HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11

Assessment 3 Term 3 2019

STUDENT NUMBER: ______________________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 80 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks –

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

Section II Pages 5 – 14

47 marks

Attempt Questions 6 – 9.

Question	***1-5***	***6***	***7***	***8***	***9***	***Total***
Total	*/5*	*/12*	*/11*	*/11*	*/13*	*/52*

This assessment task constitutes 40% of the Preliminary Higher School Certificate Course School Assessment

Section I
5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. What is the derivative of $y = x^2 e^{2x}$?

(A) $y' = 4xe^{2x}$

(B) $y' = 2e^{2x}(1+x)$

(C) $y' = 2xe^{2x}(1+x)$

(D) $y' = 2xe^2$

Q2. Which of the following pairs of equations are parametric equations for $y = x^2 - 2x$?

(A) $x = t+1,\ \ y = t^2 - 1$

(B) $x = t-1,\ \ y = t^2 + 1$

(C) $x = t,\ \ y = t - 2$

(D) $x = t,\ \ y = t^2 - 2$

Q3. Simplify $\dfrac{1+\tan 5x \tan 3x}{\tan 5x - \tan 3x}$

(A) $\cot 2x$

(B) $\tan 2x$

(C) $\dfrac{\tan 2x}{1+\tan 5x \tan 3x}$

(D) $\cot x$

Q4. How many solutions does the equation $(\cot x - 1)(\cos x + 1) = 0$ have in the domain $0 \le x \le 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

Q5. If $t = \tan\frac{\theta}{2}$ which of the following expressions is equivalent to $4\sin\theta + 3\cos\theta + 5$?

(A) $\frac{2(t+2)^2}{1-t^2}$

(B) $\frac{(t+4)^2}{1-t^2}$

(C) $\frac{2(t+2)^2}{1+t^2}$

(D) $\frac{(t+4)^2}{1+t^2}$

End of Section I

Blank page

Section II

47 marks
Attempt Questions 6-9
Allow about 70 minutes for this section

Question 6 (12 marks)

(a) What is the remainder when $P(x) = 2x^4 - 5x^3 + 12x - 7$ is divided by $(x-2)$? **1**

....

....

(b) (i) Write down the expansion for $\cos(A+B)$. **1**

....

(ii) Hence, or otherwise, find the exact value of $\sec 105°$. Express your answer with a rational denominator. **2**

....

....

....

....

....

....

(c) Solve for x: $\dfrac{x-5}{x+1} \geq 3$. **2**

....

....

....

....

....

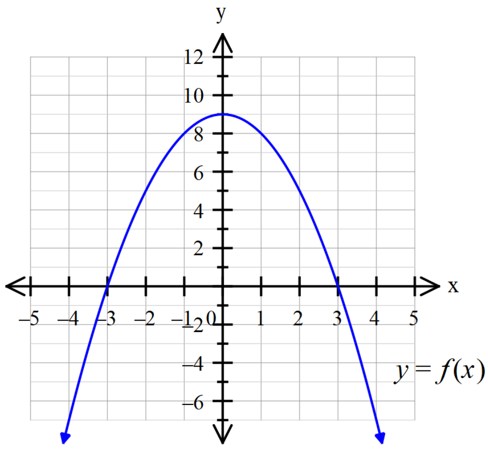
(d) State the exact value of $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)$. **2**

....

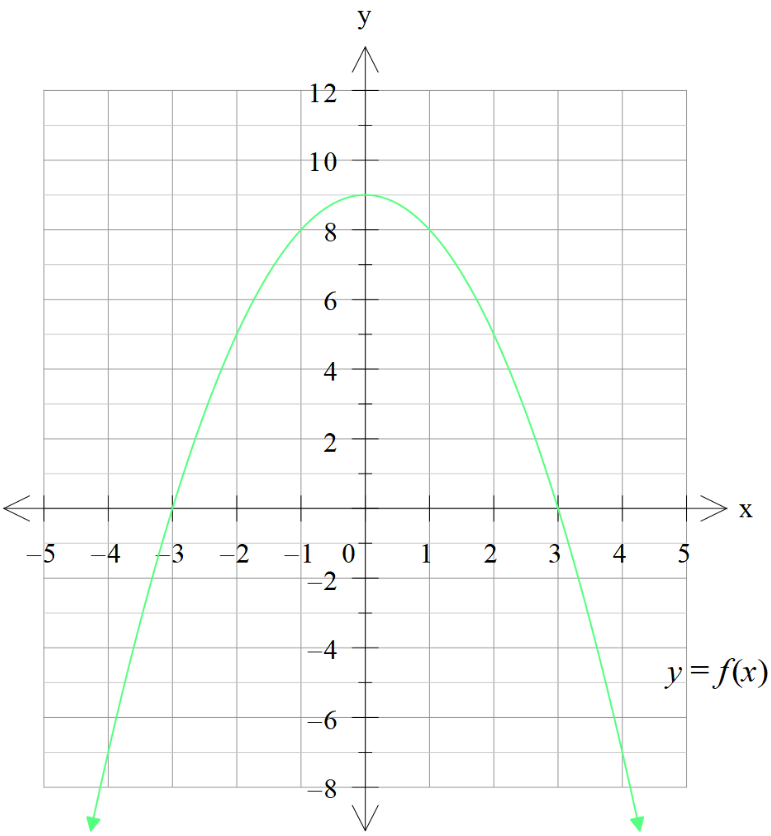
....

(e) The graph $f(x) = 9 - x^2$ is drawn below.

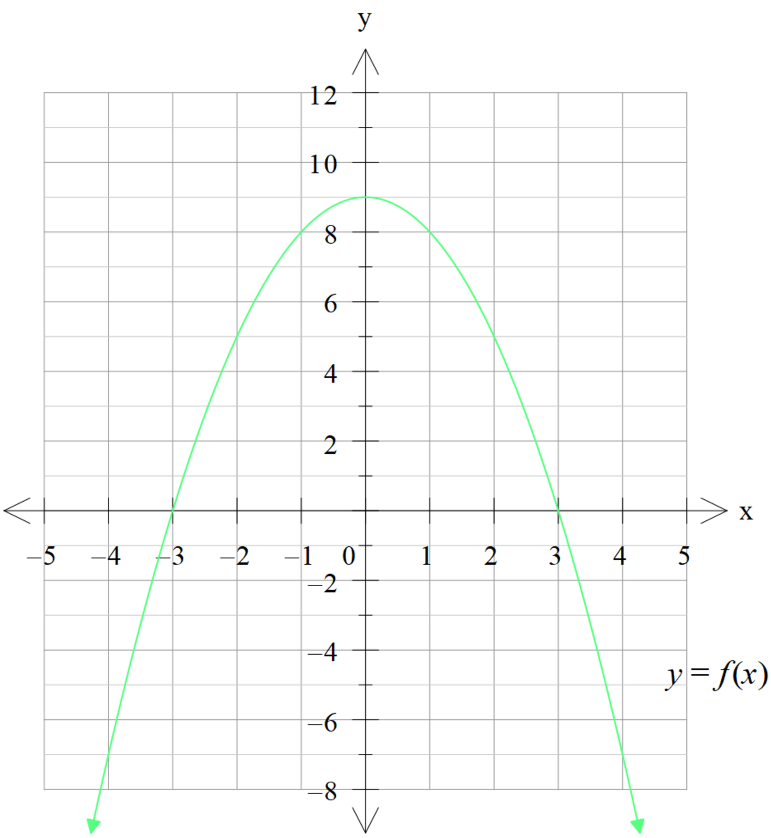
NOT TO SCALE



(i) On the graph below, sketch $y = \dfrac{1}{f(x)}$, showing all important features including intercepts with the coordinate axes and asymptotes. **2**



(ii) On the graph below, sketch $y = |f(x)| - 3$, showing all important features including intercepts with the coordinate axes and asymptotes. **2**



Question 7 (11 marks)

(a) If α, β and γ are the roots of $3x^3 - 6x^2 + 3x - 1 = 0$, find the value of **2**

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

....

....

....

....

....

....

....

....

(b) Solve for θ, $0 \le \theta \le 2\pi$, $\quad \cos 2\theta = \sin\theta$ **3**

....

....

....

....

....

....

....

....

Question 7 continues on the next page

(c) A polynomial is defined as $P(x) = (x-2)^2(x+2)^2$

(i) Neatly sketch $y = P(x)$, clearly showing all intercepts with the coordinate axes. **2**

(ii) Hence, or otherwise, find the value(s) of k for which $(x-2)^2(x+2)^2 = k$ has at least 3 solutions. **1**

....

....

(d) Let $f(x) = x^3 + kx^2 + 3x - 10$, where k is a constant. Find the values of k for which $f'(x) = 0$ has exactly two solutions. **3**

....

....

....

....

....

....

Question 8 (11 marks)

(a) Sketch the function $y = 3\cos^{-1} 2x$ **2**

(b) (i) Show that $(x+3)$ is a factor of $P(x) = x^3 - 5x + 12$ **1**

....

....

(ii) Show that $(x+3)$ is the only linear factor of $P(x)$ over the field of Real Numbers for x. **2**

....

....

....

....

....

....

....

....

Question 8 continues on the next page

(c) Find the exact value of $\sin\left(2\tan^{-1}(\frac{3}{4})\right)$ **2**

....

....

....

....

(d) The rate of growth of a seal colony is proportional to the excess of the colony's population, N, over 500 and is given by $\frac{dN}{dt} = k(N - 500)$.

(i) Show that $N = 500 + Ae^{kt}$, where t is the time in years, satisfies the differential equation $\frac{dN}{dt} = k(N - 500)$. **1**

....

....

....

(ii) The initial population of seals in the colony was 1 500. After five years, the seal colony had doubled in size. Find the exact values of A and k. **2**

....

....

....

....

....

(iii) Calculate the expected population of seals in the colony after 7 years. Answer correct to the nearest seal. **1**

....

....

Question 9 (13 marks)

(a) Let $P(x) = (x+1)(x-3)Q(x) + a(x+1) + b$ where $Q(x)$ is a polynomial where a and b are real numbers.

When $P(x)$ is divided by $(x+1)$, the remainder is -11.

When $P(x)$ is divided by $(x-3)$, the remainder is 1.

(i) What is the value of b? **1**

....

....

....

....

(ii) What is the remainder when $P(x)$ is divided by $(x+1)(x-3)$? **2**

....

....

....

....

....

....

....

....

Question 9 continues on the next page

(b) Use products to sums formulas, or otherwise, to solve **3**

$$2(\sin\theta + 2\cos 2\theta \sin\theta) = 1 \qquad \text{for} \quad 0 \le \theta \le \pi.$$

....

....

....

....

....

....

....

....

....

....

(c) A spherical balloon is being deflated at a rate of 2π cm^3/s. Calculate the rate of change of its radius at the instant the radius is 5cm. **3**

....

....

....

....

....

....

....

....

....

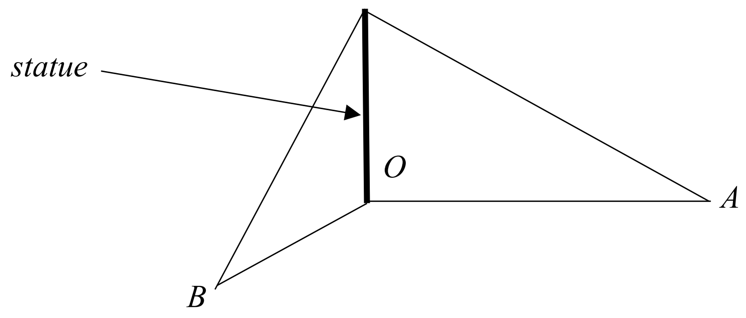
....

....

Question 9 continues on the next page

Question 9 continued:

(d) Two tourists, A and B, are observing the top of an ancient statue. Tourist A is standing due east of the statue whereas Tourist B is standing on a bearing of $210°$ from the base of the statue. The angles of elevation of the top of the statue from Tourist A and Tourist B are $43°$ and $61°$ respectively. Let the base of the statue be O.



Not to scale

(i) Explain why $\angle AOB = 120°$ **1**

....

....

....

(ii) If h is the height of the statue and the distance between the tourists is 20 metres, show that: **3**

$$h = \frac{20}{\sqrt{\tan^2 29° + \tan^2 47° + \tan 29° \tan 47°}}$$

....

....

....

....

....

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………

End of paper

HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11 Assessment 3 Term 3 2019

STUDENT NUMBER:...

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 3 Term 3 2019

STUDENT NUMBER: SOLUTIONS

General Instructions

- Reading Time – 5 minutes
- Working Time – 80 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks –

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

Section II Pages 5 – 14

47 marks

Attempt Questions 6 – 9.

Question	*1-5*	*6*	*7*	*8*	*9*	*Total*
Total	*/5*	*/12*	*/11*	*/11*	*/13*	*/52*

This assessment task constitutes 40% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. What is the derivative of $y = x^2e^{2x}$?

$y' = x^2 \cdot 2e^{2x} + 2xe^{2x}$

$y' = 2xe^{2x}(x+1)$

(A) $y' = 4xe^{2x}$

(B) $y' = 2e^{2x}(1+x)$

(C) $y' = 2xe^{2x}(1+x)$

(D) $y' = 2xe^2$

Q2. Which of the following pairs of equations are parametric equations for $y = x^2 - 2x$?

(A) $x = t+1,\ y = t^2 - 1$

(B) $x = t-1,\ y = t^2 + 1$

(C) $x = t,\ y = t-2$

(D) $x = t,\ y = t^2 - 2$

$(t+1)^2 - 2(t+1)$

$= t^2 + 2t + 1 - 2t - 2$

$= t^2 - 1$

Q3. Simplify $\dfrac{1+\tan 5x \tan 3x}{\tan 5x - \tan 3x}$

$= \dfrac{1}{\dfrac{\tan 5x - \tan 3x}{1 + \tan 5x \tan 3x}}$

$= \dfrac{1}{\tan(5x - 3x)}$

$= \dfrac{1}{\tan 2x}$

$= \cot 2x$

(A) $\cot 2x$

(B) $\tan 2x$

(C) $\dfrac{\tan 2x}{1+\tan 5x \tan 3x}$

(D) $\cot x$

Q4. How many solutions does the equation $(\cot x - 1)(\cos x + 1) = 0$ have in the domain

$0 \le x \le 2\pi$?

$\cot x = 1$, $\cos x = -1$

$\tan x = 1$ $\quad x = \pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \pi$

but $x = \pi$ is undefined for $\cot x$

$\therefore$ only 2 solutions

(A) 1

(B) 2

(C) 3

(D) 4

Q5. If $t = \tan\frac{\theta}{2}$ which of the following expressions is equivalent to $4\sin\theta + 3\cos\theta + 5$?

(A) $\frac{2(t+2)^2}{1-t^2}$

(B) $\frac{(t+4)^2}{1-t^2}$

(C) $\frac{2(t+2)^2}{1+t^2}$

(D) $\frac{(t+4)^2}{1+t^2}$

$$4\left(\frac{2t}{1+t^2}\right) + 3\left(\frac{1-t^2}{1+t^2}\right) + 5$$

$$= \frac{8t + 3 - 3t^2 + 5 + 5t^2}{1+t^2}$$

$$= \frac{2t^2 + 8t + 8}{1+t^2}$$

$$= \frac{2(t^2 + 4t + 4)}{1+t^2}$$

$$= \frac{2(t^2+2)^2}{1+t^2}$$

End of Section I

Section II

47 marks
Attempt Questions 6-9
Allow about 70 minutes for this section

Question 6 (12 marks)

(a) What is the remainder when $P(x) = 2x^4 - 5x^3 + 12x - 7$ is divided by $(x-2)$? **1**

$P(2) = 2(2)^4 - 5(2)^3 + 12(2) - 7$

$= 9$ ✓

(b) (i) Write down the expansion for $\cos(A+B)$. **1**

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

(ii) Hence, or otherwise, find the exact value of sec 105°. **2**

$\sec 105° = \dfrac{1}{\cos 105°} = \dfrac{1}{\cos(60°+45°)}$

$= \dfrac{1}{\cos 60 \cos 45 - \sin 60 \sin 45}$

$= \dfrac{1}{\frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}}$

$= \dfrac{1}{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}$

$= \dfrac{4}{\sqrt{2} - \sqrt{6}} = \dfrac{-\cancel{4}(\sqrt{2}+\sqrt{6})}{\cancel{2-6}} = -(\sqrt{2}+\sqrt{6})$.

Many students made technical error of $\dfrac{1}{\cos 60° \cos 45° - \sin 60° \sin 45°} = \sec 60° \sec 45° - \operatorname{cosec} 60° \operatorname{cosec} 45°$

(c) Solve for x: $\dfrac{x-5}{x+1} \geq 3$. **2**

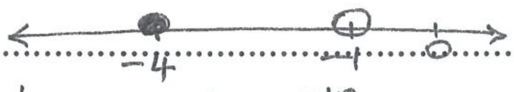
CV: $x \neq -1$

Consider: $\dfrac{x-5}{x+1} = 3$

$x - 5 = 3x + 3$

$-8 = 2x$

$\therefore x = -4$



Test: $x = -5$, $\frac{-10}{-4} \ngeq 3$ ✗

$x = -2$, $\frac{-7}{-1} \geq 3$ ✓

$x = 0$, $\frac{-5}{1} \ngeq 3$ ✗

$\therefore -4 \leq x < 1$

• Many multiplied both sides with perfect square then got lost in simplifying the inequation.
• Many forgot to exclude $x = -1$.

(d) State the exact value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. **2**

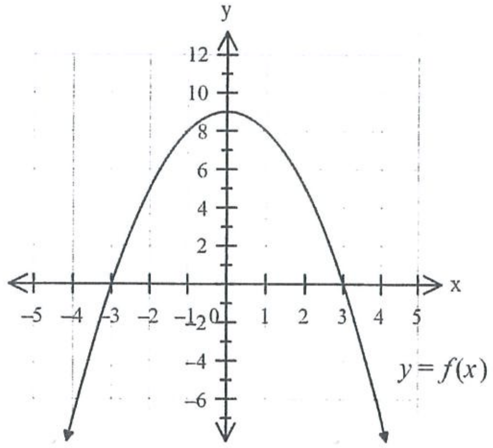
$\cos\frac{7\pi}{6} = \cos\frac{5\pi}{6}$ $\quad \therefore \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$= \frac{5\pi}{6}$

many did not understand ~~and~~ how the domain applies.

(e) The graph $f(x) = 9 - x^2$ is drawn below.

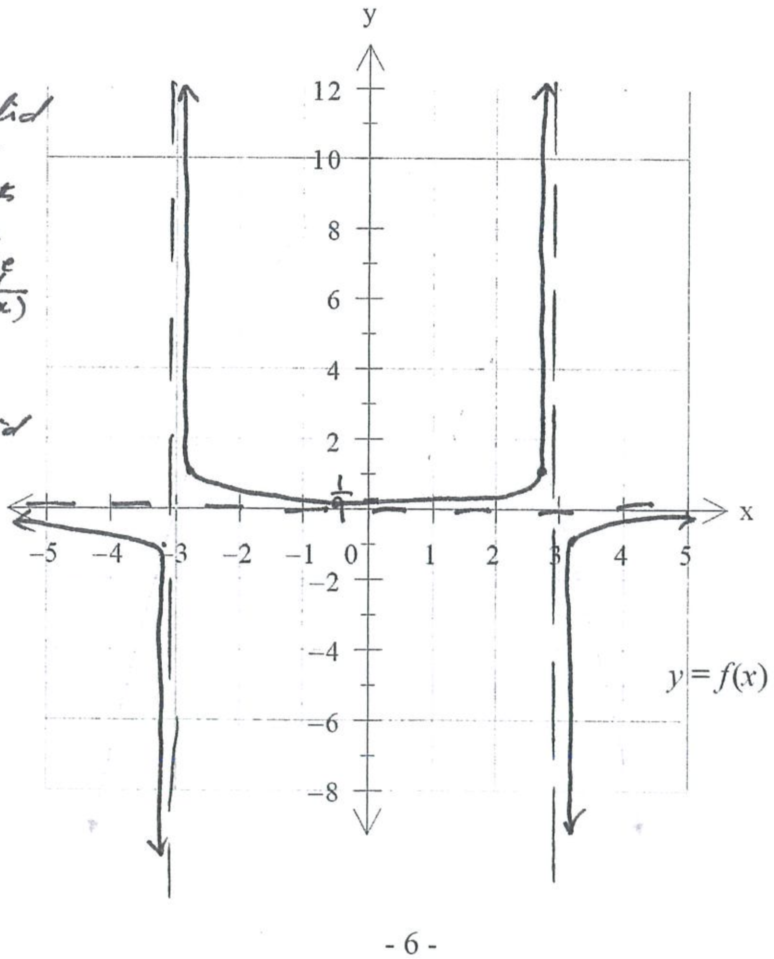
NOT TO SCALE



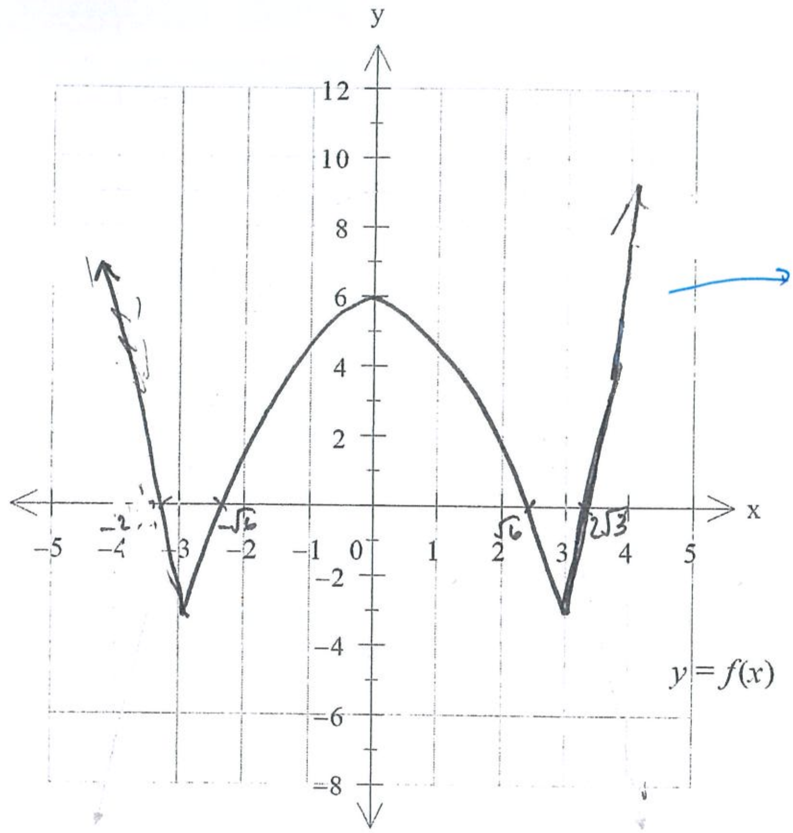
(i) On the graph below, sketch $y = \frac{1}{f(x)}$, showing all important features including intercepts with the coordinate axes and asymptotes. **2**

1) Many students did not pay attention to the fact that the points at line $y = \pm 1$ stay the same, hence the shape of the graph of $y = f(x)$ ~~will not be sharp~~. is broader

② many students did not notice that as $x \to \pm\infty$, $y \to 0$, hence the x-axis is the horizontal asymptote.



(ii) On the graph below, sketch $y = |f(x)| - 3$, showing all important features including intercepts with the coordinate axes and asymptotes. **2**



→ Many flared the amount.

$y = |9 - x^2| - 3$

$0 = 9 - x^2 - 3$ or $0 = -9 + x^2 - 3$ for x-intercepts

$0 = 6 - x^2$ $\qquad 0 = x^2 - 12$

$\therefore x = \pm\sqrt{6}$ $\qquad x^2 = 12$

$\therefore x = \pm 2\sqrt{3}$

$\therefore$ x-int. are: $x = \pm\sqrt{6}$, $x = \pm 2\sqrt{3}$

$y = |9 - 0| - 3$

$y = 6$

$\therefore$ y-int: $y = 6$

Question 7 (11 marks)

(a) If α, β and γ are the roots of $3x^3 - 6x^2 + 3x - 1 = 0$, find the value of **2**

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$$

$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{1}{\frac{1}{3}}$

$= 3$

$\alpha\beta + \beta\gamma + \gamma\alpha = 1$

$\alpha\beta\gamma = \frac{1}{3}$

Generally well done. However some students do not remember the connection b/w the roots & the coefficients.

(b) Solve for θ, $0 \le \theta \le 2\pi$, $\cos 2\theta = \sin\theta$ **3**

$1 - 2\sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\therefore \sin\theta = \frac{1}{2}, \quad \sin\theta = -1$

+ | + ref. $\angle = \frac{\pi}{6}$

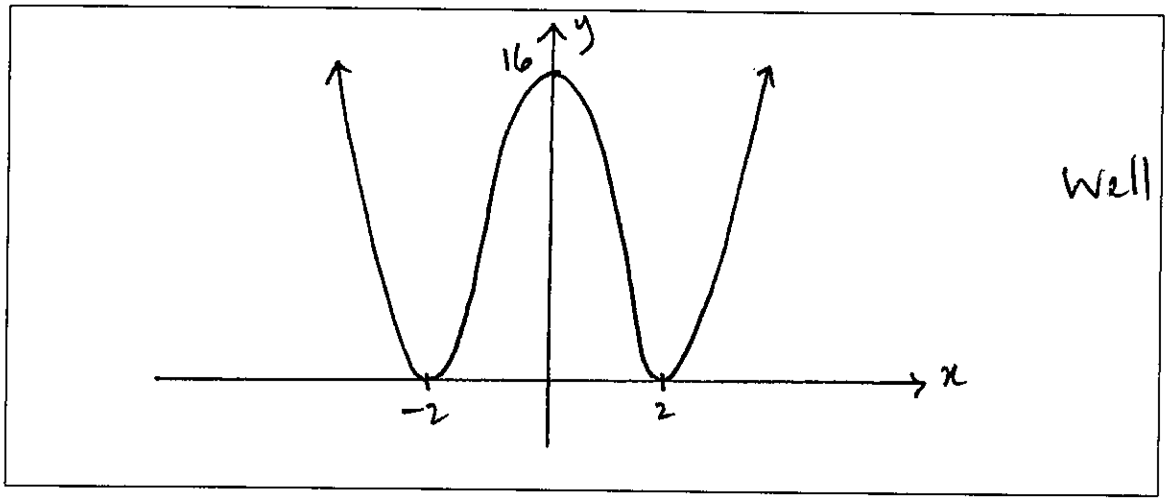
$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Poorly done. Some students wrote $\cos 2\theta$ as $1 - \sin^2\theta$. Others wrote $\cos 2\theta = \cos(90 - \theta)$ & only solved for acute values of θ. Some students forgot to convert the angles from degrees to radians

Question 7 continues on the next page

(c) A polynomial is defined as $P(x)=(x-2)^2(x+2)^2$

(i) Neatly sketch $y=P(x)$, clearly showing all intercepts with the coordinate axes. **2**



Well done!

(ii) Hence, or otherwise, find the value(s) of k for which $(x-2)^2(x+2)^2=k$ has at least 3 solutions. **1**

$\therefore 0 < k \leq 16$

Poorly done. Students did not make the connection of $y=k$ on the above graph.

(e) Let $f(x)=x^3+kx^2+3x-10$, where k is a constant. Find the values of k for which $f'(x)=0$. **3**

$f'(x) = 3x^2 + 2kx + 3$

$0 = 3x^2 + 2kx + 3$

$\Delta = 4k^2 - 4(3)(3) > 0$ for two solutions

$4k^2 - 36 > 0$

$k^2 - 9 > 0$

$\therefore (k-3)(k+3) > 0$

$\therefore k < -3,\ k > 3$

Poorly done. Very few students used $\Delta > 0$ for two solutions

Question 8 (11 marks)

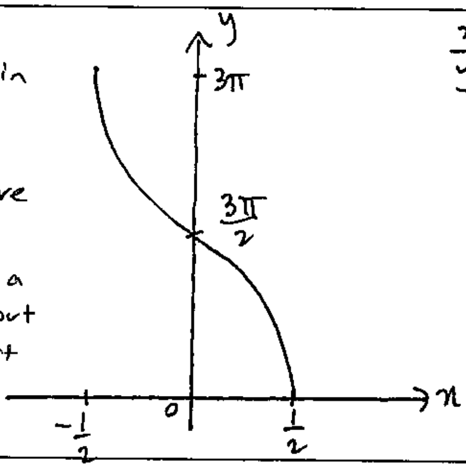
$-1 \le 2x \le 1$

(a) Sketch the function $y = 3\cos^{-1} 2x$ $\therefore -\frac{1}{2} \le x \le \frac{1}{2}$ **2**

1 for correct domain and range

1 for accurate curve

Most students had a reasonable curve but many had incorrect domain and range.



x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	3π	$\frac{3\pi}{2}$	0

The most common error was
$D: -3 \le x \le 3$
$R: 0 \le y \le \pi$

(b) (i) Show that $(x+3)$ is a factor of $P(x) = x^3 - 5x + 12$ **1**

$P(-3) = (-3)^3 - 5(-3) + 12$

$P(-3) = -27 + 15 + 12$

$P(-3) = 0$

Since $P(-3) = 0$, it follows that $(x+3)$ is a factor of $P(x)$.

(ii) Show that $(x+3)$ is the only linear factor of $P(x)$ over the field of Real Numbers for x. **2**

$$\begin{array}{r l}
 & x^2 - 3x + 4 \\
x+3 \,) & x^3 + 0x^2 - 5x + 12 \\
 & x^3 + 3x^2 \\
 & \quad -3x^2 - 5x \\
 & \quad -3x^2 - 9x \\
 & \qquad 4x + 12 \\
 & \qquad 4x + 12 \\
 & \qquad\quad 0
\end{array}$$

$\therefore P(x) = (x+3)(x^2 - 3x + 4)$

for $x^2 - 3x + 4$, $\Delta = 9 - 4(4)$

$\Delta = -7$

$\Delta < 0$

$\therefore$ no real solutions

$\therefore (x+3)$ is the only real linear factor.

Some students checked all the integral zeros $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$ and found only $x = -3$ satisfies $P(x) = 0$ and then concluded that $(x+3)$ is the only linear factor. However, this does not consider the possibility of irrational (or complex) zeros.

e.g. $P(x) = x^3 + 2x^2 - 3x - 6$ could be checked for integral zeros at ±1, ±2, ±3, ±6 and -2 works. $P(x) \div (x+2)$ gives

$P(x) = (x+2)(x^2 - 3)$

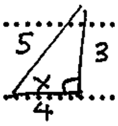
$\therefore$ the zeros are $-2, \pm\sqrt{3}$

Question 8 continues on the next page

(c) Find the exact value of $\sin\left(2\tan^{-1}(\frac{3}{4})\right)$ **2**

let $x = \tan^{-1}\frac{3}{4}$

$\therefore \tan x = \frac{3}{4}$



Must show sufficient working for a 2mark question.

$\therefore \sin 2x = 2\sin x\cos x$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

Correct answer with insufficient working was awarded one mark.

(c) The rate of growth of a seal colony is proportional to the excess of the colony's population, N, over 500 and is given by $\frac{dN}{dt} = k(N - 500)$.

Working must demonstrate that students know how to differentiate Ae^{kt}

(i) Show that $N = 500 + Ae^{kt}$, where t is the time in years, satisfies the differential equation $\frac{dN}{dt} = k(N - 500)$. **1**

Very poor setting out by a large number of students.

$N = 500 + Ae^{kt}$ $\qquad N - 500 = Ae^{kt}$ —— ①

$\frac{dN}{dt} = kAe^{kt}$ $\qquad \therefore N = 500 + Ae^{kt}$ satisfies

$= k(N-500)$ from ① $\qquad$ the differential equation

(ii) The initial population of seals in the colony was 1 500. After five years, the seal colony had doubled in size. Find the exact values of A and k. **2**

when $t=0$, $N=1500$

$1500 = 500 + Ae^0$

$\therefore \boxed{A = 1000}$

$\therefore N = 500 + 1000e^{kt}$

when $t=5$, $N=3000$

$3000 = 500 + 1000e^{5k}$

$\frac{2500}{1000} = e^{5k}$

$\therefore 5k = \ln\frac{5}{2}$

$\therefore k = \frac{1}{5}\ln\frac{5}{2}$

Many students incorrectly found that $A = 1500$ and then $k = \frac{1}{5}\ln\frac{5}{3}$ which gave $N = 3567$

(iii) Calculate the expected population of seals in the colony after 7 years. **1**

Answer correct to the nearest seal.

$N = 500 + 1500e^{7(\frac{1}{5}\ln\frac{5}{3})}$

$N \doteqdot 3567$ seals.

Question 9 (13 marks)

(a) Let $P(x) = (x+1)(x-3)Q(x) + a(x+1) + b$ where $Q(x)$ is a polynomial where a and b are real numbers.

When $P(x)$ is divided by $(x+1)$, the remainder is -11.

When $P(x)$ is divided by $(x-3)$, the remainder is 1.

(i) What is the value of b? **1**

$-11 = 0 + a(0) + b$

$\therefore b = -11$

(ii) What is the remainder when $P(x)$ is divided by $(x+1)(x-3)$? **2**

$\therefore P(x) = (x+1)(x-3)Q(x) + a(x+1) - 11$

when $x = 3$, $P(3) = 1$

$\therefore 1 = 0 + a(3+1) - 11$

$1 = 0 + 4a - 11$

$12 = 4a$

$\therefore a = 3$ 1 mark

$\therefore$ remainder is $3(x+1) - 11$

$= 3x - 8$ 1 mark

Comment: many worked out a, but not able to see the remainder being $a(x+1)+b$.

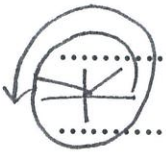
Question 9 continues on the next page

(b) Use products to sums formulas, or otherwise, to solve **3**

$$2(\sin\theta + 2\cos 2\theta \sin\theta) = 1 \qquad \text{for } 0 \le \theta \le \pi.$$

$2\cos 2\theta \sin\theta = \sin 3\theta - \sin\theta$ $\qquad 0 \le \theta \le \pi$

$\therefore 2(\sin\theta + \sin 3\theta - \sin\theta) = 1$ 1 mark



$2\sin 3\theta = 1$

$\therefore \sin 3\theta = \frac{1}{2}$ 1 mark $\qquad \therefore 0 \le 3\theta \le 3\pi$

$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\therefore \theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ 1 mark

Comment: * many couldn't see the identity
$2\cos 2\theta \sin\theta = \sin(2\theta+\theta) - \sin(2\theta-\theta)$.
$= \sin 3\theta - \sin\theta$.

* many didn't check the solutions on the domain.

(c) A spherical balloon is being deflated at a rate of 2π cm^3/s. Calculate the rate of change of its radius at the instant the radius is 5cm. **3**

$\frac{dV}{dt} = -2\pi$ $\qquad V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ 1 mark

$\frac{dV}{dr} = \frac{dV}{dt} \times \frac{dt}{dr}$

or: $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$4\pi r^2 = -2\pi \times \frac{dt}{dr}$ 1 mark

when $r = 5$,

$4\pi(5)^2 = -2\pi \times \frac{dt}{dr}$

$\frac{100\pi}{-2\pi} = \frac{dt}{dr}$

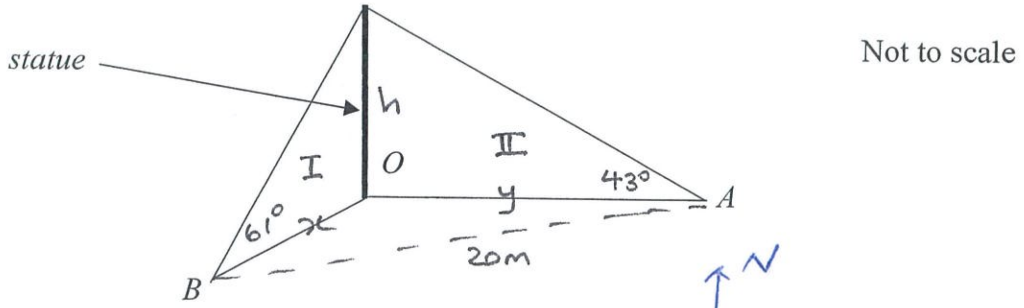
$\therefore \frac{dr}{dt} = \frac{-1}{50}$ $\qquad \therefore$ the radius is decreasing at a rate of $\frac{1}{50}$ cm/s.

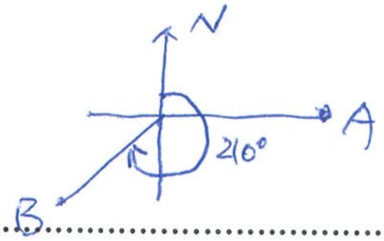
Question 9 continues on the next page

Comment: the answer should state the rate being decreasing or state $\frac{-1}{50}$ cm/s in the answer.

Question 9 continued:

(d) Two tourists, A and B, are observing the top of an ancient statue. Tourist A is standing due east of the statue whereas Tourist B is standing on a bearing of $210°$ from the base of the statue. The angles of elevation of the top of the statue from Tourist A and Tourist B are $43°$ and $61°$ respectively. Let the base of the statue be O.





(i) Explain why $\angle AOB = 120°$ **1**

$\angle AOB = 90° + 30°$

$= 120°$

Or 210° −

(ii) If h is the height of the statue and the distance between the tourists is 20 metres, show that: **3**

$$h = \frac{20}{\sqrt{\tan^2 29° + \tan^2 47° + \tan 29° \tan 47°}}$$

In ΔI, $\tan 61° = \frac{h}{x}$

$\therefore x = \frac{h}{\tan 61}$

$x = h \cot 61$

$\therefore x = h \tan 29°$

Similarly, in ΔII, $y = h \tan 47°$

$\therefore 20^2 = (h\tan 29^\circ)^2 + (h\tan 47^\circ) - 2(h\tan 29)(h\tan 47)\cos 120^\circ$

$20^2 = h^2\tan^2 29^\circ + h^2\tan^2 47^\circ - 2h^2\tan 29^\circ \tan 47^\circ\left(-\frac{1}{2}\right)$

$20^2 = h^2\tan^2 29^\circ + h^2\tan^2 47^\circ + h^2\tan 29^\circ \tan 47^\circ$

$20^2 = h^2\left[\tan^2 29^\circ + \tan^2 47^\circ + \tan 29^\circ \tan 47^\circ\right]$

$$\therefore h^2 = \frac{20^2}{\tan^2 29^\circ + \tan^2 47^\circ + \tan 29^\circ \tan 47^\circ}$$

$$h = \frac{20}{\sqrt{\tan^2 29^\circ + \tan^2 47^\circ + \tan 29^\circ \tan 47^\circ}}, \quad h > 0$$

comment: - need to show $\cos 120^\circ = -\frac{1}{2}$
- ~~many content~~
need to show $\frac{1}{\tan\theta} = \tan(90^\circ - \theta)$
or work out the
tan ratio for the
complement angle.

End of paper