STUDENT NUMBER.....................

GOSFORD HIGH SCHOOL

2020

HIGHER SCHOOL CERTIFICATE

FINAL ASSESSMENT

Year 11 PRELIMINARY MATHEMATICS – EXTENSION 1

Time Allowed - 90 minutes plus 10 minutes reading time

- Write using a black or blue pen. Black pen is preferred.
- Board approved calculators may be used
- Answers to the multiple choice are to be done on the answer sheet provided.
- Start each section on a **new** page. **Each section** will be collected **separately.**
- Show all relevant mathematical reasoning and/or calculations.
- THIS PAPER IS TO BE RE-COLLECTED AT END OF EXAM.

Marks

Section I

Attempt Questions 1-8 (8 Marks)

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1 What is the Cartesian equation of the curve with parametric equations $x = 20t$ and $y = 40t - 5t^2$?

(A) $y = 2x - \dfrac{x^2}{400}$

(B) $y = 2x - \dfrac{x^2}{200}$

(C) $y = 2x - \dfrac{x^2}{160}$

(D) $y = 2x - \dfrac{x^2}{80}$

2 What is the domain of the function $f(x) = \ln(3x - x^2)$? **1**

(A) $(0, 3)$

(B) $[0, 3)$

(C) $(0, 3]$

(D) $[0, 3]$

3 For what value of k is the polynomial $P(x) = kx^2 + (2k-1)x + k$ a perfect square? **1**

(A) $k = \frac{1}{9}$

(B) $k = \frac{1}{4}$

(C) $k = 4$

(D) $k = 9$

4 Which of the following is NOT a factor of $P(x) = x^3 + 2x^2 - 5x - 6$? **1**

(A) $(x+1)$

(B) $(x-2)$

(C) $(x-3)$

(D) $(x+3)$

Marks

5 What is the value of $\cos^{-1}(\tan x)+\sin^{-1}(\tan x)$ for $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$? **1**

(A) 0

(B) 1

(C) $\frac{\pi}{2}$

(D) π

6 Which of the following is NOT an expression for $\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)$? **1**

(A) $\sec x+\tan x$

(B) $\dfrac{1+\sin x}{\cos x}$

(C) $\dfrac{\cos x}{1-\sin x}$

(D) $(1+\tan x)^2$

7 In how many ways can a group of 6 people line up in a queue so that the oldest person is in front of the youngest person? **1**

(A) 180

(B) 360

(C) 540

(D) 720

8 What is the expansion of $\left(3-2\sqrt{2}\right)^3$? **1**

(A) $27-16\sqrt{2}$

(B) $51-34\sqrt{2}$

(C) $99-70\sqrt{2}$

(D) $171-124\sqrt{2}$

Marks

Section II

Attempt Questions 9-12
Allow about 1 hour and 45 minutes for this section.

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (12 Marks) **Use a separate writing booklet.**

(a) Solve the inequality $\left|2x+1\right|\geq 3$. 2

(b) The equation $P(x)=0$ where $P(x)=x^3+cx+d$ has a double root $x=1$. Find the values of *c* and *d*. 2

(c) Show that $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\tan x$.

(d) Solve the inequality $x+\dfrac{4}{x}\geq 5$. 3

(e) Consider the polynomial $P(x)=k^2x^3-k^3x^2-x+k$ where $k\neq 0$.

(i) Show that $(x-k)$ is a factor of $P(x)$. 1

(ii) Solve the equation $P(x)=0$. 2

Marks

Question 10 (12 Marks) **Use a separate writing booklet.**

(a) Find the number of ways that 4 students can be chosen from 6 juniors and 3 seniors

(i) without restriction. **1**

(ii) so that there are unequal numbers of juniors and seniors. **2**

(b) The equation $2x^3+6x^2+4x+1=0$ has roots $\alpha,\ \beta$ and γ.

(i) Find the value of $(1-\alpha)+(1-\beta)+(1-\gamma)$. **1**

(ii) Find the value of $(1-\alpha)(1-\beta)(1-\gamma)$. **2**

(c) Write $\cos x - 3\,sinx + 3$ using the "t" results given that $t = tan\frac{x}{2}$. **3**

d)

i. Write down the binomial expansion of $(1+x)^n$ **1**

ii. Use the expansion of $(1+x)^n$ to show that ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$ **2**

Marks

Question 11 (12 Marks) **Use a separate writing booklet.**

(a) Find the multiplicity of the root $x=-1$ of the equation $P(x)=0$ where $P(x)=x^4+4x^3+7x^2+6x+2$. **2**

(b) Sketch the graph of the function $f(x)=-2\tan^{-1}x$ showing the intercepts on the axes and the equations of the asymptotes. **2**

(c) Solve the equation $\sin^2\theta=\cos\theta+1$ where $0\le\theta\le2\pi$. **3**

(d) Find the centre and radius of the circle $x^2+y^2-6x+2y+6=0$ **2**

(e) Consider the function $f(x)=(x-2)^2,\ 0\le x\le 2$.

Find the inverse function $f^{-1}(x)$ and state its domain. **3**

Marks

Question 12 (12 Marks) **Use a separate writing booklet.**

(a) Find the number of ways in which the letters of the word TRIANGLE can be arranged in a line

(i) so that all the vowels are next to each other and all the consonants are next to each other. **1**

(ii) so that consonants are in the two end positions. **1**

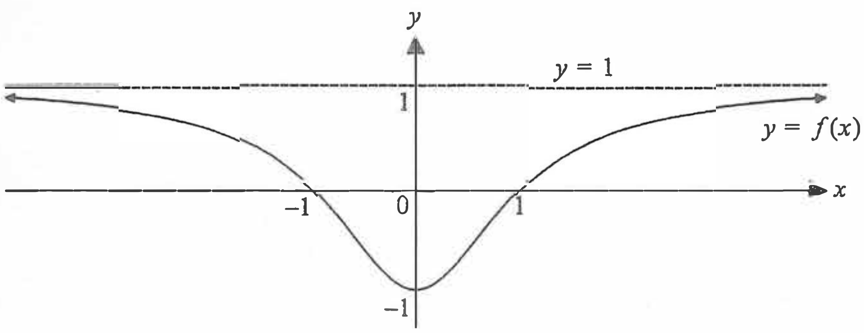
(b) If $\sec x + \tan x = k$, find in terms of k the value of $\sec x$. **2**

c) Find the number of ways in which 2 boys and 5 girls can be arranged in a circle

i. if there is no restrictions **1.**

ii. if the boys are to sit together **1 .**

(d)



The diagram shows the graph of the function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$. On separate diagrams sketch the graphs of the following curves, showing clearly the intercepts on the axes and the equations of any asymptotes.

(i) $y = |f(x)|$. **2**

(ii) $y = \dfrac{1}{f(x)}$ **2**

(e) i. Express $\cos x - \sqrt{3} sinx$ in the form $Rcos(x + \alpha)$ where $R > 0$ and $0 \le x \le 2\pi$ **2**

I/8

ANSWERS TO Yr 11 EXT ONE PRELIM FINAL 2020

MC

1/ $x = 20t \Rightarrow t = \frac{x}{20}$

$y = 40t - 5t^2$

$y = 40\left(\frac{x}{20}\right) - 5\left(\frac{x^2}{20}\right)$

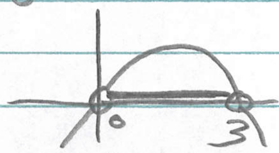
$= 2x - \frac{5x^2}{400}$

$= 2x - \frac{x^2}{80}$

D

2/ $f(x) = \ln(3x - x^2)$

for $3x - x^2 > 0$



$\therefore d: 0 < x < 3$

ie $(0, 3)$

A

3/ $P(x) = k\left(x^2 + \left(2 - \frac{1}{k}\right)x + 1\right)$

for $x^2 + \left(2 - \frac{1}{k}\right)x + 1$

to be a perfect square

$2 - \frac{1}{k} = -2$

$k = \frac{1}{4}$

B

4/ $P(x) = x^3 + 2x^2 - 5x - 6$

$P(-1) = 0$

$P(2) = 0$

$P(3) = 24$ ✓ $(x-3)$

check $P(-3) = 0$

C

5/ for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$-1 < \tan x < 1$

let $\tan x = u$

$\therefore \cos^{-1} u + \sin^{-1} u = \frac{\pi}{2}$

for all $-1 < u < 1$

$\therefore$ C

6/ Using "t" results

$\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \frac{\tan\frac{x}{2} + \tan\frac{\pi}{4}}{1 - \tan\frac{x}{2}\tan\frac{\pi}{4}}$

$= \frac{t+1}{1-t}$

A) $\sec x + \tan x = \frac{1 + t^2 + 2t}{1 - t^2}$

$= \frac{(1+t)^2}{\cancel{(1+t)}(1-t)} = \frac{1+t}{1-t}$

B) $\frac{1 + \sin x}{\cos x} = \sec x + \tan x = A$

C) $\frac{\cos x}{1 - \sin x} = \frac{\cos x(1 + \sin x)}{(1 - \sin^2 x)}$

$= \frac{\cos x(1 + \sin x)}{\cos^2 x}$

$= \frac{1 + \sin x}{\cos x}$

$= \sec x + \tan x = A$

D) $x = -\frac{\pi}{4} \Rightarrow (1 + \tan x)^2 = 0$

But

$\tan\left(-\frac{\pi}{8} + \frac{\pi}{4}\right) = \tan\frac{\pi}{8} > 0$

$\therefore$ NOTA's D

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7/ older person in front of younger

$\frac{1}{2} \times 6! = 360$

B

8/ $(3-2\sqrt{2})^3$

$= 3^3 - 3 \times 3^2 \times 2\sqrt{2} + 3 \times 3 \times (2\sqrt{2})^2 - (2\sqrt{2})^3$

$= 27 - 54\sqrt{2} + 72 - 16\sqrt{2}$

$= 99 - 70\sqrt{2}$

C

(3/8)

Section II

Q9 a) $|2x+1| \geq 3$

$2x+1 \leq -3 \qquad 2x+1 \geq 3$ (1)

$2x \leq -4 \qquad 2x \geq 2$

$x \leq -2 \qquad x \geq 1$ (1)

b) $P(x) = x^3 + cx + d.$

roots at $x = 1, 1 \,\&\, \alpha$

$1 + 1 + \alpha = \frac{-b}{a} = 0$

$\therefore \alpha = -2$

from $\alpha\beta + \alpha\delta + \beta\delta = \frac{c}{a} = c$

$-2\times1 + -2\times1 + 1\times1 = -3$ (1)

$c = -3$

from $\alpha\beta\delta = \frac{-d}{a} = -d$

$d = -(1\times1\times(-2)) = 2$

$\therefore \quad c = -3$
$\quad d = 2$ (1)

c) $\dfrac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$

$= \dfrac{2\sin^2 x + 2\sin x\cos x}{2\cos^2 x + 2\sin x \cos x}$ (1)

$= \dfrac{2\sin x(\sin x + \cos x)}{2\cos x(\cos x + \sin x)}$ (1)

$= \tan x$

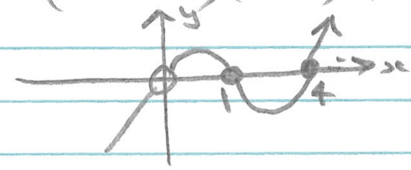
d) $x + \frac{4}{x} \geq 5 \quad (\times x^2)$

$x^3 + 4x \geq 5x^2$ (1)

$x^3 - 5x^2 + 4x \geq 0$

$x(x^2 - 5x + 4) \geq 0$

$x(x-4)(x-1) \geq 0$ (1)



(1)

$0 < x \leq 1$ and $x \geq 4$

e) $P(x)\ k^2x^3 - k^3x^2 - x + k$
$k \neq 0$

i) $P(x) = k^2x^2(x-k) - (x-k)$

$= (x-k)(k^2x^2 - 1)$

$\therefore P(k) = 0$ (1)

or $k^5 - k^5 - k + k = 0$

$\therefore (x-k)$ is a factor)

ii) $(x-k)(k^2x^2 - 1)$ (1)

$= (x-k)(kx-1)(kx+1)$ (1)

for $P(x) = 0 \quad x = k$ or $\pm\frac{1}{k}$

4/8

Q 10

a) i $^9C_4 = 126$ 1

ii $^9C_4 - {^6C_2} \times {^3C_2}$ 1

$= 126 - 45$

$= 81$ 1

b) $2x^3 + 6x^2 + 4x + 1 = 0$

i $(1-\alpha) + (1-\beta) + (1-\gamma)$

$= 3 - (\alpha + \beta + \gamma)$

$= 3 - \left(\frac{-6}{2}\right)$ 1

$= 6$

ii given that $1-\alpha$, $1-\beta$, $1-\gamma$ are roots

$\therefore$ $2x^3 + 6x^2 + 4x + 1 = 0$
is $2(1-x)^3 + 6(1-x)^2 + 4(1-x) + 1 = 0$
gives a constant of 13
with leading co-effic of -2

$\therefore$ $\alpha\beta\gamma = -\frac{d}{a} = -\left(\frac{13}{-2}\right)$

$= 6\frac{1}{2}$

c) $\cos x - 3\sin x + 3$

$\frac{1-t^2}{1+t^2} - 3\frac{2t}{1+t^2} + 3$ 1

$\frac{1-t^2 - 6t + 3 + 3t^2}{1+t^2}$ 1

$= \frac{2t^2 - 6t + 4}{1+t^2}$

$= \frac{2(t^2 - 3t + 2)}{1+t^2}$

$= \frac{2(t-2)(t-1)}{1+t^2}$ 1

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Q11 contd.

Q11 d

i)

$(1+x)^n$

$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots + {}^nC_n x^n$ 1

ii) LHS let $x=1$

$\therefore (1+1)^n = 2^n$

RHS ${}^nC_0 + {}^nC_1(1) + {}^nC_2 1^2 + {}^nC_3 1^3 + {}^nC_n 1^n$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

Question 12

a) $P(x) = x^4 + 4x^3 + 7x^2 + 6x + 2$

$P(-1) = 0$

$P'(x) = 4x^3 + 12x^2 + 14x + 6$

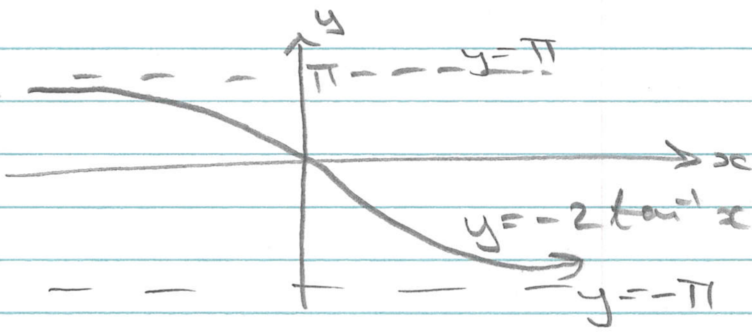
$P'(-1) = 0$

$P''(x) = 12x^2 + 24x + 14$

$P''(-1) \neq 0$

$\therefore -1$ has a multiplicity of 2 for $P(x) = 0$

b) $f(x) = -2\tan^{-1}x$



c) $\sin^2\theta = \cos\theta + 1$

$1 - \cos^2\theta = \cos\theta + 1$

$\cos^2\theta + \cos\theta = 0$

$\cos\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$ or $\cos\theta = -1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \theta = \pi$

Check $\frac{\pi}{2}$ & $\frac{3\pi}{2}$ $\qquad \therefore \theta = \pi$

6/8

Q12d) $x^2+y^2-6x+2y+6=0$

$x^2-6x\_\_\_ + y^2+2y+\_\_\_ = -6$

$x^2-6x+9 + y^2+2y+1 = -6+9+1$

$(x-3)^2 + (y+1)^2 = 4$

centre $(3,-1)$

radius 2

12 e) $f(x) = (x-2)^2$

$f(x) = y = (x-2)^2$

$f^{-1}(x) \cdot x = (y-2)^2$

$\pm\sqrt{x} = y-2$

$2 \pm \sqrt{x} = y$

$y = 2 \pm \sqrt{x}$

but for $f(x)$ $0 \leq y \leq 4$

$0 \leq x \leq 2$

$y = 2 - \sqrt{x}$

$D: [0, 4]$

$\frac{7}{8}$

Q 13

a) TRIANGLE

i) $\boxed{\text{IIAE}} \leftrightarrow \boxed{\text{TRNGL}}$

$2 \times 3! \times 5! = 1440$

ii) 1^{st} & last 5×4
rest arranged $6!$ $\Rightarrow 5 \times 4 \times 6!$ 14400

b) $\sec x + \tan x = k$

$(\sec x + \tan x)(\sec x - \tan x) = \sec^2 - \tan^2 x = 1$

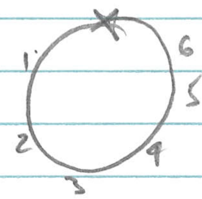
$\therefore \sec x - \tan x = \dfrac{1}{\sec x + \tan x}$

so $\sec x + \tan x = k$

$\sec x - \tan x = \dfrac{1}{k}$

$2 \sec x = k + \dfrac{1}{k}$

$\sec x = \dfrac{1}{2}\left(k + \dfrac{1}{k}\right)$

c) i) 

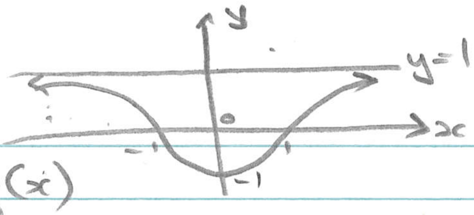
1 fixed $\therefore 6! = 720$

ii) Fix one girl $\therefore$ $2!$ ways for boys
$5!$ for 4 girls and 1 boy

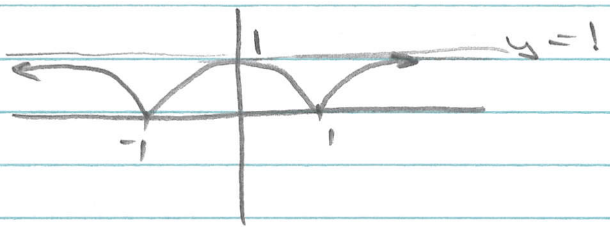
Total $5! \times 2! = 240$

8/8

d) from

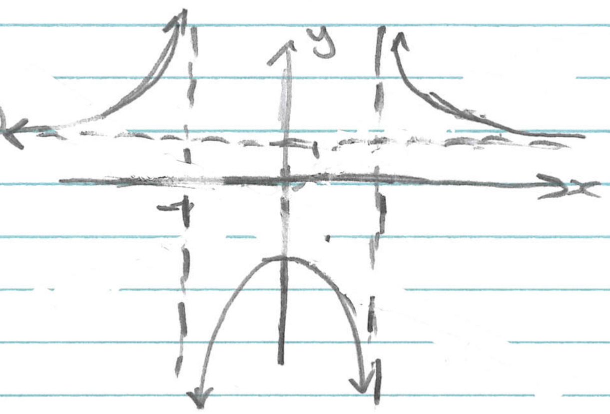


13 i) $y = |f(x)|$



① for +ve
① for shape

ii) $y = \frac{1}{f(x)}$



① for Assymptotes
① for shape

e) $\cos x - \sqrt{3}\sin x$

$R\cos(x+\alpha) \equiv \cos x - \sqrt{3}\sin x$

$R\cos\alpha = 1$ $\therefore \tan\alpha = \sqrt{3}$

$R\sin\alpha = \sqrt{3}$ $\alpha = \frac{\pi}{3}$

$\& \; R^2 = 1 + 3$

$R^2 = 4$

$R = 2$ for $R > 0$

$\therefore 2\cos\left(x + \frac{\pi}{3}\right)$