array}{r|l} & x^3 + x^2 + 2x \\ \hline x-4 & x^4 - 3x^3 - 2x^2 - 5x \\ & -\underline{(x^4 - 4x^3)} \\ & \quad x^3 - 2x^2 \\ & \quad -\underline{(x^3 - 4x^2)} \\ & \qquad 2x^2 - 5x \\ & \qquad -\underline{(2x^2 - 8x)} \\ & \qquad\qquad 3x \end{array}$$

Most common mistake is writing 3 instead of $3x$ for the remainder

$P(x) = (x-4)(x^3 + x^2 + 2x) + 3x$

$= x(x-4)(x^2 + x + 2) + 3x$

b) $P(x) = 2x^3 + 3x^2 - 29x - 60$

i) $P(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$

$= -6$

done well

Remainder $= -6$

ii) $P(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$

$= 0$

done well

$\therefore (x+3)$ is a factor of $P(x)$

iii)

$$\begin{array}{r|l} & 2x^2 - 3x - 20 \\ \hline x+3 & 2x^3 + 3x^2 - 29x - 60 \\ & -\underline{(2x^3 + 6x^2)} \\ & \quad -3x^2 - 29x \\ & \quad -\underline{(-3x^2 - 9x)} \\ & \qquad -20x - 60 \\ & \qquad \underline{-20x - 60} \end{array}$$

Some are finding roots and then writing factors which can give wrong answers. Discourage the use of this method.

$\therefore P(x) = (x+3)(2x^2 - 3x - 20)$

$= (x+3)(2x+5)(x-4)$

8 c) $P(x) = x^4 - x^3 - 7x^2 + 13x - 6$; $P'(x) = 4x^3 - 3x^2 - 14x + 13$

i) $P(1) = 1^4 - 1^3 - 7(1)^2 + 13(1) - 6$

$= 0$

Many forgot to show $P(1) = 0$

$P'(1) = 4(1)^3 - 3(1)^2 - 14(1) + 13$

$= 0$

$P(1) = P'(1) = 0$

$\therefore$ $x = 1$ is a root of multiplicity 2 of $P(x)$

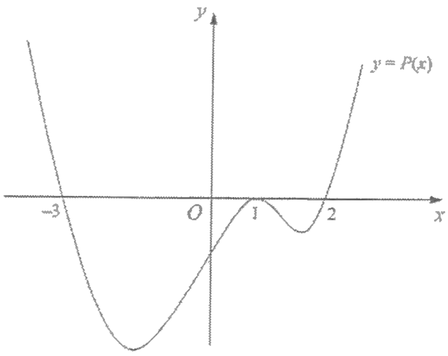
ii)

$$
\begin{array}{r l}
 & x^2 + x - 6 \\
x^2 - 2x + 1 \,\big) & x^4 - x^3 - 7x^2 + 13x - 6 \\
- & \underline{x^4 - 2x^3 + x^2} \\
 & x^3 - 8x^2 + 13x \\
- & \underline{x^3 - 2x^2 + x} \\
 & -6x^2 + 12x - 6 \\
 & \underline{-6x^2 + 12x - 6}
\end{array}
$$

students are using different method. They got full mark as long as the answer is correct.

$\therefore$ $P(x) = (x-1)^2(x-2)(x+3)$

iii)



Some students are drawing the graph of the cubic given in (b)

iv) $x \leq -3$, $x = 1$, $x \geq 2$

Some missed $x = 1$

8 d) $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$

Double root at $x = 3 \Rightarrow P(3) = 0$ and $P'(3) = 0$

$$P(3) = 8(3)^4 - 38(3)^3 + 9(3)^2 + 3a + b$$

$$0 = 648 - 1026 + 81 + 3a + b$$

$$\therefore 3a + b = 297 \quad \text{—— (1)}$$

$$P'(x) = 32x^3 + 114x^2 + 18x + a$$

$$P'(3) = 32(3)^3 + 114(3)^2 + 18(3) + a$$

$$0 = 864 - 1026 + 54 + a$$

$$a = 108$$

Mostly done well.

substituting $a = 108$ into (1)

$$3(108) + b = 297$$

$$b = -27$$

$$\therefore a = 108, \ b = -27.$$

Question 9 (17 marks)

a) $P(x) = x^3 - 10x^2 - 39x + 180$; roots α, β, γ

i) $\alpha + \beta + \gamma = \frac{-b}{a}$

$= 10$

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$= -39$

iii) $\alpha\beta\gamma = \frac{-d}{a}$

$= -180$

(iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha + \beta + \gamma)$

$= -180(10)$

$= -1800$

v) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= (10)^2 - 2(-39)$

$= 178$

Generally well done, but A LOT OF ACRONYMS (SOR, POR, SOT etc) were used. DON'T USE THESE! They are NOT mathematical conventions & CAN BE PENALISED (they weren't THIS time!).

vi) $(\alpha\beta)^{-2} + (\beta\gamma)^{-2} + (\alpha\gamma)^{-2}$

$= \frac{1}{(\alpha\beta)^2} + \frac{1}{(\beta\gamma)^2} + \frac{1}{(\alpha\gamma)^2}$

$= \frac{\gamma^2 + \alpha^2 + \beta^2}{(\alpha\beta\gamma)^2}$

$= \frac{178}{(-180)^2} = \frac{89}{16200}$

b) $P(x) = x^4 + 4x^3 - 3x^2 + 5x - 7$; roots: $\alpha, \beta, \gamma, \delta$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

$= \frac{\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta + \alpha\beta\gamma}{\alpha\beta\gamma\delta}$

$= \frac{-d/a}{e/a} = \frac{5}{7}$

9 c) $P(x) = x^3 + 3x^2 + ax + b$

Very few students used this method (which was the best way to do the question)

Let $P(x) = (x^2-1)\,Q(x) + Ax + B$

$= (x+1)(x-1)\,Q(x) + Ax + B$

Most students attempted using alternative solution (on next page).

$P(-1) = 4$ $\qquad$ $P(1) = -2$

$\therefore\ 4 = -A + B$ — ① $\qquad$ $-2 = A + B$ — ②

① + ② $\Rightarrow 2B = 2$ $\qquad$ ① − ② $\Rightarrow -2A = 6$

$B = 1$ $\qquad$ $A = -3$

$\therefore$ Remainder $= -3x + 1$

d) $x^3 + 3ax + b = 0$; roots : α, α, β

$P(x) = x^3 + 3ax + b$

$P'(x) = 3x^2 + 3a$

$P'(\alpha) = 0$

$\Rightarrow 3\alpha^2 + 3a = 0$

$\alpha^2 = -a$

Alternative for finding $\alpha^2 = -a$.

Sum (2 at a time) $\alpha^2 + 2\alpha\beta = 3a$

Using Sum Of Roots: $\beta = -2\alpha$.

$\alpha^2 + 2\alpha \times -2\alpha = 3a$

$-3\alpha^2 = 3a$

$\alpha^2 = -a$.

Sum of the roots :

$2\alpha + \beta = 0$

$\beta = -2\alpha$

Product of roots :

$\alpha^2\beta = -b$

$-2\alpha^3 = -b$ $\quad (\beta = -2\alpha)$

$4\alpha^6 = b^2$

$4(-a)^3 = b^2$ $\quad (\alpha^2 = -a)$

$\therefore\ 4a^3 + b^2 = 0$

Very poorly done. Students could use sum of roots to get $\beta = -2\alpha$ but didn't know what else to do.

A lot of fudging done.

Alternative solution to (9)(c)

$P(x) = x^3 + 3x^2 + ax + b$

$P(-1) = 4$

$(-1)^3 + 3(-1)^2 + a(-1) + b = 4 \Rightarrow -a + b = 2$ (1)

$+$

$P(1) = -2$:

$1^3 + 3(1)^2 + a(1) + b = -2 \Rightarrow a + b = -$ (2)

$2b = -4 \Rightarrow b = -2$.

(2)−(1) $2a = -8 \Rightarrow a = -4$.

A lot of students did this part, but then thought they HAD the remainder when all they had done was to complete P(x).

$\therefore P(x) = x^3 + 3x^2 - 4x - 2$.

Finding remainder:

$$
\begin{array}{r}
x + 3 \\
x^2 + 0x - 1 \overline{\big)\, x^3 + 3x^2 - 4x - 2} \\
\underline{x^3 + 0x^2 - x } \\
3x^2 - 3x - 2 \\
\underline{3x^2 + 0x - 3} \\
-3x + 1.
\end{array}
$$

Remainder $= -3x + 1$.

Question 10

a) $(2x-y)^5 = {}^5C_0(2x)^5 + {}^5C_1(2x)^4(-y) + {}^5C_2(2x)^3(-y)^2$

$+ {}^5C_3(2x)^2(-y)^3 + {}^5C_4(2x)(-y)^4 + {}^5C_5(-y)^5$

* Mostly done well.

* $(2x^5) = 32x^5$ not $2x^5$

* some did not simplify

$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

b) $\left(\frac{x}{2}-3\right)^4 = {}^4C_0\left(\frac{x}{2}\right)^4 + {}^4C_1\left(\frac{x}{2}\right)^3(-3) + {}^4C_2\left(\frac{x}{2}\right)^2(-3)^2$

$+ {}^4C_3\left(\frac{x}{2}\right)(-3)^3 + {}^4C_4(-3)^4$

* Mostly done well

* some did not simplify.

$= \frac{x^4}{16} - \frac{3x^3}{2} + \frac{27x^2}{2} - 54x + 81$

c) $(a+2x)^7$

Term in $x^4 = {}^7C_3\, a^3 (2x)^4$

$= 560\, a^3 x^4$

* Mostly done well

$\therefore\ 560a^3 = 8750$

$a^3 = \frac{8750}{560}$

* $\sqrt[3]{\frac{125}{8}} = \frac{5}{2}$ not $\pm 5/2$

$= \frac{125}{8}$

$\therefore\ a = \frac{5}{2}$

10. d) $(2-\sqrt{3})^5 = {}^5C_0\, 2^5 + {}^5C_1\, 2^4(-\sqrt{3}) + {}^5C_2\, 2^3(-\sqrt{3})^2$

$+ {}^5C_3\, 2^2(-\sqrt{3})^3 + {}^5C_4\, 2(-\sqrt{3})^4 + {}^5C_5(-\sqrt{3})^5$

Some mistakes made in this step

$= 32 - 80\sqrt{3} + 240 - 120\sqrt{3} + 90 - 9\sqrt{3}$

$= 362 - 209\sqrt{3}$

$\therefore a = 362,\ b = -209$

e) $(1-3x+2x^3)(1-2x)^6$

$= (1-3x+2x^3)\left[{}^6C_0 + {}^6C_1(-2x) + {}^6C_2(-2)^2 + {}^6C_3(-2)^3 + {}^6C_4(-2)^4 + {}^6C_5(-2)^5 + {}^6C_6(-2)^6\right]$

Terms in x^5:

$2x^3 \times {}^6C_2(-2x)^2 + (-3x)\times {}^6C_4(-2x)^4 + {}^6C_5(-2x)^5$

$= (120 - 720 - 192)x^5$

mostly done well

$= -792x^5$

coefficient of $x^5 = -792$

f) i) $\left(x + \frac{1}{x}\right)^8$

General Term : ${}^8C_r\, x^{8-r}(x^{-1})^r$

Mostly done well.

$= {}^8C_r\, x^{8-2r}$

ii) For the term in x^2

$8 - 2r = 2$

$r = 3$

Term $= {}^8C_3\, x^5\left(\frac{1}{x}\right)^3 = 56x^2$

$\therefore$ coefficient of $x^2 = 56$

Question 11

a) $^{n}C_2 + {}^{n+1}C_1 = 22$

$$\frac{n!}{2!(n-2)!} + \frac{(n+1)!}{1!\,n!} = 22$$

$$\frac{n(n-1)}{2} + n+1 = 22$$

$$n(n-1) + 2(n+1) = 44$$

$$n^2 + n - 42 = 0$$

$$(n+7)(n-6) = 0$$

$$n = -7, 6$$

$$n = 6 \quad (n>0)$$

Some students did not think to simplify the factorials before taking common denominators. This made it quite difficult to solve

Some did not realise a noninteger answer was not valid.

Bald answer got ①

Some did not ignore negative answer

b) $(1+x)^{11} \Rightarrow$ General term $= {}^{11}C_r\, x^r$

$\therefore\ {}^{11}C_5$ is the coefficient of x^5

$$(1+x)^3 = {}^3C_0 + {}^3C_1 x + {}^3C_2 x^2 + {}^3C_3 x^3$$

$$(1+x)^8 = {}^8C_0 + {}^8C_1 x + {}^8C_2 x^2 + {}^8C_3 x^3 + {}^8C_4 x^4 + {}^8C_5 x^5 + \dots$$

$\therefore$ Coefficient of x^5 in $(1+x)^3(1+x)^8$

$$= {}^3C_0 \times {}^8C_5 + {}^3C_1 \times {}^8C_4 + {}^3C_2 \times {}^8C_3 + {}^3C_3 \times {}^8C_2$$

$$= {}^8C_5 + {}^3C_1\,{}^8C_4 + {}^3C_2\,{}^8C_3 + {}^8C_2$$

Equating coefficients :

$$^{11}C_5 = {}^8C_5 + {}^3C_1\,{}^8C_4 + {}^3C_2\,{}^8C_3 + {}^8C_2$$

Poorly done.
Very poor setting out by many students.
Insufficient working or reasoning was penalised.

Very poorly done

11 c) $(5x+2)^{20}$

many students did realise to make $(5x)^k$

General Term : $(2+5x)^{20}$

Term in $x^k = {}^{20}C_k \; 2^{20-k} \; (5x)^k = {}^{20}C_k \; 2^{20-k} \; 5^k \; x^k$

Term in $x^{k+1} = {}^{20}C_{k+1} \; 2^{19-k} \; (5x)^{k+1} = {}^{20}C_{k+1} \; 2^{19-k} \; 5^{k+1} \; x^{k+1}$

Equating coefficients,

$$\frac{20!}{k!\,(20-k)!} \times 5^k \times 2^{20-k} = \frac{20!}{(k+1)!\,(19-k)!} \cdot 2^{19-k} \times 5^{k+1}$$

$$\frac{2}{k!\,(20-k)!} = \frac{5}{(k+1)!\,(19-k)!}$$

$$2(k+1) = 5(20-k)$$

$$2k+2 = 100-5k$$

$$7k = 98$$

$$k = 14$$

d) $(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

Differentiating both sides,

$$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \dots + n\,{}^nC_n x^{n-1}$$

Substituting $x=1$

$$n(2)^{n-1} = {}^nC_1 + 2\,{}^nC_2(1) + \dots + n\,{}^nC_n (1)^{n-1}$$

$$n\,2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \dots + n\,{}^nC_n$$

$${}^nC_1 + 2\,{}^nC_2 + \dots + n\,{}^nC_n = n\,2^{n-1}$$

Generally well done

Some students substituted first then differentiated. This received zero.

Very poorly done

11 e) $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_n x^n + \dots$

$+ {}^{2n}C_{2n} x^{2n}$

Substituting $x = 1$

$2^{2n} = {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \dots + {}^{2n}C_n + \dots + {}^{2n}C_{2n}$

Many did not get past the first step.

$2^{2n} = 2 \times {}^{2n}C_0 + 2 \times {}^{2n}C_1 + 2 \times {}^{2n}C_2 + \dots +$

often insufficient reasoning

$2\,{}^{2n}C_n - {}^{2n}C_n$

$\left[\text{Since } {}^{n}C_r = {}^{n}C_{n-r}\right]$

$\therefore\ 2^{2n} = 2 \times \sum_{r=0}^{n} {}^{2n}C_r - {}^{2n}C_n$

f) i) $(q+p)^n - (q-p)^n$

$= \left[{}^nC_0 q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p + {}^nC_3 q^{n-3} p^3 + \dots\right] -$

$\left[{}^nC_0 q^n - {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 - {}^nC_3 q^{n-3} p^3 + \dots\right]$

$= 2\,{}^nC_1 q^{n-1} p + 2\,{}^nC_3 q^{n-3} p^3 + \dots$

ii) If n is odd, the last term will be

$2\,{}^nC_n q^0 p^n = 2p^n$

iii) If n is even, the last term will be

$2\,{}^nC_{n-1} q^1 p^{n-1} = 2nqp^{n-1}$

Poorly done overall