HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 3 Term 3 2021
Online

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 40 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- In Questions 6 – 9, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape

Total marks – 25

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Clearly write your answers for Questions 1 – 5 on one sheet of paper for scanning.

You may wish to print off, fill in the objective response answer sheet if and upload a copy of that for section I

Section II Pages 4 – 5

20 marks

Attempt Questions 6 – 9.

Clearly write your answers for Questions 6 – 9 on a separate pages of writing paper for scanning.

Question	1-5	6	7	8	9	Total
Total	/5	/5	/5	/5	/5	/25

This assessment task constitutes 25% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Clearly write your answers for Questions 1 – 5 on one sheet of paper for scanning

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Q1. If $(x+m)$ is a factor of $8x^3 - 14x^2 - m^2x$ where $m \in R,\ m \neq 0$, then the value of m is

(A) 7

(B) 4

(C) 1

(D) –2

Q2. Which polynomial has a multiple root at $x = 1$?

(A) $x^5 - x^4 - x^2 + 1$

(B) $x^5 - x^4 - x - 1$

(C) $x^5 - x^3 - x^2 + 1$

(D) $x^5 - x^3 - x + 1$

Q3. Which of the following is the domain of $f(x) = \sin^{-1}\left(\frac{3}{x}\right)$

(A) $(-\infty, -3] \cup [3, \infty)$

(B) $[-3, 3]$

(C) $\left(-\infty, -\frac{6}{\pi}\right] \cup \left[\frac{6}{\pi}, \infty\right)$

(D) $\left[-\frac{6}{\pi}, \frac{6}{\pi}\right]$

Q4 The angle θ satisfies $\sin\theta = \frac{12}{13}$ for $\frac{\pi}{2} < \theta < \pi$.

The value of $\cos 2\theta$ is…

(A) $\frac{10}{13}$

(B) $-\frac{10}{13}$

(C) $\frac{119}{169}$

(D) $-\frac{119}{169}$

Q5. Which expression is equivalent to $\frac{\tan 3x - \tan x}{1 + \tan 3x \tan x}$?

(A) $\tan 2x$

(B) $\tan 4x$

(C) $\tan 6x$

(D) $\frac{\tan 3x - 1}{1 + \tan 3x}$

End of Section I

Section II

20 marks
Attempt Questions 6-9
Allow about 30 minutes for this section
Clearly write your answers for Questions 6 – 9
on separate pages for scanning

Question 6 (5 marks) START A NEW PAGE

(a) Find all zeros of $P(x) = x^3 - 6x^2 + 5x + 12$ **3**

(b) The roots of $x^3 - 3x + 1 = 0$ are α, β and γ. Find the value of

(i) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **1**

Question 7 (5 Marks) START A NEW PAGE

(a) Find the exact value of $\sin\left(2\cos^{-1}\left(\dfrac{2}{3}\right)\right)$ in simplest surd form. **2**

(b) Prove that $\tan^{-1}\left(\dfrac{5}{12}\right) + \cos^{-1}\left(\dfrac{5}{13}\right) = \dfrac{\pi}{2}$ **3**

Question 8 (5 Marks) START A NEW PAGE

(a) By letting $t = \tan\frac{\theta}{2}$ or otherwise, show **2**

$$\cos ec\,\theta + \cot\theta = \cot\frac{\theta}{2}$$

(b) (i) Show $\sin x + \sin 3x = 2\sin 2x\cos x$ **1**

(ii) Hence solve $\sin x + \sin 2x + \sin 3x = 0$ for $0 \le x \le 2\pi$ **2**

Question 9 (5 marks) START A NEW PAGE

(a) If the roots of $P(x) = 2x^3 - 7x^2 - 12x + 45$ are α, β and γ, **2**
Find the polynomial whose roots are double that of *P(x)*.

(b) Solve $x^3 - 5x^2 + 3x + 9 = 0$ given it has a root of multiplicity 2 **3**

End of paper

Mathematics Extension 1

Year 11 Assessment 3 Term 3 2021

STUDENT NUMBER:…………………………………………………………………

Section I – Objective Response Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

Assessment 3 Solutions Ex1 2021

Q1 $8(-m)^3 - 14(-m)^2 - m^2(-m) = 0$

$-8m^3 - 14m^2 + m^3 = 0$

$-7m^3 - 14m^2 = 0$

$-7m^2(m+2) = 0$ (D)

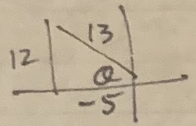
$\therefore m = 0$ or $m = -2$

$m \neq 0$ $\therefore m = -2$

Q2 Graph is a shift of a $\sin^{-1}x$ half unit to the right ~~left~~ and half as wide. $\therefore 2(x - \frac{1}{2})$

$\therefore y = \sin^{-1}(2x-1)$ (B)

Q3. $\sin\theta = \frac{12}{13}$ 2nd Quad



$\therefore \cos 2\theta = 2\cos^2\theta - 1$

$= 2 \times \left(-\frac{5}{13}\right)^2 - 1$ (D)

$= -\frac{119}{169}$

Q4 $\dfrac{\tan 3x - \tan x}{1 + \tan 3x \tan x} = \tan(3x - x) = \tan 2x$ (A)

Q5. $\tan^{-1}\left(\tan\left(-\frac{4\pi}{3}\right)\right) = \cancel{\tan^{-1}\left(\frac{\pi}{3}\right)}$ (B)

$= \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)$

$= -\frac{\pi}{3}$

Question 6

(a) $P(x) = x^3 - 6x^2 + 5x + 12$

$P(1) = 1 - 6 + 5 + 12 \neq 0$

$P(-1) = -1 - 6 - 5 + 12 = 0$

$\therefore$ $(x+1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 - 7x + 12 \\
\hline
x+1 & x^3 - 6x^2 + 5x + 12 \\
 & x^3 + x^2 \\
\hline
 & -7x^2 + 5x \\
 & -7x^2 - 7x \\
\hline
 & 12x + 12 \\
 & 12x + 12 \\
\hline
 & 0
\end{array}
$$

$\therefore$ $P(x) = x^3 - 6x^2 + 5x + 12$
$= (x+1)(x^2 - 7x + 12)$
$= (x+1)(x-3)(x-4)$

$\therefore$ zeros are $-1, 3, 4$

25% of students did not show the substitution of P(-1) to obtain zero and fails to communicate what it implies

A few students did not pay attention to the difference between a factor and a zero.

Majority did well showing, and communicating what they are doing.

(b) $x^3 - 3x + 1 = 0$

$\alpha + \beta + \gamma = 0$
$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-3}{1}$
$\alpha\beta\gamma = -\frac{1}{1}$

(i) $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{-3}{1} = -3$

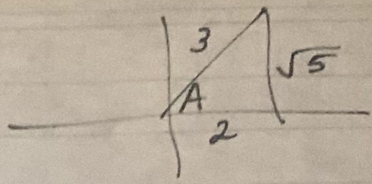
Many did these well.

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$

$= \frac{-3}{-1}$

$= 3$

Question 7(a) $\sin\left(2\cos^{-1}\left(\frac{2}{3}\right)\right)$



$= \sin 2A$

$= 2\sin A\cos A$

$= 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$

$= \frac{4\sqrt{5}}{9}$

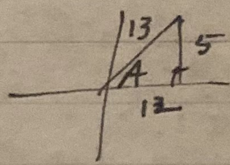
Generally well done. Some students not able to use double angle formula.

(b) RTP $\tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{5}{13}\right) = \frac{\pi}{2}$

ie RTP say $\cos\left(\tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right) = \cos\frac{\pi}{2}$

$LHS = \cos\left(\tan^{-1}\frac{5}{12} + \cos^{-1}\frac{5}{13}\right)$

$= \cos(A+B)$ where $A = \tan^{-1}\frac{5}{12}$

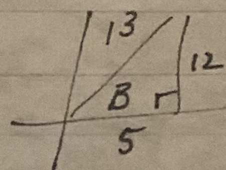


$= \cos A\cos B - \sin A\sin B$

$= \frac{12}{13}\cdot\frac{5}{13} - \frac{5}{13}\cdot\frac{12}{13}$

$= 0$

and where $B = \cos^{-1}\frac{5}{13}$



$RHS = \cos\frac{\pi}{2}$

$= 0$

$\therefore LHS = RHS$

$\therefore \tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{5}{13}\right) = \frac{\pi}{2}$

Generally well done. Many other solutions are also acceptable, eg. using Pythagoras or $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{5}{13} = \frac{\pi}{2}$.

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Q 8 a) By letting $t = \tan\frac{\theta}{2}$ or otherwise, show $\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$

$$\begin{aligned} LHS &= \operatorname{cosec}\theta + \cot\theta \\ &= \frac{1}{\sin\theta} + \frac{1}{\tan\theta} \\ &= \frac{1+t^2}{2t} + \frac{1-t^2}{2t} \\ &= \frac{1+\cancel{t^2}+1-\cancel{t^2}}{2t} \\ &= \frac{2}{2t} \\ &= \frac{1}{t} \\ &= \frac{1}{\tan\frac{\theta}{2}} \\ &= \cot\frac{\theta}{2} \\ &= RHS \end{aligned}$$

1 for correct substitution of t results

1 for showing the required result

b) (i) show $\sin x + \sin 3x = 2\sin 2x \cos x$

$$\begin{aligned} LHS &= \sin x + \sin 3x \\ &= \sin 3x + \sin x \\ &= 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) \\ &= 2\sin\frac{4x}{2}\cos\frac{2x}{2} \\ &= 2\sin 2x \cos x \end{aligned}$$

$LHS = RHS$

1 mark

students need to ensure that they give sufficient working to demonstrate understanding to the marker.

or

$$\begin{aligned} LHS &= \sin x + \sin 3x \\ &= \sin x + \sin(2x+x) \\ &= \sin x + \sin 2x\cos x + \cos 2x \sin x \\ &= \sin x(1+\cos 2x) + 2\sin x\cos x\cdot\cos x \\ &= \sin x(\cancel{1}+2\cos^2 x - \cancel{1}) + 2\sin x\cos^2 x \\ &= 2\sin x\cos^2 x + 2\sin x\cos^2 x \\ &= 4\sin x\cos^2 x \\ &= 2\times 2\sin x\cos x\times\cos x \\ &= 2\sin 2x\cos x \end{aligned}$$

$LHS = RHS$

or

$$\begin{aligned} LHS &= \sin x + \sin 3x \\ &= \sin 3x + \sin x \\ &= \sin(2x+x) + \sin(2x-x) \\ &= 2\sin 2x\cos x \end{aligned}$$

using $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

(ii) Hence solve $\sin x + \sin 2x + \sin 3x = 0$ for $0 \le x \le 2\pi$

$\sin x + \sin 2x + \sin 3x = 0$

$(\sin x + \sin 3x) + \sin 2x = 0$

$2\sin 2x\cos x + \sin 2x = 0$ from part i

$\sin 2x(2\cos x + 1) = 0$

1.

$\sin 2x = 0$

since $0 \le x \le 2\pi$

$0 \le 2x \le 4\pi$

$2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

Some students missed the last two solutions

or

$2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

S | A
T | C

$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Some students gave an additional two solutions

$x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, 2\pi$

1.

* Method 3 (Longest method)
students factorised original. Doubled all its roots then rewrote ~~the~~ required polynomial in factored form $y = (x-6)^2(x+5)$

Question 9 (a)

(a) If $P(x) = 2x^2 - 7x - 12x + 45$ then a polynomial with roots double of this:

Method 1 let say $X = 2x \quad \therefore x = \frac{X}{2}$

$$\therefore P(x) = 2\left(\frac{X}{2}\right)^3 - 7\left(\frac{X}{2}\right)^2 - 12\left(\frac{X}{2}\right) + 45$$

$$= \frac{X^3}{4} - \frac{7X^2}{4} - 6X + 45$$

$\therefore$ a Polynomial that has the roots double of $P(x)$ is any of the form

$$P(X) = X^3 - 7X^2 - 24X + 180$$

If students used this method it was done well

Although a few put in $2X$ rather than $\frac{X}{2}$

Some did X^2 then $\sqrt{X}$

Method 2 If $P(x) = 2x^2 - 7x - 12x + 45$

$$\alpha + \beta + \gamma = \frac{7}{2}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = -6$$

$$\alpha\beta\gamma = -45$$

$$\therefore 2\alpha + 2\beta + 2\gamma = 7$$

$$2\alpha . 2\beta + 2\alpha . 2\gamma + 2\beta 2\gamma = -24$$

$$\text{or } 4(\alpha\beta + \alpha\gamma + \beta\gamma) = -24$$

$$2\alpha . 2\beta . 2\gamma = 8(\alpha\beta\gamma) = -180$$

$$\therefore P(x) = x^3 - 7x^2 - 24x + 180$$

Mixed results using this method.
A lot of students only doubled roots 2 at a time and 4 at a time

Two at a time is $\times 4$ NOT $\times 2$

Three at a time is $\times 8$ NOT $\times 2$

※ Method 3 (longest method)
students factored original. Doubled all its roots then rewrote ~~the~~ required polynomial in factored form $y=(x-6)^2(x+5)$

Question 9

(a) If $P(x)=2x^2-7x-12x+45$ then a polynomial with roots double of this:

Method 1 let say $X=2x \therefore x=\frac{X}{2}$

$$\therefore P(x)=2\left(\frac{X}{2}\right)^3-7\left(\frac{X}{2}\right)^2-12\left(\frac{X}{2}\right)+45$$

$$=\frac{X^3}{4}-\frac{7X^2}{4}-6X+45$$

If students used this method it was done well

Although a few put in $2x$ rather than $\frac{x}{2}$
Some did X^2 then $\sqrt{X}$

$\therefore$ a Polynomial that has the roots double of $P(x)$ is any of the form

$$P(X)=X^3-7X^2-24X+180$$

Method 2 If $P(x)=2x^2-7x-12x+45$

$$\alpha+\beta+\gamma=\frac{7}{2}$$
$$\alpha\beta+\alpha\gamma+\beta\gamma=-6$$
$$\alpha\beta\gamma=-45$$

Mixed results using this method.
A lot of students only doubled roots 2 at a time and 4 at a time

$$\therefore 2\alpha+2\beta+2\gamma=7$$
$$2\alpha.2\beta+2\alpha.2\gamma+2\beta 2\gamma=-24$$
$$\text{or } 4(\alpha\beta+\alpha\gamma+\beta\gamma)=-24$$

Two at a time is $\times 4$
NOT $\times 2$

$$2\alpha.2\beta.2\gamma=8(\alpha\beta\gamma)=-180$$

Three at a time is $\times 8$
NOT $\times 2$

$$\therefore P(x)=x^3-7x^2-24x+180$$