



Student Full Name : _____

Teacher Name : _____

2019 YEAR 11 YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 56

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 46 marks (pages 6 – 15)

- Attempt Questions 11 - 23
- Allow about 1 hour and 15 minutes for this section

Section I

Attempt Questions 1–10 (10 Marks)

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. Max has a drawer which contains forty unmatched socks of identical style, with equal numbers of white, blue, grey and brown socks.

Max reaches into the drawer in the dark to retrieve a pair of socks.

What is the minimum number of socks that Max must take from the drawer to ensure that his selection includes a pair of matching socks of any colour

(A) 3

(B) 4

(C) 5

(D) 6

2. A particle moves along the x -axis such that its displacement (x metres) from the origin at a time t seconds is given by the equation:

$$x = 5t^2 - \frac{4}{t^5}$$

What is the acceleration (in m/s^{-2}) of the particle when $t=1$?

(A) -120

(B) -110

(C) 0

(D) 30

3. Solve $|3 - 2x| \leq 1$

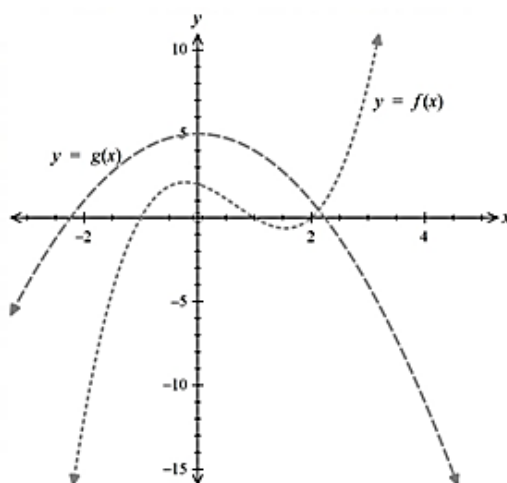
(A) $1 \leq x \leq 2$

(B) $x \leq 1, x \geq 2$

(C) $-2 \leq x \leq -1$

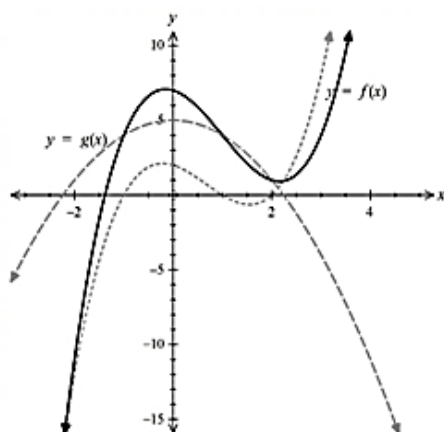
(D) $x \leq -2, x \geq -1$

4. The graph of $y = f(x)$ and $y = g(x)$ are shown on the grid below.

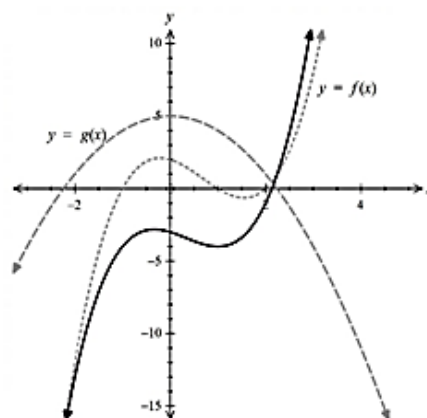


In which diagram has the graph of $y = f(x) + g(x)$ been drawn on the same grid?

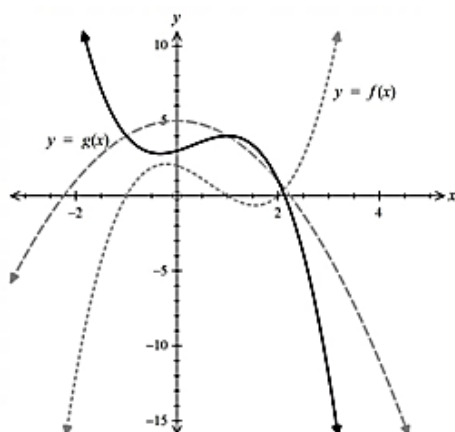
A.



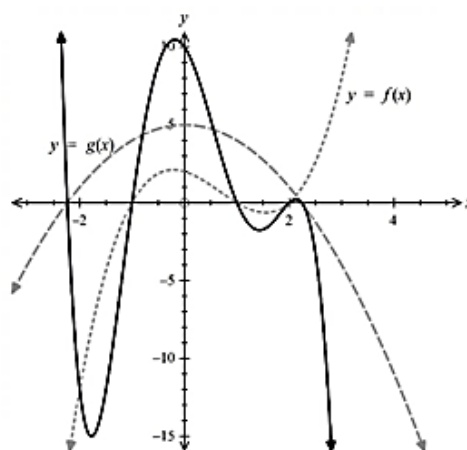
B.



C.



D.



5. How many distinct arrangements are there of the letters of the word IONISATION?
- (A) 151 200
- (B) 604 800
- (C) 907 200
- (D) 3 628 800
6. A population grows according to the equation $N(t) = 120e^{0.025t}$ where t is measured in months.
- By how many individuals, does the population grow over the first 3 years?
- (A) 120
- (B) 125
- (C) 175
- (D) 295
7. Given that $\cos^{-1}(\sqrt{1-t^2}) = \alpha$, which expression represents $\sin \alpha$.
- (A) $\sqrt{1-t^2}$
- (B) $\sqrt{1+t^2}$
- (C) 1
- (D) t

8. What is the inverse of the function $y = \frac{8x^3}{5} + 2$?
- (A) $y = \frac{\sqrt[3]{5x-10}}{8}$
- (B) $y = \frac{\sqrt[3]{5x-10}}{2}$
- (C) $y = \frac{5\sqrt[3]{x-2}}{2}$
- (D) $y = \frac{\sqrt{5x-10}}{2}$
9. Find the coefficient of x^6 in the expansion of $\left(5x^2 - \frac{3}{x}\right)^6$.
- (A) $-\binom{6}{4} \cdot 5^8 \cdot 3^2$
- (B) $-\binom{6}{4} \cdot 5^6 \cdot 3^6$
- (C) $\binom{6}{4} \cdot 5^8 \cdot 3^2$
- (D) $\binom{6}{4} \cdot 5^4 \cdot 3^2$
10. The expansion of $\cos(2x + 60^\circ)$ equals:
- (A) $\frac{\cos 2x}{2} + \frac{\sqrt{3}\sin 2x}{2}$
- (B) $\frac{\cos 2x}{2} - \frac{\sqrt{3}\sin 2x}{2}$
- (C) $\frac{\sqrt{3}\cos 2x}{2} + \frac{\sin 2x}{2}$
- (D) $\frac{\sqrt{3}\cos 2x}{2} - \frac{\sin 2x}{2}$

End of section I

Section II Answer Booklet

Student name

Student Number

Teacher Name

46 marks

Attempt questions 11-23

Allow about 1 hour and 15 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 Marks) Questions 11-16 have a cumulative total of 15 marks.

2

The equation $x^3 - 6x^2 + 4x - 3 = 0$ has roots α, β and γ .

Find the value of $\alpha^2 + \beta^2 + \gamma^2$

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Question 12 (2 Marks)

Solve for x :

$$\frac{2x}{x-1} \leq 7.$$

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Question 13 (3 Marks)

Find the Cartesian equation that is represented by the pair of parametric equations below:

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$$\begin{array}{ll} x = 2p + 3 & \dots \textcircled{1} \\ y = p^2 - 6p + 9 & \dots \textcircled{2} \end{array}$$

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Question 14 (3 Marks)

- (a) For the function $f(x) = (x - 3)^2$, write an equation which is its inverse. 1

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- (b) Describe how the domain and range could be restricted so that the inverse is a function. 2

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Question 15 (2 Marks)

- Using the identity $\cos(A + B) + \cos(A - B) = 2\cos A \cos B$, find the exact value of: 2

$$\cos \frac{5\pi}{12} + \cos \frac{\pi}{12}$$

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Question 16 (3 Marks)

For the function $f(x) = 4 \sin^{-1}\left(\frac{x-5}{2}\right)$

- (a) Give the domain and range of the function.

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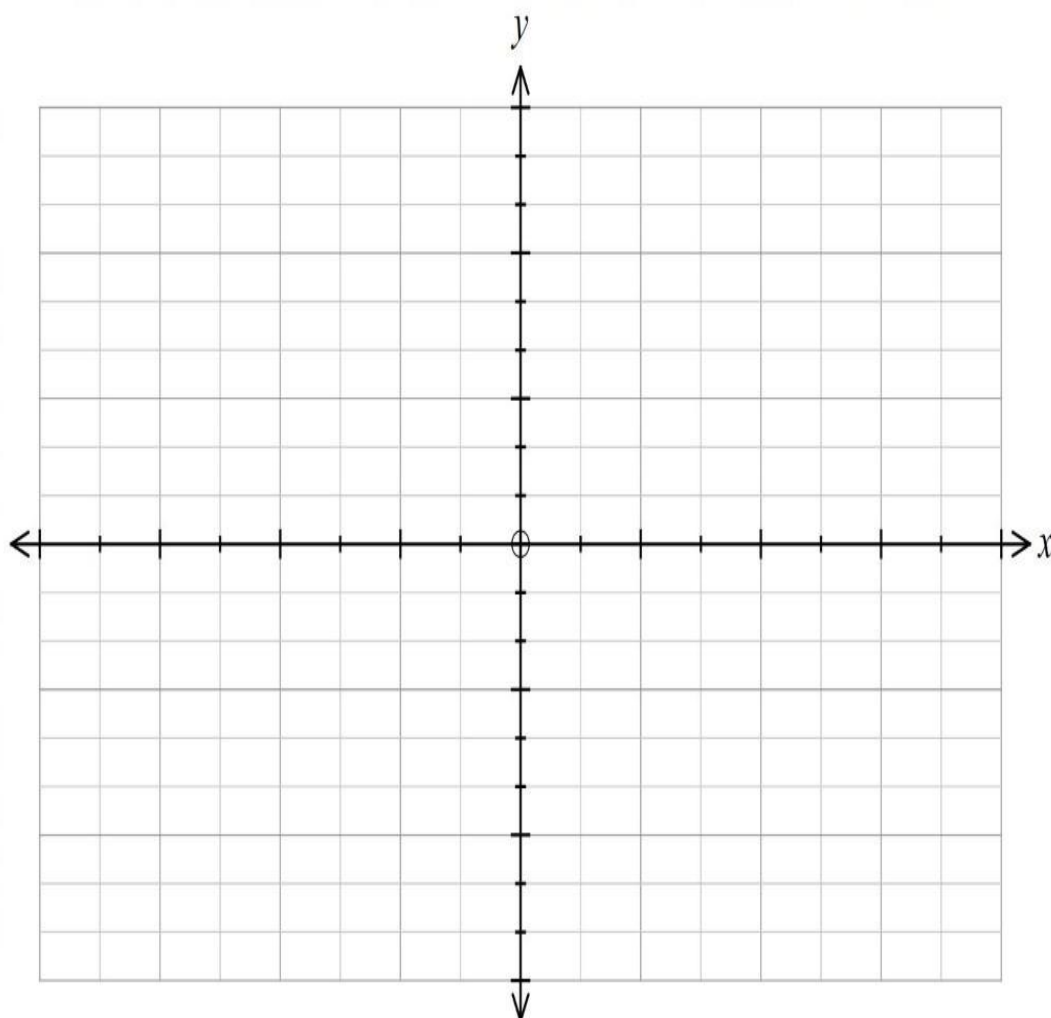
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- (b) Draw a sketch of the function.

1



Question 17 (4 Marks) Questions 17-20 have a cumulative total of 17 marks.

- (a) Find the exact value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \tan^{-1}(-\sqrt{3})$ 2

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- (b) Find the exact value of $\tan\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$ 2

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Question 18 (5 Marks)

Avijit has a six character password consisting of four lower case letters followed by two digits. No letter or number can be repeated

- (a) How many possible passwords exist? 2

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- (b) Avijit has forgotten his password, but remembers it has a 't' and a 'w' in it.
What is the probability that he guesses it correctly on his first attempt? 3

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Question 19 (3 Marks)

There are 8 girls and 7 boys in a class.

- (a) The class elects a captain and a vice captain. In how many ways is this possible? 1

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- (b) The class elects four representatives for the student council. How many different groups of representatives are possible? 1

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- (c) How many groups of representatives for the student council are possible if the class decides that they want two girls and two boys in the group? 1

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Question 20 (5 Marks)

The displacement of a particle moving along the x -axis is given by $x = (t - 1)^3$, where t is in seconds and x is in cm

- (a) Find the equations for velocity ($\dot{x}$) and acceleration ($\ddot{x}$). 2

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- (b) Find the acceleration of the after 10 seconds?. 1

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- (c) When will the particle become stationary? 1

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- (d) What is the minimum acceleration of the particle? 1

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Question 21 (4 Marks) Questions 21-23 have a cumulative total of 14 marks.

- (a) Show that $(1 - x)^5 \left(1 + \frac{1}{x}\right)^5 = \left(\frac{1}{x} - x\right)^5$ 1

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- (b) Write the expansion of $\left(\frac{1}{x} - x\right)^5$, leaving coefficients in the form $\binom{n}{r}$ 1

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- (c) Determine the coefficient of x^3 in the expansion of $(1 - x)^5 \left(1 + \frac{1}{x}\right)^5$ and hence show that $\binom{5}{4}\binom{5}{1} - \binom{5}{5}\binom{5}{2} - \binom{5}{3}\binom{5}{0} = \binom{5}{4}$ 2

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Question 22 (6 Marks)

The blood temperature of a live animal is 38.6°C .

An animal is found dead with a blood temperature of 31.1°C , in a barn with a constant temperature $P^{\circ}\text{C}$. 10 minutes later, the animal's blood temperature is 30.7°C .

The cooling rate is proportional to the difference between the blood temperature and the room temperature

- (a) Using Newton's Law of cooling, $T = P + Ae^{kt}$, where T is the temperature at time t , find an expression for k in terms of A . 2

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- (b) If the temperature in the barn, P , is a constant 26 degrees show that $k = -0.008$ to three decimal places. 2

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Question 22 continues on next page.

- (c) Given that the blood temperature of a live animal is 38.6°C and that the animal was found at 8 pm, evaluate the time of death. Answer to the nearest minute. 2

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Question 23 (4 Marks)

The volume of water in a tank is given by the following equation:

$$V = \frac{1}{16}t^2 - 35t + 4900$$

where V is the volume, in litres, and t is the time, in minutes

- (a) Find the rate at which the water is flowing after 16 minutes. 2

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(b) Find the time taken for the tank to be completely empty

2

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End of examination paper 😊

Extra writing space

If you use this space, clearly indicate which question you are answering.

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Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Q 11.

$$x^3 - 6x^2 + 4x - 3 = 0$$

$$\alpha + \beta + \gamma = -\frac{b}{a} = 6$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = 4$$

$$\alpha\beta\gamma = -\frac{d}{a} = 3$$

$$\begin{aligned}\alpha^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) \\ &= 6^2 - 2(4) \\ &= 36 - 8 \\ &= 28\end{aligned}$$

2

2 marks for correct answer

1 mark for working which shows

Q 12.

Multiply both sides by $(x - 1)^2$ ($x \neq 1$).

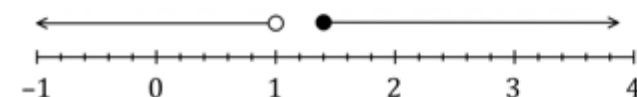
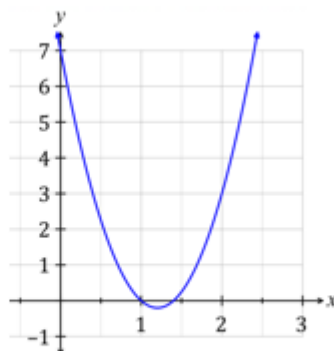
$$(x - 1)^2 \times \frac{2x}{x - 1} \leq 7 \times (x - 1)^2$$

$$(x - 1) \times 2x - 7(x - 1)^2 \leq 0$$

$$(x - 1)[2x - 7(x - 1)] \leq 0$$

$$(x - 1)(5x - 7) \geq 0$$

$$\therefore 1 > x \geq \frac{7}{5}$$



3 marks: Correct answer.

2 marks: Solves the inequality.

1 mark: Multiplies both sides of the inequality by $(x - 1)^2$ ($x \neq 1$).

Q.13.

$$x = 2p + 3 \quad (1)$$

$$y = p^2 - 6p + 9 \quad (2)$$

Factorise (2)

$$y = (p - 3)^2 \quad (3)$$

Make $p - 3$ the subject of (1)

$$x - 3 = 2p$$

$$\frac{x - 3}{2} = p$$

$$\frac{x - 3}{2} - 3 = p - 3$$

$$p - 3 = \frac{x - 9}{2}$$

Substitute in (3)

$$y = \left(\frac{x - 9}{2}\right)^2$$

$$OR \quad y = \frac{(x - 9)^2}{4}$$

$$OR \quad 4y = (x - 9)^2$$

$$OR \quad 4y = x^2 - 18x + 81$$

Or sub directly into (2)

$$y = \left(\frac{x-3}{2}\right)^2 - 6\left(\frac{x-3}{2}\right) + 9$$

$$4y = (x - 3)^2 - 12(x - 3) + 36$$

$$4y = x^2 - 6x + 9 - 12x + 36 + 36$$

$$4y = x^2 - 18x + 81$$

3

3 marks for correct equation in any of the forms provided

2 marks for correct working toward answer with minor error

1 mark for an attempt to rearrange equations so that one can be substituted in the other

Q.14.

a) $f(x) = (x - 3)^2$
Inverse $x = (y - 3)^2$
 $\pm\sqrt{x} = y - 3$
 $y = 3 \pm \sqrt{x}$

1

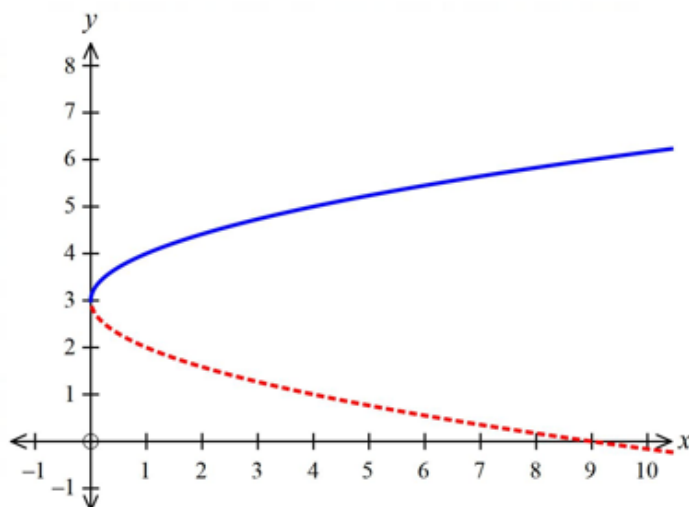
1 mark for correct equation of inverse in either form

b) For it to be a function we need to restrict the range. There are two ways of doing this, to give either the top or bottom half of the parabola.

2

1 mark for domain

$$f^{-1}(x) = 3 + \sqrt{x} \quad OR \quad f^{-1}(x) = 3 - \sqrt{x}$$



Domain $x \geq 0$

Range $y \geq 3 \quad OR \quad y \leq 3$

1 mark for range

15.

13 (b) (ii)

Criteria	Marks
• Correct solution	3
• Correct method with 1 error	2
• Uses part i) to rewrite expression as $= 2 \cdot \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6}$	1

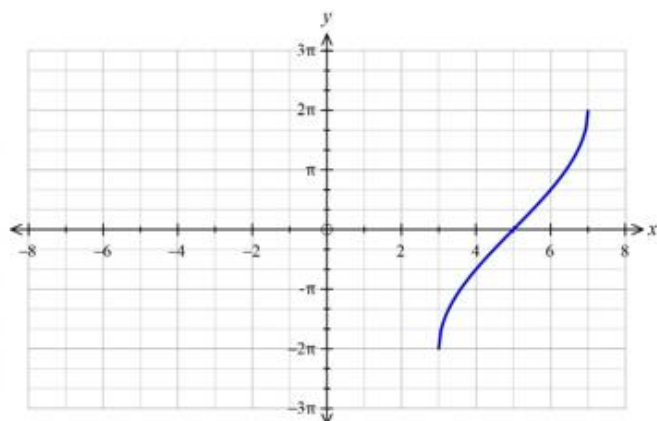
Sample answer

$$\begin{aligned}
 & \cos \frac{5\pi}{12} + \cos \frac{\pi}{12} \\
 &= \cos \left(\frac{\pi}{4} + \frac{\pi}{6} \right) + \cos \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \\
 &= 2 \cdot \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} \quad \text{using } i) \\
 &= 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{3}}{\sqrt{2}} \\
 &= \frac{\sqrt{6}}{2}
 \end{aligned}$$

Q.16.

a)	$y = 4\sin^{-1}\left(\frac{x-5}{2}\right)$ Domain $-1 \leq \frac{x-5}{2} \leq 1$ $-2 \leq x-5 \leq 2$ $3 \leq x \leq 7$ Domain $[3, 7]$ Range $-\frac{\pi}{2} \leq \sin^{-1}\left(\frac{x-5}{2}\right) \leq \frac{\pi}{2}$ $4\left(-\frac{\pi}{2}\right) \leq 4\sin^{-1}\left(\frac{x-5}{2}\right) \leq 4\left(\frac{\pi}{2}\right)$ $-2\pi \leq y \leq 2\pi$ Range $[-2\pi, 2\pi]$	2	2 marks for correct domain and range 1 mark for correct domain or range
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b)



Q 17.

a)

$$\begin{aligned}\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \tan^{-1}(-\sqrt{3}) &= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}(\sqrt{3}) \\ &= \frac{\pi}{6} + \frac{\pi}{3} \\ &= \frac{\pi}{2}\end{aligned}$$

1

1 mark for
correct answer

b)

$$\begin{aligned}&\tan\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right) \\ &= \tan\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) \\ &= \tan\left(\pi - \frac{\pi}{4} + \frac{\pi}{4}\right) \\ &= \tan(\pi) \\ &= 0\end{aligned}$$

2

2 marks for
correct answer1 mark for
correct use of
identities or of
exact value
results

Q.18.

Criteria	Marks
• Correct answer	2
• Significant progress towards solution	1

Sample answer

$$26 \times 25 \times 24 \times 23 \times 22 \times 10 \times 9 = 32\,292\,000$$

Criteria	Marks
• Correct answer	3
• Significant progress towards solution: - Calculates total combinations = 596 160 - Or, Calculates probability, with 1 error	2
• Some progress towards solution	1

Sample answer

Total Combinations with t and w : ${}^4C_2 \times 2 \times 24 \times 23 \times 10 \times 9 = 596\,160$

Therefore, probability that he guess correctly first time = $\frac{1}{596\,160}$

Q 19.

(a)	15 students available for selection and 2 students to select (order) ${}^{15}P_2 = 210$ or $15 \times 14 = 210$ $\therefore$ There are 210 ways to choose a captain and vice-captain.	1 mark: Correct answer.
(b)	15 students available for selection and 4 students to select (unorder) ${}^{15}C_4 = 1365$ or $\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$ $\therefore$ There are 1365 ways to choose the student council.	1 mark: Correct answer.
(c)	8 girls available for section and 2 girls to select (unorder) 7 boys available for section and 2 boys to select (unorder) ${}^8C_2 \times {}^7C_2 = 588$ $\therefore$ There are 588 ways to choose the student council.	1 mark: Correct answer.

Q 20.

12 (b) (i)

Criteria	Marks
• Both equations correct	2
• One equation correct	1

Sample answer

$$x = (t - 1)^3$$

$$\dot{x} = 3(t - 1)^2$$

$$\ddot{x} = 6(t - 1)$$

12 (b) (ii)

Criteria	Mark
• Correct solution	1

Sample answer

at $t = 10$

$$\ddot{x} = 6(10 - 1)$$

$$= 54 \text{ cm s}^{-2}$$

12 (b) (iii)

Criteria	Mark
• Correct solution	1

Sample answer

for stationary points, $\dot{x} = 0$

$$\therefore t = 1 \text{ s}$$

12 (b) (iv)

Criteria	Mark
• Correct solution	1

Sample answer

$$\ddot{x} = 6(t - 1)$$

this is a linear function with a positive gradient

$\therefore$ minimum acceleration when $t = 0$.

$$\therefore \ddot{x} = 6(0 - 1)$$

$$= -6 \text{ cm s}^{-2}$$

Q 21.

a)	$(1-x)^5 \left(1 + \frac{1}{x}\right)^5 = \left((1-x) \left(1 + \frac{1}{x}\right)\right)^5$ $= \left(1 + \frac{1}{x} - x - 1\right)^5$ $= \left(\frac{1}{x} - x\right)^5$	1	1 mark for correct expansion and simplification
b)	$\left(\frac{1}{x} - x\right)^5 = \binom{5}{0} \left(\frac{1}{x^5}\right) - \binom{5}{1} \left(\frac{1}{x^4} \cdot x\right) + \binom{5}{2} \left(\frac{1}{x^3} \cdot x^2\right)$ $- \binom{5}{3} \left(\frac{1}{x^2} \cdot x^3\right) + \binom{5}{4} \left(\frac{1}{x} \cdot x^4\right) - \binom{5}{5} \cdot x^5$ $\left(\frac{1}{x} - x\right)^5 = \binom{5}{0} \left(\frac{1}{x^5}\right) - \binom{5}{1} \left(\frac{1}{x^3}\right) + \binom{5}{2} \left(\frac{1}{x}\right)$ $- \binom{5}{3} (x) + \binom{5}{4} (x^3) - \binom{5}{5} \cdot x^5$	1	1 mark for correct expansion and simplification

c)	$(1-x)^5 \left(1 + \frac{1}{x}\right)^5 = \left(\binom{5}{0} - \binom{5}{1}x + \binom{5}{2}x^2 - \binom{5}{3}x^3 + \binom{5}{4}x^4 - \binom{5}{5}x^5 \right)$ $\times \left(\binom{5}{0} + \binom{5}{1}\frac{1}{x} + \binom{5}{2}\frac{1}{x^2} + \binom{5}{3}\frac{1}{x^3} + \binom{5}{4}\frac{1}{x^4} + \binom{5}{5}\left(\frac{1}{x^5}\right) \right)$ Coefficients of x^3 come from: $x^5 \times \frac{1}{x^2}$, $x^4 \times \frac{1}{x}$ and $x^3 \times 1$. Coefficient of $x^3 = -\binom{5}{5}\binom{5}{2} + \binom{5}{4}\binom{5}{1} - \binom{5}{3}\binom{5}{0}$ Equating with coefficient of x^3 from other expansion $-\binom{5}{5}\binom{5}{2} + \binom{5}{4}\binom{5}{1} - \binom{5}{3}\binom{5}{0} = \binom{5}{4}$ $\binom{5}{4}\binom{5}{1} - \binom{5}{5}\binom{5}{2} - \binom{5}{3}\binom{5}{0} = \binom{5}{4}$	2	2 marks for expansion and equating the coefficients to obtain the correct result 1 mark for correct expansion or equivalent merit
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Q 22.

Criteria	Marks
• Correct solution	2
• Correctly applies simultaneous equations	1

Sample answer

when $t = 0$ $T = 31.1$

$$\therefore 31.1 = P + Ae^{k \cdot 0}$$

$$\therefore 31.1 = P + A \quad (1)$$

when $t = 10$ $T = 30.7$

$$\therefore 30.7 = P + Ae^{10k} \quad (2)$$

$$(1) - (2)$$

$$0.4 = A - Ae^{10k}$$

$$Ae^{10k} = A - 0.4$$

$$e^{10k} = \frac{A - 0.4}{A}$$

$$10k = \ln\left(\frac{A - 0.4}{A}\right)$$

$$k = \frac{\ln\left(\frac{A - 0.4}{A}\right)}{10}$$

Criteria	Mark
• Correct solution	1

Sample answer

when $t = 0$ $T = 31.1$

$$\therefore 31.1 = 26 + Ae^{k \cdot 0}$$

$$\therefore A = 5.1$$

$\therefore$ using $i)$

$$k = \frac{\ln\left(\frac{5.1 - 0.4}{5.1}\right)}{10}$$

$$k = -0.00816....$$

$$k = -0.008 \text{ (3 d.p.)}$$

Criteria	Marks
• Correct solution	2
• Correctly calculates t	1

Sample answer

when $T = 38.6$

$$38.6 = 26 + 5.1 \cdot e^{-0.008t}$$

$$12.6 = 5.1 \cdot e^{-0.008t}$$

$$\frac{12.6}{5.1} = e^{-0.008t}$$

$$\ln\left(\frac{12.6}{5.1}\right) = -0.008t$$

$$t = -\frac{\ln\left(\frac{12.6}{5.1}\right)}{0.008}$$

$$t = -113.057.... \text{ minutes}$$

$\therefore$ time of death = 8 pm - 113.057... minutes

= 6 :07 pm

Q 23.

(a)	$V = \frac{1}{16} t^2 - 35t + 4900$ $\frac{dV}{dt} = \frac{1}{8} t - 35$ After 16 minutes $\frac{dV}{dt} = \frac{1}{8} \times 16 - 35$ $= -33$ $\therefore$ Water is flowing out of the tank at a rate of 33 litres per minute.	2 marks: Correct answer. 1 mark: Finds $\frac{dV}{dt}$
(b)	Tank is completely empty when $\frac{dV}{dt} = 0$ $\frac{1}{8} \times t - 35 = 0$ $\frac{1}{8} t = 35$ $t = 280 \text{ minutes}$ (Alternative method is to use $V = 0$) $\therefore$ The tank is empty after 280 minutes.	2 marks: Correct answer. 1 mark: Correct interpretation by using $\frac{dV}{dt} = 0$ or $V = 0$.