Girraween High School

2024 Year 11 Mathematics Extension 1

Yearly Examination
September-2024

General Instructions

- Working Time - 120 minutes
- Reading Time 10 minutes
- Write using black or blue pen only
- Calculators approved by NESA may be used
- For questions in Section II, Show relevant Mathematical reasoning and/ or calculations

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Total Marks: 90

Section I - 5 marks (pages 1 and 2)

- Attempt questions 1-5
- Allow about 10 minutes for this section

Section II - 85 marks (pages 3-9)

- Attempt questions 6-11,
- **Start each question on a new page**
- Allow about 110 minutes for this section

Section I

Multiple Choice Questions

Question 1 (1 mark)

What is the domain of the function $f(x) = \sqrt{x^2 - 2x}$

A. $x \geq 2$

B. $x > 2$

C. $x \leq 0$ or $x \geq 2$

D. $x < 0$ or $x > 2$

Question 2 (1 mark)

Given $f(x) = \frac{1}{4}x^3 + 2$, what is the value of $f^{-1}(4)$?

A. 2

B. 5

C. 8

D. 18

Question 3 (1 mark)

In how many ways can 7 boys be arranged in a line if Peter, Jeffery, and Kevin have to stand together at the beginning of the line?

A. 4!

B. 7!

C. $4! \times 3!$

D. $7! \times 3!$

Question 4 (1 mark)

Which expression is equivalent to $\dfrac{1+\tan 5x \tan 3x}{\tan 5x - \tan 3x}$

A. $\cot 2x$

B. $\tan 2x$

C. $\dfrac{\tan 2x}{1+\tan 5x \tan 3x}$

D. $\cot x$

Question 5 (1 mark)

The roots of the equation $2x^3 - 5x^2 + 8x + 2 = 0$ are α, β and γ. Find the value of $\alpha(\beta^2+\gamma^2)+\beta(\alpha^2+\gamma^2)+\gamma(\alpha^2+\beta^2)$

A. -4

B. $\dfrac{13}{2}$

C. 7

D. 13

Exam continues on the next page

Mathematics Extension 1

Section II

85 Marks

Attempt Questions 6-11

Start each question on a new page

Allow about 110 minutes for this section

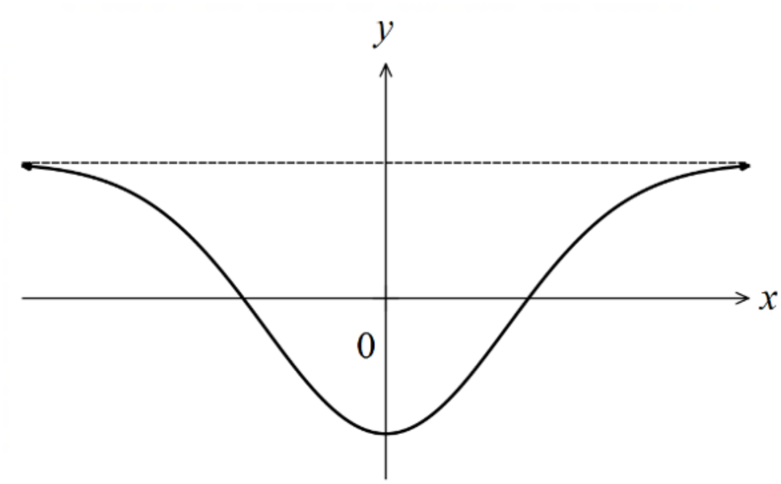
Question 6 (13 marks)

(a) Solve the inequality $\dfrac{x}{2x-1} \leq 5$ [3]

(b) Find the cartesian equation of the curve defined by the parametric equations: [2]

$$x = 2\sin\theta$$
$$y = 3 + 2\cos 2\theta$$

(c) The diagram below shows the graph of $y = f(x)$ where $f(x) = 1 - 2e^{-x^2}$.

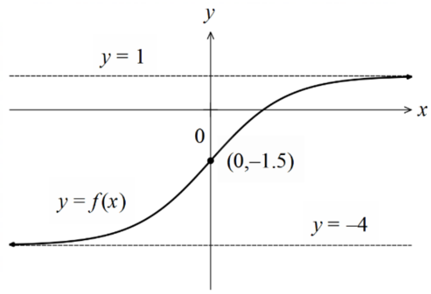


i. Find the equation of the horizontal asymptote, showing relevant working out. [1]

ii. A copy of the graph $y = f(x)$ is on page 4 of your writing booklet. On the same set of axes, sketch the graph of $y = \sqrt{f(x)}$. Showing clearly the asymptote and any points of intersection between $y = f(x)$ and $y = \sqrt{f(x)}$. [2]

Exam continues on the next page

(d) The graph of the function $f(x) = \dfrac{e^x - 4}{e^x + 1}$ is shown below.



i. Explain why $f(x)$ has an inverse function for all values of x. [1]

ii. On the same set of axes which is on the page 5 of you solution booklet, sketch the graphs of $y = x$, $y = f(x)$ and $y = f^{-1}(x)$ [2]

iii. Show that the equation of the inverse function is $f^{-1}(x) = ln\left(\dfrac{x+4}{1-x}\right)$ [2]

Question 7 (12 marks)

(a) What is the remainder when $P(x) = 2x^4 - 5x^3 + 12x - 7$ is divided by $x - 2$? [1]

(b) Consider the polynomial $p(x) = x^3 - px^2 + qx - pq$, where $q > 0$.

i. Show that $(x - p)$ is a factor of $P(x)$. [1]

ii. Show that the equation $P(x) = 0$ has only one real root. [2]

(c) Find the multiplicity of the root 1 of the equation $P(x) = 0$ where $P(x) = x^4 - 5x^3 + 9x^2 - 7x + 2$ [2]

(d) Consider the polynomial $P(x) = 2x^3 - 19x^2 + 60x - 63$. It is given that $P(x)$ has a double root at $x = 3$, factorise $P(x)$ completely. [2]

(e) The zeros of $x^3 - 5x + 3 = 0$ are α, β and γ, find a cubic equation with integer coefficients whose zeros are 2α, 2β and 2γ. [4]

Exam continues on the next page

Question 8 (14 marks)

(a) Write $\dfrac{1}{(n+1)!} - \dfrac{1}{(n-1)!}$ as a single fraction [2]

(b) If each distinct arrangement of the letters of DELETED is called a word:

i. How many words are possible? [1]

ii. In how many of these words will the D's be seperated? [1]

(c) A five-person team is to be chosen at random from eight women and nine men.

i. In how many ways can this team be chosen? [1]

ii. What is the probability that the team will consist of five men? [1]

(d) Find the number of ways in which the letters of the word HEXAGON can be arranged in a straight line so that:. [2]

i. the vowels are all next to each other. [1]

ii. no two consonants are next to each other [1]

(e) In how many ways can a committee of 5 be selected from 10 people, consisting of 5 men and 5 women, if one man, Ken, refuses to work in the same committee with Sue? [2]

(f) Consider the digits 7, 6, 5, 4, 3, 2, 1 and 0. Find how many four-digit numbers are possible if the digits are in:

i. descending order [1]

ii. ascending order [1]

Exam continues on the next page

Question 9 (13 marks)

(a) Using the expansion of $\cos(A+B)$, find the exact value of $\sec 105^o$ [3]

(b) i. Prove that $\tan\theta = \sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}$ [2]

ii. Hence show the exact value of $\tan 22\frac{1}{2}^o = \sqrt{2}-1$ [2]

(c) By making the substitution $t = \tan\dfrac{\theta}{2}$. [2]

Show that $cosec\,\theta + cot\,\theta = cot\,\dfrac{\theta}{2}$

(d) Show that $\sin(A-2B)\cos(A+2B) = \dfrac{1}{2}(\sin 2A - \sin 4B)$ [2]

Note: Sine is an odd function and Cosine is an even function.

(e) Prove the identity $\dfrac{2\sin^3 A + 2\cos^3 A}{\sin A + \cos A} = 2 - \sin 2A$ [2]

Exam continues on the next page

Question 10 (17 marks)

(a) There are four couples: [2]

Aaryan and Betty

Chen and Daisy

Elias and Fiona

Gus and Helena

They go to a cinema and decide to sit in the last row. How many different arrangements are possible if Aaryan and Betty want to sit together but Elias and Fiona do NOT want to sit together?

(b) Ken wants to celebrate his 18^{th} birthday by having a dinner party for himself and nine of his friends (five girls and four boys).

In how many ways can the people be seated at a round table if:

i. the boys and girls alternate? [1]

ii. Ken to be seated between two particular girls? [1]

(c) In how many ways 8 different gifts can be distributed to two children so that each child receives an odd number of gifts? [2]

(d) Five points are scattered over a 2m by 2m square. Show that there is at least one pair of points which are a distance at most $\sqrt{2}$ m apart. [2]

(e) Ten members of a sporting team have the numbers 1 to 10 on the back of their jumpers. What is the minimum number of players that must be chosen to guarantee that there is at least one pair of players whose numbers add to 11? [1]

(f) i. Expand and simplify $(3+x^2)^5$. [2]

ii. Hence find the coefficient of x^4 in $(x+2)^2(3+x^2)^5$ [3]

(g) Use binomial theorem to find an expression for the constant term in the expansion of $\left(3x^2 - \dfrac{1}{x}\right)^{15}$. Hence find the constant term. [3]

Exam continues on the next page

Question 11 (16 marks)

(a) By using $(1+x)^{10} = (1+x)^5(1+x)^5$, show that [2]

$$\binom{10}{3} = \binom{5}{3}\binom{5}{0} + \binom{5}{2}\binom{5}{1} + \binom{5}{1}\binom{5}{2} + \binom{5}{0}\binom{5}{3}$$

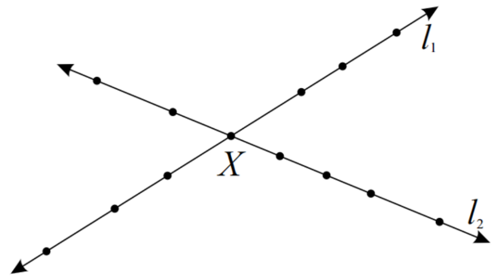
(b) In the binomial expansion of $\left(1+\dfrac{x}{k}\right)^n$, the coefficient of x^3 is twice the coefficient of x^2. Prove that $n = 6k+2$ [2]

(c) i. Write down the binomial expansion of $(1+x)^n$. [1]

ii. Hence, show that $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + ...+\binom{n}{n}=2^n - 1$. [2]

iii. By substituting a suitable value for x in the expansion $(1+x)^n$, show that: [2]
$1+ 2\binom{n}{1} + 4\binom{n}{2}+ \; ... \; + 2^{n-1}\binom{n}{n-1}=3^n - 2^n$

Exam continues on the next page



(d) The diagram above shows two distinct intersecting lines l_1 and l_2 lying on a plane. Each line has n distinct points marked on it, with the point X lying on both lines. The case above shows a possible situation when $n = 7$, and there are a total of thirteen points marked.

Consider the general case where there are n distinct points marked on each line, for $n \geq 3$.

i. In how many ways can three points be chosen on l_1? [1]

ii. By considering the total number of points on both lines and your answer to part(i), determine the number of triangles that can be formed using the marked points on either lines as vertices. [2]

iii. Of the total number of possible triangles in part (ii):

α) how many have X as a vertex? [1]

β) how many have two vertices that lie on l_1? [1]

iv. Use your answers to part (ii) and part (iii) to show that: [2]

$$\binom{2n-1}{3} = 2 \times \binom{n}{3} + (n-1)^3.$$

End of Examination

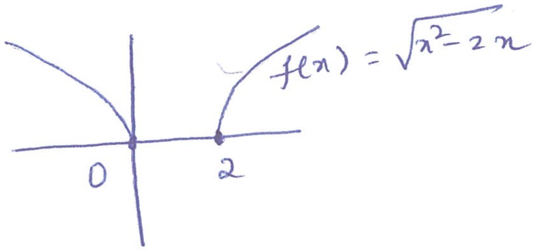
Mathematics Assessment Solutions
Girraween High School
2024

Task: Y11 Yearly Exam | Year: 2024

Suggested Solutions

Question 1

C $x \le 0$ or $x \geq 2$



1	2	3	4	5
C	A	C	A	D

Question 2

A

$$y = \frac{1}{4}x^3 + 2$$

$$x = \frac{1}{4}y^3 + 2$$

$$y^3 = 4x - 8$$

$$y = \sqrt[3]{4x - 8}$$

$$f^{-1}(4) = \sqrt[3]{4(4) - 8}$$

$$= 2$$

Marker's Feedback

Suggested Solutions

Question 3

C $4! \times 3!$

Question 4 A

$$\frac{1+\tan 5x \tan 3x}{\tan 5x - \tan 3x}$$

$$= \frac{1}{\frac{\tan 5x - \tan 3x}{1+\tan 5x \tan 3x}}$$

$$= \frac{1}{\tan(5x-3x)}$$

$$= \frac{1}{\tan 2x}$$

$$= \cot 2x$$

Marker's Feedback

Suggested Solutions

Question 5 D

$\alpha+\beta+\gamma=\frac{5}{2}$

$\alpha\beta+\alpha\gamma+\beta\gamma=4$

$\alpha\beta\gamma=-1$

$\alpha\beta^2+\alpha\gamma^2+\beta\alpha^2+\beta\gamma^2+\gamma\alpha^2+\gamma\beta^2$

$\alpha\beta^2+\beta\alpha^2+\alpha\gamma^2+\gamma\alpha^2+\beta\gamma^2+\gamma\beta^2$

$\alpha\beta(\alpha+\beta)+\alpha\gamma(\alpha+\gamma)+\beta\gamma(\gamma+\beta)$

$\alpha\beta(\alpha+\beta+\gamma)-\alpha\beta\gamma+\alpha\gamma(\alpha+\beta+\gamma)-\alpha\beta\gamma+\beta\gamma(\alpha+\beta+\gamma)-\alpha\beta\gamma$

$(\alpha+\beta+\gamma)(\alpha\beta+\alpha\gamma+\beta\gamma)-3(\alpha\beta\gamma)$

$4\left(\frac{5}{2}\right)-3(-1)$

$=13$

Marker's Feedback

Page ______

Suggested Solutions

Question 6

(a) $\dfrac{x}{2x-1} \le 5 \qquad x \ne \frac{1}{2}$

$(2x-1)^2 \times \dfrac{x}{(2x-1)} \le 5(2x-1)^2$

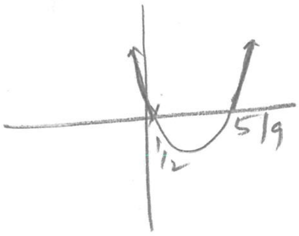
$x(2x-1) \le 5(2x-1)^2$ —— (1)

$0 \le 5(2x-1)^2 - x(2x-1)$

$0 \le (2x-1)\left[5(2x-1)-x\right]$

$0 \le (2x-1)(9x-5)$

$(2x-1)(9x-5) \geq 0$



—— (1)

$x < \frac{1}{2}$ and $x \geq \frac{5}{9}$ & $x \ne \frac{1}{2}$. —— (1)

Marker's Feedback

Some students who did not show the diagram as part of the above shown working ~~out~~ have lost a mark

→ Some students forgot to omit $x = \frac{1}{2}$ as part of the solution.

Page ______

Suggested Solutions

Question 6 b

$x = 2\sin\theta$

$x^2 = 4\sin^2\theta \rightarrow (1)$

$y = 3 + \cos 2\theta$

$y = 3 + 1 - 2\sin^2\theta$

$y = 4 - 2\sin^2\theta \rightarrow (2)$

From (1) and (2)

$\sin^2\theta = \dfrac{x^2}{4}$ sub into eq (2) — 1M

$y = 4 - 2\left(\dfrac{x^2}{4}\right)$

$y = 4 - \dfrac{x^2}{2}$

$\therefore 2y = 8 - x^2$ — 1M

Marker's Feedback

Students who did not realise the double angle or couldn't use the double angle formula have lost marks.

Suggested Solutions

Question 6C

i, let $y = f(x)$

$$y = 1 - \frac{2}{e^{x^2}}$$

As $x \to \infty$ $\quad \frac{-2}{e^{x^2}} \to 0^-$

$$1 - \frac{2}{e^{x^2}} \to 1^-$$

As $x \to -\infty$ $\quad \frac{-2}{e^{x^2}} \to 0^-$

$$1 - \frac{2}{e^{x^2}} \to 1^-$$

$\therefore$ Horizontal asymptote is $y = 1$ (1M)

Marker's Feedback

Mostly done well; however some students did no write the equation $y=1$; instead wrote Horizontal asymptote $= 1$; It is recommended to always write the equation.

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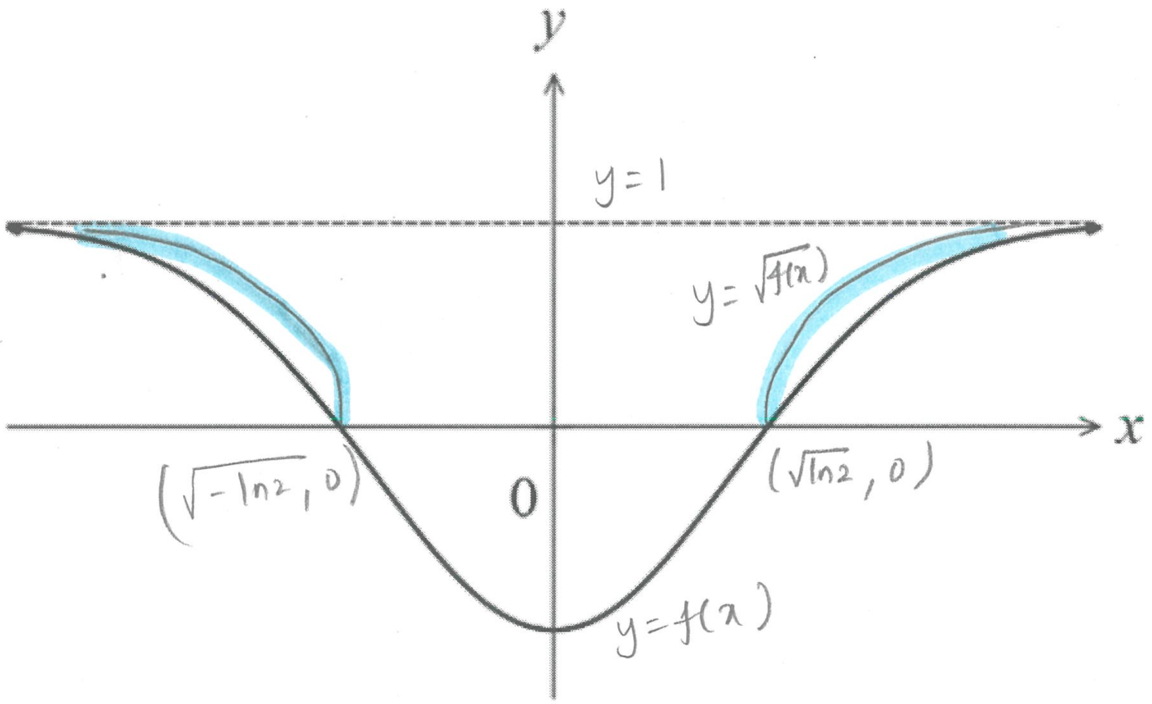
Q6cii



Correct shape or position (1m)

Correct shape or position relative to
f(x) → (2m)

Mostly done well otherwise lost marks.

Suggested Solutions

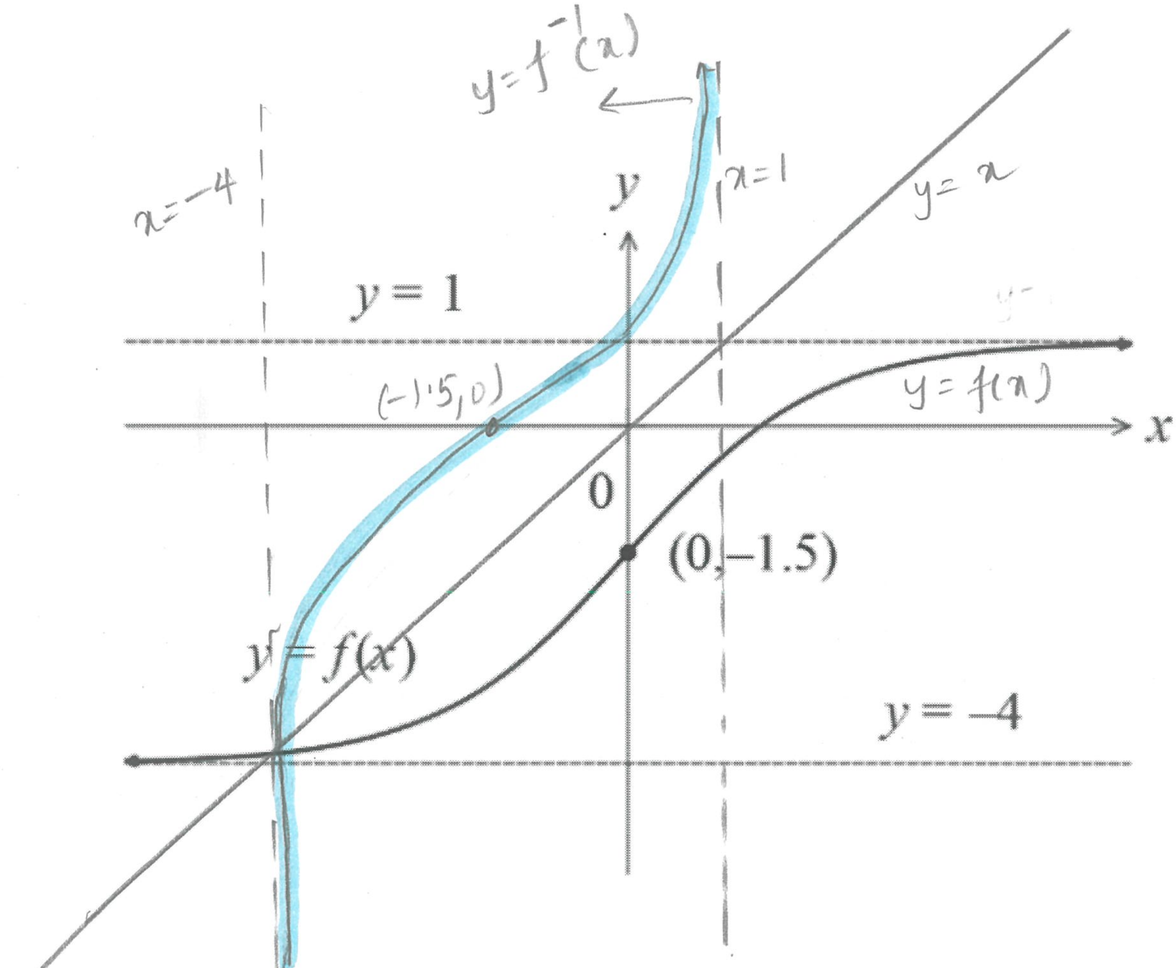
Question 6d

i, $f(x)$ is a one-one-one function or a montonically increasing function.

Marker's Feedback

Mostly done well; Most students either wrote one-one-one function or passes Horizontal line test and Vertical line test.

Q6 d ii)



Few students lost all marks.
Students advised to revise similar style Question

Suggested Solutions

Question 6d (iii)

$$f(x) = y = \frac{e^x - 4}{e^x + 1}$$

$$f^{-1}(x) = x = \frac{e^y - 4}{e^y + 1} \quad — \quad 1m$$

$$xe^y + x = e^y - 4$$

$$(x-1)e^y = -4 - x$$

$$e^y = \frac{x+4}{1-x}$$

$$y = \ln\left(\frac{x+4}{1-x}\right) \text{; as required.} \rightarrow 1M$$

Marker's Feedback

Mostly done well!

Page ______

Suggested Solutions

Question 7a

$p(2) = 2(2)^4 - 5(2)^3 + 12(2) - 7$

$= 9$ — 1m

Well done.
Some students used division which is not efficient.

7b i, $p(x) = x^3 - px^2 + qx - pq$; $q > 0$

if $(x-p)$ is a factor then
$p(p)$ will be equal to zero.

$p(p) = \cancel{p^3} - \cancel{p(p)^2} + \cancel{q(p)} - \cancel{pq}$ — 1

$= 0$

$\therefore (x-p)$ is a factor

well done

7b ii $p(x) = x^3 - px^2 + qx - pq$

$= x^2(x-p) + q(x-p)^2$

$= (x^2+q)(x-p)$ — 1

Now $x^2+q=0$ has no real roots — 1

$\therefore p(x)=0$ has only one real root i.e p.

Students had trouble with this.

must provide some sort of reasoning to get second mark.

Marker's Feedback

Page _____

Suggested Solutions

Question 7c

$p(x) = x^4 - 5x^3 + 9x^2 - 7x + 2$; $p(1) = 0$

$p'(x) = 4x^3 - 15x^2 + 18x - 7$; $p'(1) = 0$

$p''(x) = 12x^2 - 30x + 18$; $p''(1) = 0$ } 1

$p'''(x) = 24x - 30x$; $p'''(1) \neq 0$ → 1

$\therefore$ 1 is a root of multiplicity $\underline{\underline{3}}$.

Generally well done

7d consider the polynomial

$p(x) = 2x^3 - 19x^2 + 60x - 63$

Given that $x = 3$ is a double root,

$\therefore p(x) = (x-3)^2 (ax+b)$ —— 1M

$= (x^2 - 6x + 9)(ax+b)$

comparing the coefficients of x^3 : $a = 2$

comparing the constant term, $b = -7$

$\therefore p(x) = (x-3)^2 (2x-7)$ —— 1M

well done.

Marker's Feedback

Alternate solution for Q7d let the roots be

$\alpha, 3, 3$

$\therefore \alpha + 3 + 3 = \frac{19}{2}$

$\therefore \alpha = \frac{7}{2}$

$P(x) = (x-3)^2(2x-7)$

A common error was

$P(x) = (x-3)^2(x - \frac{7}{2})$

Page ______

Suggested Solutions

Question 7e

$\alpha+\beta+\gamma = \frac{-b}{a} = 0$

$2\alpha+2\beta+2\alpha = 2(\alpha+\beta+\gamma)$

$= 0$

1M

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = -5$

$2\alpha 2\beta + 2\alpha 2\gamma + 2\beta \cdot 2\gamma = 4(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= -20$

1M

$\alpha\beta\gamma = \frac{-d}{a} = -3$

$2\alpha \cdot 2\beta \cdot 2\gamma = 8\alpha\beta\gamma$

$= -24$

1M

$\therefore$ New cubic equation with integer coefficients is $x^3 - 20x + 24 = 0$ 1M

Was not too poorly done.

Common errors $2\alpha \cdot 2\beta \cdot 2\gamma = 2\alpha\beta\gamma$

$2\alpha\beta + 2\beta\gamma + 2\alpha\gamma = 2(\ldots)$

Marker's Feedback

Page _____

Suggested Solutions

Question 8a

$$\frac{1}{(n+1)!} - \frac{1}{(n-1)!}$$

$$\frac{1}{(n-1)!}\left[\frac{1}{n(n+1)} - 1\right] \quad — \text{1M}$$

$$\frac{1}{(n-1)!}\left[\frac{1-(n(n+1))}{n(n+1)}\right]$$

$$\frac{1}{(n-1)!}\left[\frac{1-n^2-n}{n(n+1)}\right]$$

$$= \frac{1-n^2-n}{(n+1)!} \quad — \text{1M}$$

Marker's Feedback

Generally well done, although many students stopped at $\frac{1-n(n+1)}{(n+1)!}$ → note: this was still paid full marks.

overexpanded this to $\frac{1-n^2+n}{(n+1)!}$

Page ______

Suggested Solutions

Question 8b(i)

N.o of ways $= \dfrac{7!}{2!\,3!}$

$= 420$ — 1M

8b(ii) All D's together $= \dfrac{6!}{3!} = 120$ ways

D's separated is $420 - 120 = 300$ ways

→ 1M

8c(i) $^{17}C_5 = 6188$

8c(ii) $\dfrac{^{9}C_5}{^{17}C_5} = \dfrac{126}{6188} = \dfrac{9}{442}$

Marker's Feedback

(8)(b)&(c) Generally well done by most students. The few students who didn't get these right generally didn't understand at all.

Suggested Solutions

Question 8d i,

Vowels : $\boxed{E, A, O}$ & H X G N can be arranged in 5! ways

$\boxed{EAO}$ can be arranged in 3! ways

$\therefore$ $5! \times 3! = 720$ ways.

<u>8d ii</u> No consonants are next to each other

V – Vowels ; C – Consonants

Can only be arranged as

C V C V C V C

C, arrangements $= 4!$

V arrangements $= 3!$

$\therefore$ $4! \times 3! = 144$ ways.

Marker's Feedback

Once again, generally well done with the few students who didn't understand being completely lost.

Page ______

Suggested Solutions

Question 8e

K, Not Sue $^8C_4$ ways

Not Ken, Sue $^8C_4$ ways

No Ken, No Sue $^8C_5$ ways

1M

$\therefore$ Number of ways = 196 —— 1M

8f i, descending Order

7, 6, 5, 4, 3, 2, 1 & 0

Selecting any four digits and then arranged in the required order ; $^8C_4 = 70$ 1M

8f ii Ascending Order:

Cannot start with zero

$\therefore$ $^7C_4 = 35$ → 1M

Marker's Feedback

Alternative solution was (for (e))

Total − both together

$= {}^{10}C_5 - {}^8C_3$

$= 196.$

Students did both this way & the above method well.

Also in (e); Some students had the right idea but put $^9C_4$ for Ken not Sue (forgetting that BOTH Ken & Sue are accounted for so there are only 8 people left).

(f) Generally well done. Most students recognised that a COMBINATION could be used & that you could not start with 0 for ascending order.

The students who didn't get this generally had no idea (again).

Suggested Solutions

Question 9a

$$\sec 105^\circ = \frac{1}{\cos(105^\circ)} = \frac{1}{\cos(60^\circ+45^\circ)} \quad 1M$$

$$= \frac{1}{\cos 60 \cos 45 - \sin 60 \sin 45} \quad 1m$$

$$= \frac{1}{\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{2\sqrt{2}}{1-\sqrt{3}} \quad \text{ok} \quad 1m$$

$$\frac{2\sqrt{2}\,(1+\sqrt{3})}{1-3}$$

$$= -\sqrt{2}-\sqrt{6}$$

Marker's Feedback

Well done.

Suggested Solutions

Question 9bi,

$$\tan\theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$RHS = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$RHS = \sqrt{\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}} \quad \text{— 1M}$$

$$= \sqrt{\frac{\cancel{1}-\cancel{1}+2\sin^2\theta}{\cancel{1}+2\cos^2\theta-\cancel{1}}}$$

$$= \sqrt{\frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}} \quad \text{1M}$$

$= \tan\theta$; as required

Marker's Feedback

Mostly done well.

Page ______

Suggested Solutions

Question 9bii, I

$$\tan 22\tfrac{1}{2} = \sqrt{\frac{1-\cos\left(22\frac{1}{2}\times 2\right)}{1+\cos\left(22\frac{1}{2}\times 2\right)}}$$

$$= \sqrt{\frac{1-\cos 45}{1+\cos 45}} \quad 1M$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}}$$

$$= \sqrt{(\sqrt{2}-1)^2}$$

$$= \sqrt{2}-1 \quad 1M$$

Marker's Feedback

Some students didn't show that $\tan 22\frac{1}{2} = \sqrt{2}-1$

Some got $\tan 22\frac{1}{2}^\circ = \sqrt{3-2\sqrt{2}}$ or $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

This is a "show that" question, so '1' mark deducted if they didn't show full working out to get the final answer $\sqrt{2}-1$.

Suggested Solutions

Question 9C

$t = \tan\frac{\theta}{2}$

Show that $\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$.

LHS: $\dfrac{1+t^2}{2t} + \dfrac{1-t^2}{2t}$

$= \dfrac{\cancel{2}}{\cancel{2}t}$ — 1

$= \dfrac{1}{t}$

$= \dfrac{1}{\tan\frac{\theta}{2}}$ — 1

$= \cot\frac{\theta}{2}$

Marker's Feedback

Done well.

Page ______

Suggested Solutions

Question 9d

LHS: $\sin(A-2B)\cos(A+2B)$ $\rightarrow$ 1M

$= \frac{1}{2}\left[\sin(A-2B+A+2B) + \sin(A-2B-A-2B)\right]$

$= \frac{1}{2}\left[\sin 2A + \sin(-4B)\right]$ $\rightarrow$ 1M

$= \frac{1}{2}\left[\sin 2A - \sin 4B\right]$ $\underbrace{(\text{Sine is odd})}_{\text{no mark assigned.}}$

$=$ RHS ; as required

Marker's Feedback

Generally well done.

Page _____

Suggested Solutions

Question 9e

LHS: $\dfrac{2\sin^3 A + 2\cos^3 A}{\sin A + \cos A}$

$\dfrac{2(\sin^3 A + \cos^3 A)}{\sin A + \cos A}$

$= \dfrac{2(\sin A + \cancel{\cos A})(\sin^2 A - \sin A\cos A + \cos^2 A)}{(\sin A + \cancel{\cos A})}$ 1M

$= 2(\sin^2 A + \cos^2 A - \sin A\cos A)$

$= 2(1 - \sin A\cos A)$ 1M

$= 2 - 2\sin A\cos A$

$= 2 - \sin 2A$

$=$ RHS

Marker's Feedback

Some students had trouble with this question.

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Suggested Solutions

Question 10a

$\boxed{AB}$ C D E F GH

Separate

working out

1M

All arrangements with $\boxed{AB}$ − $\boxed{AB}$ $\boxed{EF}$ arrangements

$7! \times 2 - 6! \times 2! \times 2!$

$= 7200$ — 1M

10bi

place one boy first, the rest can be arranged in $4! \times 5! = 2880$ ways.

1M

10bii

Place Ken and the two girls first

This can be done in $2!$ ways

The remainder can be arranged in $7!$ ways

1M

$\therefore$ N.o of ways $= 2! \times 7! = 10080$ ways

Marker's Feedback

Most of the students did well but some students got all Perms and combs questions wrong.

Suggested Solutions

Question 10C

A	1	3	5	7
B	7	5	3	1

→ 1M

N.o of ways: ${}^{8}C_{1} \cdot {}^{7}C_{7} + {}^{8}C_{3} \cdot {}^{5}C_{5} + {}^{8}C_{5} \cdot {}^{3}C_{3} + {}^{8}C_{7} \cdot {}^{1}C_{1}$

$= 128$ ways → 1M

Marker's Feedback

Lot of Students gave 4 as the answer

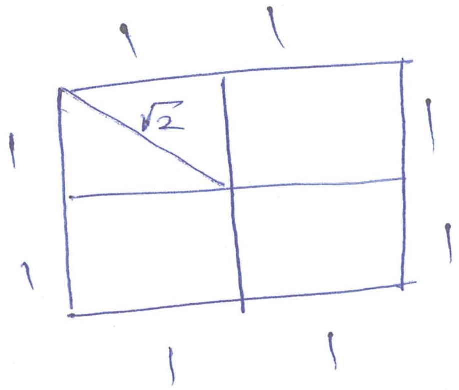
Suggested ~~Solutions~~

Question 10d

Divide the square into 1m x 1m squares

By the pigeonhole principle, at least one of the squares must contain 2 points. The furthest distance between the 2 points is the diagonal of the smaller square which is $\sqrt{2}$ m

2m



Marker's Feedback

Majority got it right - well done.

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Suggested Solutions

Question 10e

$\{1, 10\}, \{2, 9\}, \{3, 8\}, \{4, 7\}, \{5, 6\}$

worst case scenario

Choosing 5 players whose numbers do not add to 11.

But choosing 1 more would guarantee that there is at least one pair of players whose numbers add to 11.

$\therefore$ 6 is the minimum number of players required.

1M

Marker's Feedback

Mostly done well.

Suggested Solutions

Question

10g $\left(3x^2 - \frac{1}{x}\right)^{15}$

General term: ${}^{15}C_k \, (3x^2)^{15-k} \left(-\frac{1}{x}\right)^k$

${}^{15}C_k \, 3^{15-k} \, x^{30-2k} \, (-1)^k \, x^{-k}$

${}^{15}C_k \, 3^{15-k} \, x^{30-3k} \, (-1)^k$ 1m

Constant term occurs when $30 - 3k = 0$

$30 = 3k$

$k = 10 \rightarrow$ 1m

$\therefore$ constant term $= {}^{15}C_{10} \times 3^{15-10} \times (-1)^{10}$

$= 729\,729 \longrightarrow$ 1m

Marker's Feedback

Mostly done well.

Some had -729729 as answer and lost 1 mark.

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Suggested Solutions

Question 10f i

$(3+x^2)^5$

$= {}^5C_0 \cdot 3^5 + {}^5C_1 \cdot 3^4 \cdot x^2 + {}^5C_2 \cdot 3^3 \cdot (x^2)^2 + {}^5C_3 \cdot 3^2 (x^2)^3 +$

${}^5C_4 \cdot 3 \cdot (x^2)^4 + {}^5C_5 \cdot (x^2)^5$ — 1M

$= 243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10}$

1M

10f ii

Coefficient of x^4

$(x+2)^2 \, (3+x^2)^5$ 1M

$(x^2+4x+4) \, (3+x^2)^5$ using (g.i,)

$405x^2 \cdot x^2 + 4 \times 270 \cdot x^4$ → 1M

Coefficient of x^4 is $\underline{\underline{1485}}$.

1M

Marker's Feedback

Majority did well.

10f ii) Some put answer as $1485x^4$ rather than 1485.

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Suggested Solutions

Question 11a

$$(1+x)^{10} = (1+x)^5 (1+x)^5$$

LHS $(1+x)^{10} = {}^{10}C_{10} + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + \ldots$

coefficient of $x^3 = {}^{10}C_3 \longrightarrow$ 1m

RHS: $(1+x)^5 (1+x)^5 = ({}^5C_0 + {}^5C_1 x + {}^5C_2 x^2 + {}^5C_3 x^3 + {}^5C_4 x^4 + {}^5C_4 x^5)$
$({}^5C_0 + {}^5C_1 x + {}^5C_2 x^2 + {}^5C_3 x^3 + {}^5C_4 x^4 + {}^5C_5 x^5)$

coefficient of $x^3 = {}^5C_0 \cdot {}^5C_3 + {}^5C_1 \cdot {}^5C_2 + {}^5C_2 \cdot {}^5C_1 + {}^5C_3 \cdot {}^5C_0 \longrightarrow$ 1m

Equating coefficients of x^3

$${}^{10}C_3 = {}^5C_3\,{}^5C_0 + {}^5C_2\,{}^5C_1 + {}^5C_1\,{}^5C_2 + {}^5C_0\,{}^5C_3$$

$${}^{10}C_3 = 2\left[{}^5C_0\,{}^5C_3 + {}^5C_1\,{}^5C_2\right]$$

Marker's Feedback

Some students did n't expand the LHS and RHS. Without expanding and writing you can't equate coefficients. They lost 1 mark.

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Suggested Solutions

Question 11b

$$2 \cdot {}^nC_2 \frac{x^2}{k^2} = {}^nC_3 \frac{x^3}{k^3}$$

$$\frac{2n!}{(n-2)!\,2!\,k^2} = \frac{n!}{(n-3)!\,3!\,k^3}$$

$$\frac{\cancel{2}n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,\cancel{2!}\,\cancel{k^2}} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}\,3!\,\cancel{k^3}\,k}$$

$$\cancel{n}(n-\cancel{1}) = \frac{\cancel{n}(n-\cancel{1})(n-2)}{3!\,k}$$

$$6k = n-2$$

$$\therefore n = 6k+2$$

Marker's Feedback

Working out very messy. students do not know the difference between term and coefficient.

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Suggested Solutions

Question 11 C

i, $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

ii, Sub $x = 1$ in i, 1M

$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_2$

$2^n = 1 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n \longrightarrow$ 1M

$\therefore {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1$

iii, Sub $x = 2$ in i, 1M

$3^n = {}^nC_0 + 2 \cdot {}^nC_1 + 4\,{}^nC_2 + \dots + 2^{n-1} \cdot {}^nC_{n-1} + 2^n \cdot {}^nC_n$

$3^n = \underline{{}^nC_0} + 2 \cdot {}^nC_1 + \dots + 2^{n-1} \cdot {}^nC_{n-1} + \underline{2^n}$ 1M

$\therefore 3^n - 2^n = 1 + 2\binom{n}{1} + 4\binom{n}{2} + \dots + 2^{n-1}\binom{n}{n-1}$;

as required

Marker's Feedback

(i) done well

(ii) done well

(iii) Some students subtracted the value of 2^n from (ii) and lost mark.

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Suggested Solutions

Question 11d

(i) $^{n}C_3$ ways → 1m

(ii) There are $2n-1$ points in total, and any 3 non-collinear points can be chosen to form the vertices of a triangle.

$\therefore$ There are $^{2n-1}C_3 - 2 \times {}^{n}C_3$ possible triangles.

(iii) (α) If 'x' is assigned as a vertex:

→ 1 vertex must be chosen from l_1 which happens in $(n-1)$ ways.

→ 1 vertex must be chosen from l_2 $(n-1)$ ways.

$\therefore (n-1)^2$ triangles have X as vertex.

Marker's Feedback

(i) Some students wrote $^{7}C_3$. They were not given mark.

(ii) Hard question. Very few got correct answer.

(iii) Hard question. Not many correct answers.

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Suggested Solutions

Question 11d (Continued)

iii β → ${}^nC_2$ ways to choose the vertices on l_1

→ $(n-1)$ ways to choose the remaining vertex on l_2

${}^nC_2 \times {}^{(n-1)}C_1$ triangles.

iv The total number of triangles that can be formed can be considered as:

$$\begin{pmatrix}\text{Triangles with 2} \\ \text{vertices on } l_1\end{pmatrix} + \begin{pmatrix}\text{Triangles with 2} \\ \text{vertices on } l_2\end{pmatrix} - \begin{pmatrix}\text{Triangles that} \\ \text{have X as vertex} \\ \text{which are counted} \\ \text{twice}\end{pmatrix}$$

∴ From iii, Total Δ^{les}

$$= {}^nC_2(n-1) + {}^nC_2(n-1) - (n-1)^2$$

$$= 2 \times \frac{n!}{2!(n-2)!}(n-1) - (n-1)^2$$

$$= n(n-1)^2 - (n-1)^2$$

$$= (n-1)^2(n-1) = (n-1)^3$$

Marker's Feedback

(iii) β hard question.

(iv) Very hard questions.

Suggested Solutions

Question

11d iv, continued

Equating this result with result from (b) ii :

$$^{2n-1}C_3 - 2 \times {}^{n}C_3 = (n-1)^3$$

$$^{2n-1}C_3 = 2 \times {}^{n}C_3 + (n-1)^3$$; as required.

Marker's Feedback

Only one or two got the correct answer.