Student Name: ___________________________

Teacher Name: ___________________________

2019

HURLSTONE
AGRICULTURAL
HIGH SCHOOL
YEAR 11

Mathematics Extension 1

Yearly Examination

Examiners

- Mrs P Biczo
- Ms J Fares
- Ms M Sabah
- Ms D Crancher
- Mr G Huxley

General Instructions

- Reading time – 3 minutes
- Working time – 60 minutes
- Write using black/blue pen
- NESA-approved calculators may be used
- A formulae and data sheet is provided
- In Questions 5 – 8, show relevant mathematical reasoning and/or calculations

Total marks: 52

Section I – 4 marks

- Attempt Questions 1 – 4
- Allow about 5 minutes for this section

Section II – 48 marks

- Attempt Questions 5 – 8
- Allow about 55 minutes for this section

2019 Year 11 Yearly Examination ~ Mathematics Extension 1

Section I

4 marks
Attempt Questions 1 - 4
Allow about 5 minutes for this section

Use the **Multiple Choice Answer Sheet** provided for Questions 1 – 4.

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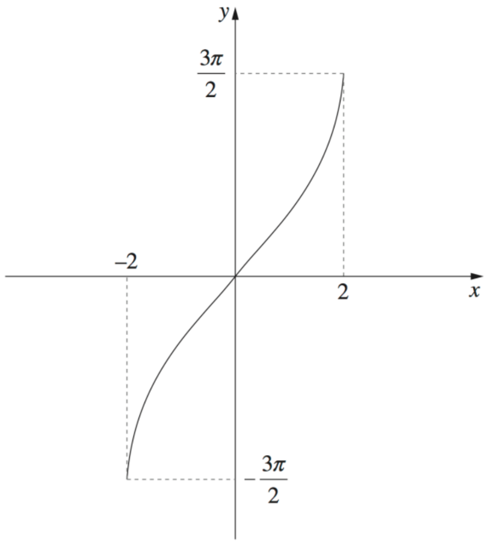
1. Which of the following functions best describes the graph below?



A. $y = 3\sin^{-1}\dfrac{x}{2}$

B. $y = \dfrac{3}{2}\sin^{-1}2x$

C. $y = 3\sin^{-1}2x$

D. $y = \dfrac{3}{2}\sin^{-1}\dfrac{x}{2}$

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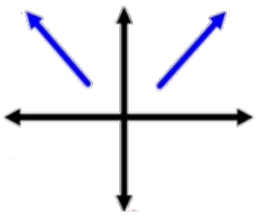
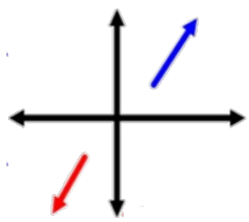
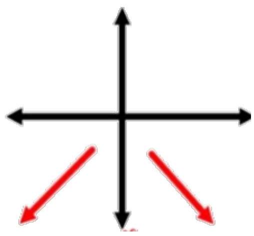
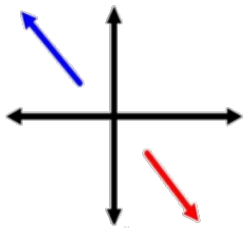
2. Which of the following diagrams shows the end behaviour of a polynomial with leading term $-3x^4$?

A. 

B. 

C. 

D. 

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3. What type of graph do the parametric equations $x = 3\cos t$ and $y = 3\sin t$ represent?

A. a linear graph

B. an hyperbola

C. a parabola

D. a circle

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4. How many unique arrangements of the letters in the word MATHEMATICS are possible?

A. $11\,!$

B. $\frac{11!}{6}$

C. $\frac{11!}{8}$

D. $8\,!$

End of Section I

Section II

48 marks
Attempt Questions 5 - 8
Allow about 55 minutes for this section

Answer the questions in the space provided. Extra space can be found at the end of each question and at the end of the booklet.
ALL necessary working should be shown for each question for **FULL MARKS** to be awarded.

QUESTION 5: (12 marks) **Marks**

(a) Solve $|2-5x| \le 1$ **2**

(b) Consider the function $f(x) = \dfrac{1}{x^2+1}$ for $x \le 0$

(i) Find the domain and range for $f^{-1}(x)$ **2**

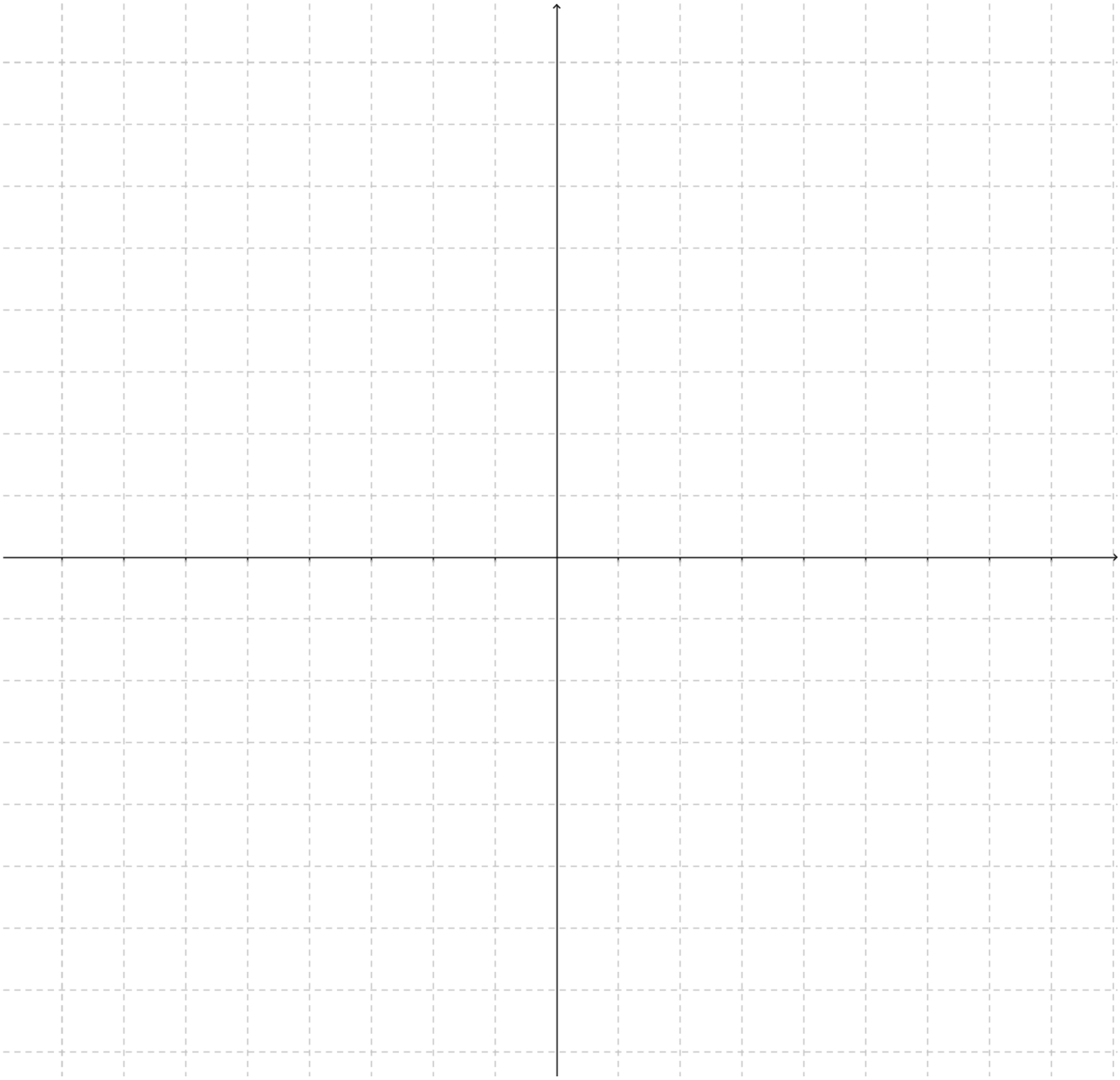
(ii) Find the inverse function $f^{-1}(x)$ **2**

(c) Consider the function $f(x) = (x-2)(x+2)$

(i) Sketch $y = f(x)$ **1**

(ii) Sketch the reciprocal function $y = \dfrac{1}{f(x)}$ on the same graph as part (i) **3**

Show any asymptotes and the positions of any turning points and points of intersection with the curve $y = f(x)$

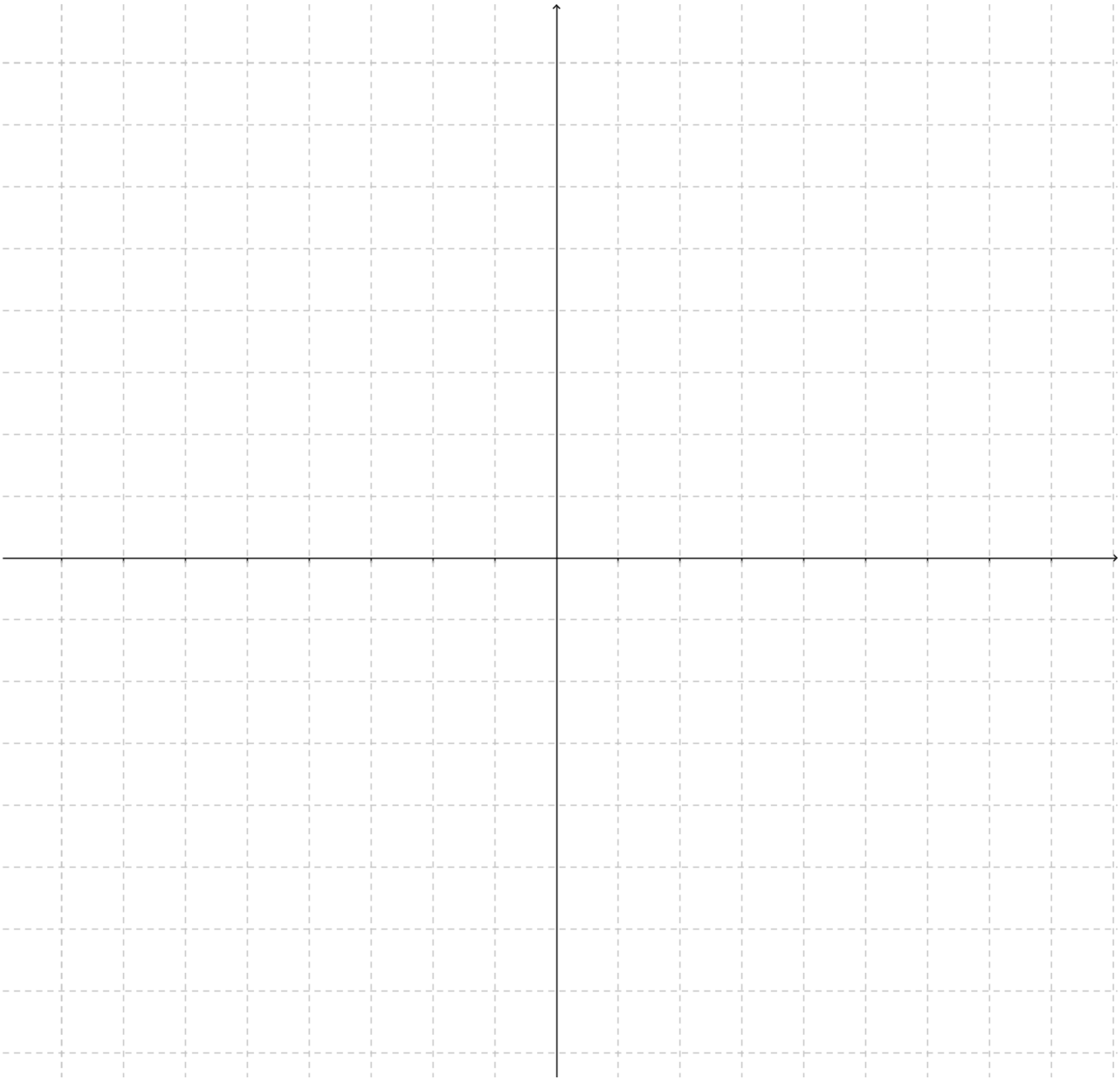


Another grid is available at the end of this question if required.

(d) A curve is represented parametrically by the equations $x = 2t,\ y = \dfrac{1}{t+1}$. **2**

Find the Cartesian equation of this curve.

Extra space to complete question 5. Clearly label any parts completed here.



End of Question 5

QUESTION 6: (12 marks) **Marks**

(a) Evaluate $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$, giving your answer in terms of π **1**

(b) Sketch the graph of the function $f(x) = 2\cos^{-1} x$, clearly indicating the domain and range of the function **2**

(c) Show that $f(x) = \cos\left(\sin^{-1}(x)\right)$ is an even function **2**

(d) If $\cos A = \dfrac{7}{9}$ and $\sin B = \dfrac{1}{3}$ where $0 \leq A \leq \dfrac{\pi}{2}$ and $0 \leq B \leq \dfrac{\pi}{2}$, find the value of $\sin(A - B)$ in simplest form **2**

(e) If $\tan A$ and $\tan B$ are the roots of the equation $3x^2 - 5x - 1 = 0$, find the value of $\tan(A + B)$ **2**

(f) Prove that $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$ and hence find the value of $\tan\dfrac{\pi}{8}$ in simplest surd form **3**

This space may be used to complete question 6. Clearly label any parts completed.

End of Question 6

QUESTION 7: (12 marks) **Marks**

(a) The polynomial $P(x) = x^3 + 3x^2 + 2x + 1$ can be written in the form $(x-2)(x^2 + bx + c) + r$, where b, c and r are real numbers.
Find the values of b, c and r. **3**

(b) Consider the polynomial equation $x^3 + 6x^2 - x - 30 = 0$.
One of the roots of this equation is equal to the sum of the other two roots.
Find the three roots of the equation. **3**

(c) Consider the polynomial $P(x) = 2x^3 + 3x^2 - 11x - 6$.

(i) Factorise $P(x)$ **2**

(ii) Solve $P(x) = 0$ **1**

(d) If α and β are the roots of $2x^2 + x - 8 = 0$, find the value of $|\alpha - \beta|$ **3**

Unused space above may be used to complete question 7. Clearly label any parts completed.

This space may be used to complete question 7. Clearly label any parts completed.

End of Question 7

QUESTION 8: (12 marks) **Marks**

(a) Find the co-efficient of x in $\left(x+\frac{4}{x}\right)^5$ **2**

(b) How many unique arrangements of the letters in the word SWANS are possible, if the two S's are not next to each other? **2**

(c) Twenty students, ten male and ten female, are travelling from school to the pool for the annual swimming carnival. Twelve of them go in a minibus, six of them on bikes and two of them walk.

(i) Write a mathematical expression that will give the number of ways the students can be divided for the trip. **1**

(ii) Show all working to give the numerical answer for the number of ways the students can be distributed, if none of the boys walk. **2**

(d) A committee of 5 members is to be selected at random from 5 Australians and 3 New Zealanders. Determine the probability that there will be a majority of Australians on the committee. Show all working out. **3**

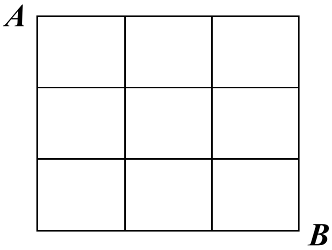
(e) Average speed cameras in NSW measure the amount of time it takes a vehicle to drive between two points. The average speed of the vehicle is then calculated. If the time taken between these points is quicker than could be done at the speed limit, a speeding fine is issued. **1**

This is an example of an application of the Pigeonhole Principle, where division is used to decide whether at least one “pigeonhole” will contain an excess amount of “pigeons”.

Which quantity represents the “pigeons” and which quantity represents the pigeonholes”, when considering average speed?

(f) An ant is to crawl along the black lines in the grid below to get from corner ***A*** to corner ***B***. However, the ant can only travel along the lines either towards the right or in a downwards direction on the page. **1**

How many different routes from ***A*** to ***B*** are possible?



This space may be used to complete any question. Clearly label any parts completed.

This space may be used to complete any question. Clearly label any parts completed.

END OF EXAMINATION

Year 11 Mathematics Extension 1 – 2019 Yearly

Section I – Multiple Choice Answer Sheet

Name / Student Number : ___________________________________

Teacher : ___________________________________

Allow about 5 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ correct C ○ D ○

Start Here ➔ **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

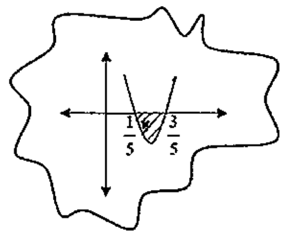
Year 11 Yearly Mathematics Extension 1 Examination 2019
Multiple Choice Solutions and Marking Guidelines

Outcomes Addressed in this Question		
Outcome	**Solutions**	**Marking Guidelines**
ME11-3	1. Answer: A The graph shown is a sine inverse graph. For $y=\sin^{-1}x$, we will only get a y value if $-1\le x\le 1$. For the domain to be $-2\le x\le 2$, need $\sin^{-1}\frac{x}{2}$, so that $-1\le\frac{x}{2}\le 1$. The range of $y=\sin^{-1}x$ is $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$ so that $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$. For $-\frac{3\pi}{2}\le y\le\frac{3\pi}{2}$, need to multiply $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$ by 3. Hence graph is $y=3\sin^{-1}\frac{x}{2}$	1 mark : correct answer
ME11-2	2. Answer: C If leading term is negative, the graph starts from the bottom right corner. As the power of the leading term is an even number, the arrows at the end of the graph should point the same direction.	
ME11-1	3. Answer: D As $x=3\cos t$ and $y=3\sin t$, then $\cos t=\frac{x}{3}$ and $\sin t=\frac{y}{3}$ $\sin^2 t+\cos^2 t=1,\ \therefore\left(\frac{y}{3}\right)^2+\left(\frac{x}{3}\right)^2=1$ $\therefore x^2+y^2=9$ which is a circle	
ME11-5	4. Answer: C MATHEMATICS has 11 letters, but some are repeated. Need to divide the number of arrangements of 11 distinct letters by the number of ways the repeating letters could have been arranged amongst themselves, to avoid doubling up. There are 2 M's, 2 A's & 2 T's. $\therefore\frac{11!}{2!2!2!}=\frac{11!}{8}$	

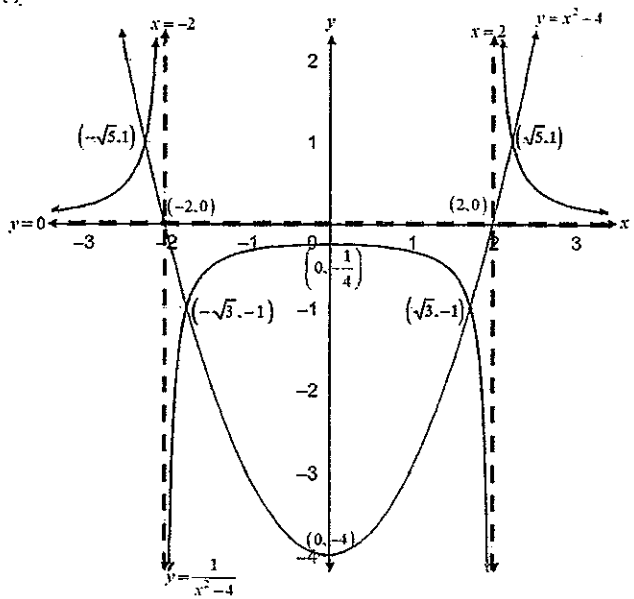
Year 11 Mathematics Extension 1 2019 Yearly Exam
Question No. 5 Solutions and Marking Guidelines

Outcomes Addressed in this Question:

ME11-1 Uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

Part	Solutions	Marking Guidelines
a)	$\lvert 2-5x\rvert\le 1$ $\sqrt{(2-5x)^2}\le 1$ squaring both sides $(2-5x)^2\le 1$ $25x^2-20x+4\le 1$ $25x^2-20x+3\le 0$ $(5x-3)(5x-1)\le 0$ Solution is $\frac{1}{5}\le x\le\frac{3}{5}$ 	2 Marks for complete correct solution 1 Mark for any correct working that could lead to a correct solution
b)	(i) $f(x)=\frac{1}{x^2+1}$ for $x\le 0$ Let $y=\frac{1}{x^2+1}$ exchange x and y $x=\frac{1}{y^2+1}$ for $y\le 0$ $y^2+1=\frac{1}{x}$ $y^2=\frac{1}{x}-1$ for $y\le 0$ $\therefore y=-\sqrt{\frac{1}{x}-1}$ Therefore, the inverse function is $y=-\sqrt{\frac{1}{x}-1}$.	2 Marks for complete correct solution 1 Mark for substantial working that could lead to a correct solution
b)	(ii) Range is $(-\infty,0]$ Domain is $(0,1]$	1 Mark for correct range 1 Mark for correct domain

c) (i)



1 Mark for correct sketch

c) (ii)

Features of $y=\dfrac{1}{f(x)}$ are:

- Vertical asymptotes at $x=-2, x=2$ which are the zeros of $f(x)=x^2-4$.
- Horizontal asymptote at $y=0$ since $f(x)\to\infty$ as $x\to\pm\infty$.
- Point $\left(0,-\dfrac{1}{4}\right)$ is local maximum.
- $y=\dfrac{1}{f(x)}$ intersects with $f(x)=x^2-4$ at $(-\sqrt{5},1),(\sqrt{5},1),(-\sqrt{3},-1),(\sqrt{3},-1)$ since $f(x)=x^2-4$ intersects $x=1$ and $x=-1$ at these points.
- For $x<-2$ $y=\dfrac{1}{f(x)}$ is increasing.

  For $-2<x<-2$ $y=\dfrac{1}{f(x)}$ is decreasing.

  For $x>2$ $y=\dfrac{1}{f(x)}$ is increasing.
- As $x\to-2^-$ $y\to\infty$.

  As $x\to 2^+$ $y\to\infty$.

  As $x\to-\infty$ $y\to 0$.

  As $x\to\infty$ $y\to 0$.
- Let $g(x)=\dfrac{1}{f(x)}=\dfrac{1}{x^2-4}$

  Now, $g(-x)=\dfrac{1}{(-x)^2-4}=\dfrac{1}{x^2-4}=g(x)$

  $\therefore$ Since $g(x)=g(x)$ then $y=\dfrac{1}{f(x)}$ is an even function and hence has line symmetry about the y-axis.

The graph of $y=\dfrac{1}{f(x)}$ is drawn above in part (i) and coloured blue with the above features displayed on it.

3 Marks for complete correct sketch

2 Marks for substantial work that could lead to a correct sketch with only one minor error.

1 Mark for any correct working that could lead to a correct sketch

d)

$x=2t$

$\therefore t=\dfrac{x}{2}$ ....(1)

$y=\dfrac{1}{t+1}$ ....(2)

sub (1) into (2)

$y=\dfrac{1}{\left(\frac{x}{2}\right)+1},\qquad x\neq-2$

$\therefore y=\dfrac{2}{x+2},\qquad x\neq-2$

Therefore the Cartesian equation of this curve is $y=\dfrac{2}{x+2},\qquad x\neq-2$

2 Marks for complete correct solution

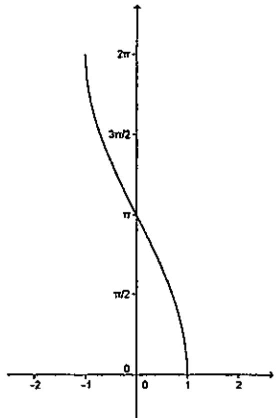
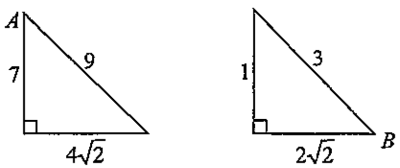
1 Mark for any correct working that could lead to a correct solution

Year 11 Yearly — Mathematics Extension 1 — Examination 2019

Question No. 6 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

ME11-1 Uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

Outcome	Solutions	Marking Guidelines
ME11-1	(a) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)$ (as $\sin\left(\pi-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}$) $=\frac{\pi}{3}$ (as $\frac{\pi}{3}$ is in the domain for which sin has an inverse)	1 mark : correct solution
ME11-1	(b) $\cos^{-1}x$ has domain $-1\le x\le 1$ and $0\le\cos^{-1}x\le\pi$, so $0\le 2\cos^{-1}x\le 2\pi$  	2 marks : correct graph 1 mark : substantial progress towards correct graph
ME11-1	(c) $f(x)=\cos\left(\sin^{-1}(x)\right)$ $f(-x)=\cos\left(\sin^{-1}(-x)\right)$ $=\cos\left(-\sin^{-1}x\right)$ (as $\sin^{-1}(-x)=-\sin x$ since $f(x)=\sin x$ is an odd function) $=\cos\left(\sin^{-1}x\right)$ (as $f(x)=\cos x$ is an even function, $\cos(-a)=\cos a$) Since $f(-x)=f(x)$, $f(x)=\cos\left(\sin^{-1}(x)\right)$ is an even function	2 marks : correct solution 1 mark : substantial progress towards correct solution
ME11-1	(d) As $\cos A=\frac{7}{9}$ and $\sin B=\frac{1}{3}$ where $0\le A, B\le\frac{\pi}{2}$ then A and B are acute.  $\sin(A-B)=\sin A\cos B-\sin B\cos A$ $=\frac{4\sqrt{2}}{9}\times\frac{2\sqrt{2}}{3}-\frac{1}{3}\times\frac{7}{9}$ $=\frac{1}{3}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
ME11-1	(e) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ As $\tan A$ and $\tan B$ are the roots of $3x^2-5x-1=0$, which has sum of roots $=\frac{-b}{a}$ and product of the roots $=\frac{c}{a}$ Then $\tan A+\tan B=\frac{5}{3}$ and $\tan A\tan B=\frac{-1}{3}$ $\therefore\tan(A+B)=\frac{\frac{5}{3}}{1-\left(-\frac{1}{3}\right)}=\frac{5}{4}$	2 marks : correct solution 1 mark : substantial progress towards correct solution
ME11-1	(f) $\frac{\sin 2x}{1+\cos 2x}=\frac{2\sin x\cos x}{1+2\cos^2 x-1}$ $=\frac{2\sin x\cos x}{2\cos^2 x}$ $=\frac{\sin x}{\cos x}$ $\therefore\frac{\sin 2x}{1+\cos 2x}=\tan x$ Substituting $x=\frac{\pi}{8}$, $\frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}=\tan\frac{\pi}{8}$ $\therefore\tan\frac{\pi}{8}=\frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $=\frac{1}{\sqrt{2}+1}$ $=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$ $=\sqrt{2}-1$	3 marks : correct solution 2 marks : prove required identity and make substantial progress towards correct value of $\tan\frac{\pi}{8}$ or incomplete proof and correct solution to find $\tan\frac{\pi}{8}$ 1 mark: either prove required inequality only or make substantial progress towards finding the value of $\tan\frac{\pi}{8}$

Year 11 **Mathematics Extension 1** **2019 Yearly Exam**

Question No. 7 **Solutions and Marking Guidelines**

Outcomes Addressed in this Question:

ME11-2 manipulates algebraic expressions and graphical functions

Part	Solutions	Marking Guidelines

a)

$$(x-2)(x^2+bx+c)+r$$
$$=x^3+bx^2+cx-2x^2-2bx-2c+r$$
$$=x^3+(b-2)x^2+(c-2b)x-2c+r$$

Now $x^3+3x^2+2x+1=x^3+(b-2)x^2+(c-2b)x-2c+r$

$b-2=3$

$\therefore\ b=5$

$c-2b=2$

$\therefore\ c=12$

$-2c+r=1$

$\therefore\ r=25$

Marking Guidelines a)
3 marks: Correct solution with full working out shown
2 marks: Substantial progress.
1 mark: Some significant progress

b)

Let the roots be α, β and $\alpha+\beta$

Sum of roots: $\alpha+\beta+(\alpha+\beta)=-\frac{6}{1}=-6$

$2\alpha+2\beta=-6$

$\alpha+\beta=-3 \qquad ----(1)$

Product of roots: $\alpha\beta(\alpha+\beta)=-\frac{-30}{1}=30$

$\alpha\beta(-3)=30$

$\alpha\beta=-10 \qquad ----(2)$

from (1):

$\alpha=-3-\beta \qquad ----(3)$

sub (3) into (2):

$(-3-\beta)\beta=-10$

$-\beta^2-3\beta+10=0$

$\beta^2+3\beta-10=0$

$(\beta+5)(\beta-2)=0$

$\beta=-5$ or $\beta=2$

$\therefore\ \alpha=-5, \quad \beta=2$

$\therefore$ the roots are -5, 2 and -3

Marking Guidelines b)
3 marks: Correct solution with full working out shown
2 marks: Substantial progress.
1 mark: Some significant progress

c)

i)

Factors of -6 are $\pm1,\ \pm2,\ \pm3,\ \pm6$

$P(2)=0$

$\therefore (x-2)$ is a factor of $P(x)$

$$\begin{array}{r}
2x^2+7x+3 \\
(x-2)\overline{)\,2x^3+3x^2-11x-6} \\
\underline{2x^3-4x^2\qquad\qquad} \\
7x^2-11x\quad\ \\
\underline{7x^2-14x}\quad\ \\
3x-6 \\
\underline{3x-6} \\
0
\end{array}$$

$\therefore P(x)=(x-2)(2x^2+7x+3)$

$\quad=(x-2)(x+3)(2x+1)$

Marking Guidelines c i)
2 marks: Correct solution with full working out shown
1 marks: Substantial progress.

ii)

$P(x)=0$

$(x-2)(x+3)(2x+1)=0$

$\therefore\ x=2,\ x=-3,\ x=-\frac{1}{2}$

Marking Guidelines c ii)
1 mark: Correct solution

d)

$(\alpha-\beta)^2=\alpha^2+2\alpha\beta+\beta^2-4\alpha\beta$

$\quad=(\alpha+\beta)^2-4\alpha\beta$

$|\alpha-\beta|=\sqrt{\left(-\frac{1}{2}\right)^2-4\left(-\frac{8}{2}\right)}$

$\quad=\frac{\sqrt{65}}{2}$

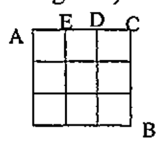
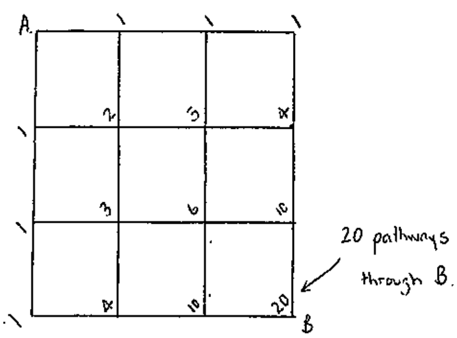
Marking Guidelines d)
3 marks: Correct solution with full working out shown
2 marks: Substantial progress.
1 mark: Some significant progress

Year 11 Mathematics Extension Task 3, 2018

Question No. 8 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

ME 11-5 Uses Concepts of permutations and combinations to solve problems involving counting or ordering.

	Solutions	Marking Guidelines
(a)	General Term = ${}^5C_r\, x^{5-r}\left(\dfrac{4}{x}\right)^r = {}^5C_r\, 4^r\, x^{5-2r}$ For $5-2r=1 \rightarrow r=2$ So, coefficient = ${}^5C_2\, 4^2 = 160$	**(a) 2 marks:** correct solution **1 mark:** Considerable relevant progress.
(b)	There are 6 cases where the two S's are not alongside each other. Each case can have the other letters arranged in 3! ways. Solution = $6\times 3! = 36$ Alternately: (Total possible arrangements of 5 objects minus those with two objects adjacent) divided by 2 gives: $(5!-4!\times 2!)\div 2 = 36$	**(b) 2 marks:** correct solution **1 mark:** Considerable relevant progress.
(c)	(i) ${}^{20}C_{12}\times {}^8C_6 \times {}^2C_2$ filling minibus, then bikes, then walking. Other rearrangements are possible. (ii) Girls walking in ${}^{10}C_2$ ways. Solution = ${}^{10}C_2 \times {}^{18}C_{12}\times {}^6C_6 = 45\times 18564\times 1 = 835380$	**(c) (i) 1 mark:** Correct expression **(ii) 2 marks:** Correct answer with working. **1 mark:** Correct expression for girls walking or equivalent.
(d)	$P(\text{Majority}) = P(3 \text{ or } 4 \text{ or } 5 \text{ Australians})$ $= \dfrac{({}^5C_3\times {}^3C_2)+({}^5C_4\times {}^3C_1)+{}^5C_5}{{}^8C_5}$ $= \dfrac{30+15+1}{56}$ $= \dfrac{46}{56}$	**(d) 3 marks:** Correct solution with working. **2 marks:** One aspect of solution incorrect. **1 mark:** Only one correct aspect.
(e)	Pigeons ÷ Pigeonholes translates to Distance ÷ Time respectively.	**(e) 1 mark:** Both correct.
(f)	Answer = 20 (This is a version of organising outcomes from a diagram.) By tracing paths, there's: 1 passing through C Another 3 passing through D Another 6 passing through E  1+3+6 = 10 represents half the pathways from A, so answer = 20. Note: there is also an application of adding as per Pascal's Triangle:  	**(f) 1 mark:** Correct answer.