Student Name: ______________________________

Teacher: ______________________________

2024

Hurlstone Agricultural High School
Year 11 Assessment Task 3

Mathematics Extension 1

Examiners

- **Mr G Rawson (Senior Examiner)**
- Mrs P Biczo
- Mr G Huxley
- Ms M Sabah
- Ms T Tarannum

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided for your use
- In Questions 11 to 15, show relevant mathematical reasoning and/or calculations

Total marks: 60

Section I – 10 marks (pages 2–4)
- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II – 50 marks (pages 5–8)
- Attempt Questions 11 - 15
- Allow about 75 minutes for this section.
- For each question use a separate answer booklet.

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

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1. There are 26 cards in a bag, each has a different letter of the alphabet written on them.

   A game consists of drawing cards one at a time, without replacement, until two consecutive letters of the alphabet have been drawn. A and Z are not consecutive letters.

   For example, suppose B is drawn first and M is drawn second. If the third card drawn is then either A, C, L or N, the game would stop there as A and B, or B and C, or L and M, or M and N form a consecutive pair of letters. There would then be 23 letters left in the bag.

   What is the least number of cards that can be left in the bag at the end of the game?

   A. 11 **B.** 12

   C. 13 **D.** 14

2. Eight chairs are equally spaced around a circular table. What is the number of ways four teachers, and four students can be seated at the table so that no student is directly opposite a teacher?

   A. 36 **B.** 144

   C. 432 **D.** 5040

3. Which expression is equivalent to $\frac{2\cos^2\theta}{1-\sin\theta}$?

   A. $2 - 2\sin\theta$ **B.** $2 + 2\sin\theta$

   C. $2 - 2\cos\theta$ **D.** $2 + 2\cos\theta$

4. Which of the following is equivalent to $\cos 4\theta$?

   A. $\cos^2\theta - \sin^2\theta$ **B.** $2\sin^2 2\theta - 1$

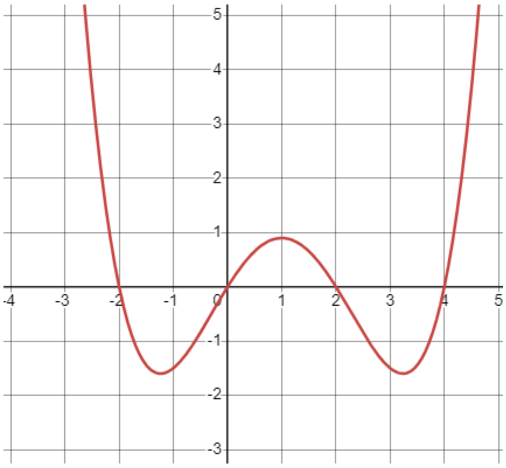
   C. $\cos^2 2\theta + \sin^2 2\theta$ **D.** $2\cos^2 2\theta - 1$

5. How many solutions are there to the equation $\tan 2x = 3$ when $-\pi \le x \le \pi$?

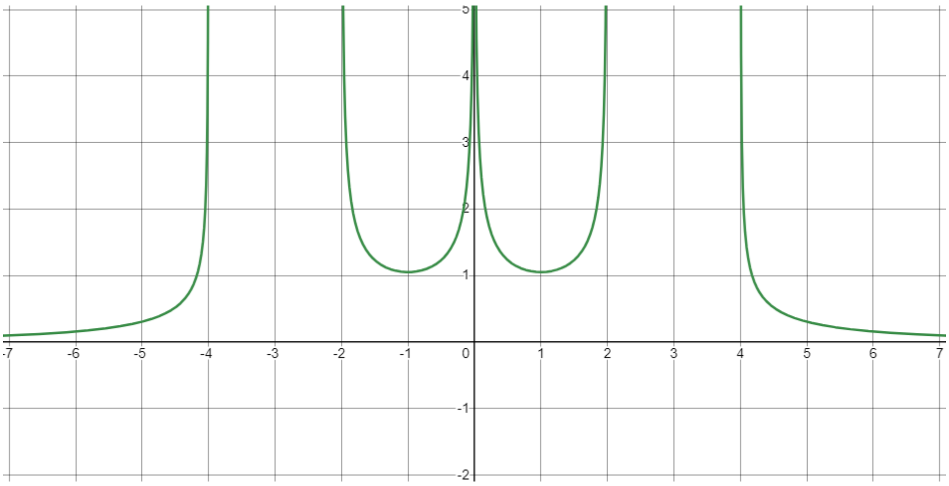
A. 2 **B.** 4

C. 6 **D.** 8

6. Below is the graph of $y = f(x)$.



A second graph, shown below, is drawn based on the graph of $y = f(x)$.



Which of the following is the equation of the second graph?

A. $y = \sqrt{f(x)}$ **B.** $y = \frac{1}{\sqrt{f(x)}}$

C. $y = \sqrt{f(|x|)}$ **D.** $y = \frac{1}{\sqrt{f(|x|)}}$

7. Which of the following is the domain of reciprocal function of $f(x) = x^2 - 1$?

A. $(-\infty, 1) \cup (1, \infty)$ **B.** $(-\infty, -1) \cup (-1, \infty)$

C. $(-\infty, \infty)$ **D.** $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

8. A curve is defined parametrically by $\begin{cases} x^2 = 9 - t \\ y = 7 - t \end{cases}$.

Which of the following best describes this curve on the Cartesian plane?

A. a straight line with gradient 16

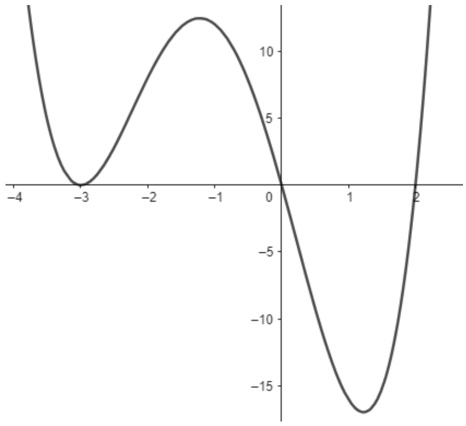
B. a parabola with x-intercepts at $x = -3$, and $x = 4$.

C. a parabola with vertex $(0, -2)$

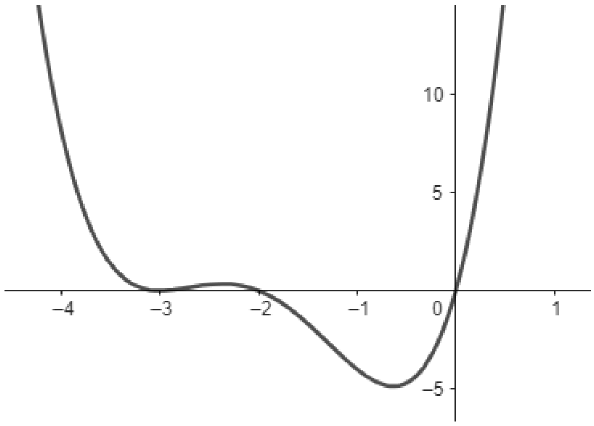
D. a hyperbola with asymptote at $x = -2$.

9. Which of the following is the graph of $y = x(x+3)^2(2-x)$?

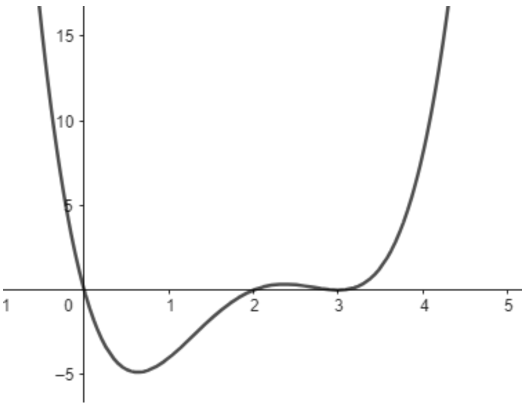
A.



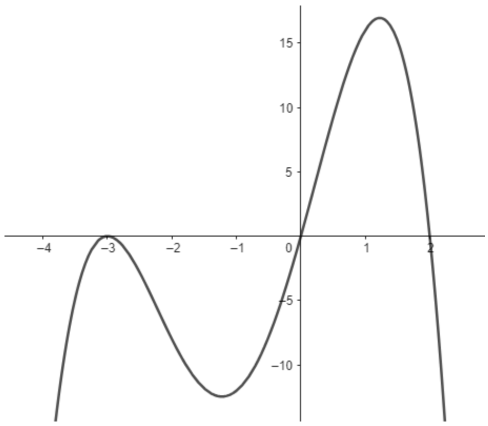
B.



C.



D.



10. The polynomial $P(x)$ is divided by $(x-p)(x-q)$, where p and q are real numbers.
Hence, $P(x) = (x-p)(x-q)Q(x) + ax + b$, where $Q(x)$ is a polynomial.
When $P(x)$ is divided by $x - p$, the remainder is p^2.
Which of the following statements are true?

A. $b = p - ap^2$

B. $b = p^2 - ap$

C. $b = p^2 + ap$

D. $b = ap - p^2$

~ END OF SECTION I ~

Section II

50 marks
Attempt Questions 11 to 15
Allow about 75 minutes for this section

***Answer each question in a new answer booklet*.**

All necessary working should be shown in every question.

Question 11 (10 marks) **Marks**

(a) The letters of the word **PERSEVERE** are arranged in a row.
How many different arrangements are possible if all of the four **E**'s remain together. **2**

(b) Three integers are chosen at random from the integers 1 to 20 (inclusive)
In how many ways can these integers be chosen if their sum is even? **2**

(c) Solve the equation $16(n-1)! = 5n! + (n+1)!$ where n is any positive integer. **3**

(d) 12 cards are drawn from a regular deck of 52 playing cards.
It is known that out of these 12 cards, exactly 2 are kings.

What is the probability that there will be more queens than kings? **3**

Note: a regular deck of 52 playing cards consists four suits,
each containing the cards 2, 3, 4, …, Jack, Queen, King, Ace.

~ End of question 11 ~

Question 12 (10 marks) **Marks**

(a) Find the exact value of $\tan 105°$. **2**

(b) With the use of t- results, prove that: **2**

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\frac{\theta}{2}$$

(c) If $\sin x=\frac{2}{3}$, where $\frac{\pi}{2}\le x\le\pi$ and $\cos y=\frac{3}{4}$, where $0\le y\le\frac{\pi}{2}$,

what is the exact value of $\sin(x+y)$? **3**

(d) (i) Prove that $\frac{\sin 2A}{1-\cos 2A}=\cot A$ **1**

(ii) Hence find the values of a and b if $\cot\frac{3\pi}{8}=a+\sqrt{b}$ for integers a and b. **2**

~ End of question 12 ~

Question 13 (10 marks) **Marks**

(a) (i) Show that $\cos^2\left(\frac{\pi}{4}-x\right)-\sin^2\left(\frac{\pi}{4}-x\right)=\sin 2x$. **2**

(ii) Hence, solve $\cos^2\left(\frac{\pi}{4}-x\right)-\sin^2\left(\frac{\pi}{4}-x\right)=\frac{\sqrt{3}}{2}$, for $0\le x\le\pi$. **2**

(b) Solve $\sec^2\alpha-\tan\alpha-1=0$, for $0\le\alpha\le 2\pi$ **3**

(c) Convert $\sin 5x\cos x$ to a sum of sine functions. **1**

(d) Show, without using a calculator, that

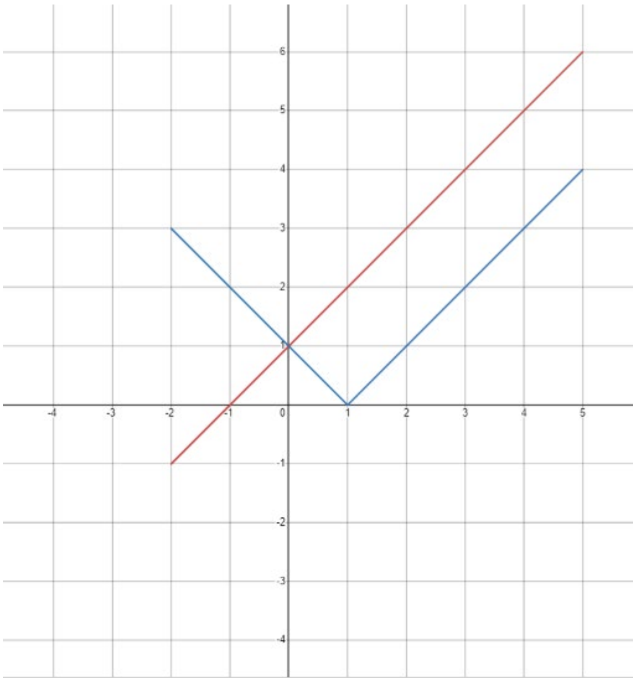
$$2\cos 95°\cos 55°-2\sin 50°\sin 10°=\frac{1}{2}\left(1-\sqrt{3}\right).$$ **2**

~ End of question 13 ~

Question 14 (10 marks) **Marks**

(a) Consider the parametric equations $x = 4\cos\theta - 3$ and $y = 4\sin\theta + 2$.
Write the corresponding cartesian equation of the above parametric equations. **2**

(b) The graphs of the functions $f(x) = x + 1$ and $g(x) = |1 - x|$ in the domain $[-2, 5]$ are shown below.

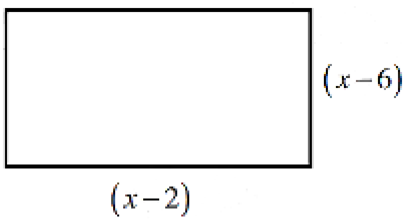


On page 4 of your answer booklet, on the diagram provided, sketch of the graph of $h(x) = f(x) - g(x)$, $[-2, 5]$. **2**

(c) Solve the inequality **3**

$$\frac{3x}{x-2} \leq 1.$$

(d) The rectangle below has dimensions $(x - 2)$ cm by $(x - 6)$ cm.



The area of the rectangle is at most 60 cm^2 and its perimeter at least 14 cm.
Using inequalities, determine the range of the possible values of x . **3**

~ End of question 14 ~

Question 15 (10 marks) **Marks**

(a) For what value of m is $2x^4 + mx^2 - 2$ divisible by $x + 2$? **1**

(b) (i) Use long division to divide the polynomial $P(x) = x^4 - x^3 + x^2 - x + 1$ by the polynomial $x^2 + 4$. **2**

(ii) Hence, find the values of a and b such that $x^4 - x^3 + x^2 + ax + b$ is divisible by $x^2 + 4$. **2**

(c) If α, β, γ are the roots of the equation $2x^3 - 5x^2 - 3x + 1 = 0$,

find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$. **2**

(d) It is given that the polynomial $Q(x) = 4x^3 - 8x^2 + 5x - 1$ has a zero of multiplicity 2.

Express $Q(x)$ as a product of linear factors. **3**

~ End of question 15 ~

~ END OF SECTION II ~

~ END OF PAPER ~

NSW GOVERNMENT

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

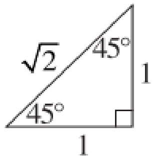
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

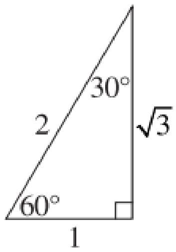
$A = \dfrac{1}{2}ab\sin C$



$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

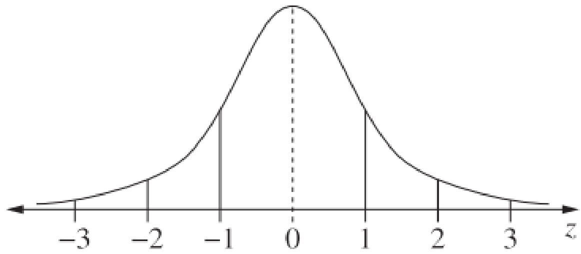
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \displaystyle\int_a^r f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Student Name: ______________________

Teacher: ______________________

2024 Year 11 Mathematics Extension 1
Assessment Task 3

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

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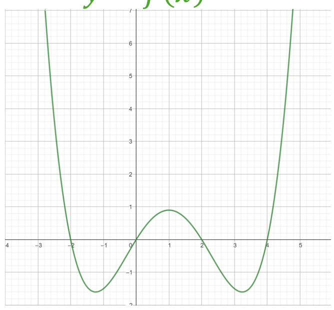
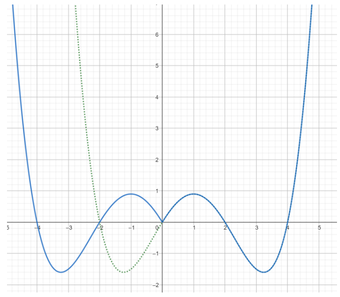
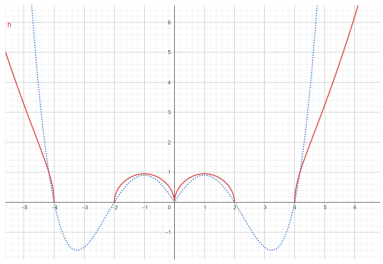
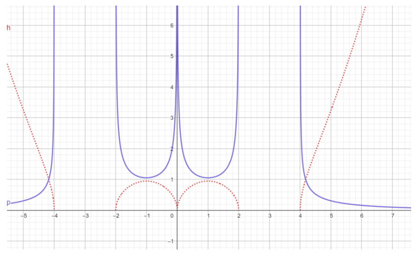
Year 11	Mathematics Extension1 2024	TASK 3
	Solutions and Marking Guidelines	

MULTIPLE CHOICE

Part / Outcome	**Solutions**	**Marking Guidelines**
Q1 ME11-5	Break the alphabet into 13 consecutive pairs *AB, CD, ….YZ.* There are 13 pigeon holes. There can be up to 14 cards drawn before a consecutive pair is made, leaving 12 cards in the bag.	**1 mark – B**
Q2 ME11-5	After first student is seated, seat a student opposite this student in 3 ways, then seat remaining 2 students opposite each other in 2×3 ways, then seat the 4 teachers in 4! Ways. This gives $3 \times 2 \times 3 \times 4! = 432\ ways$.	**1 mark – C**
Q3 ME11-3	$\dfrac{2\cos^2\theta}{1-\sin\theta} = \dfrac{2(1-\sin^2\theta)}{1-\sin\theta} = \dfrac{2(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta}$ $= 2 + 2\sin\theta$	**1 mark – B**
Q4 ME11-3	$\cos^2 nx = \frac{1}{2}(1+\cos 2nx)$ $2\cos^2 nx = 1 + \cos 2nx$ $1 + 2\cos^2 nx = \cos 2nx$ $1 + 2\cos^2 2\theta = \cos 4\theta$ (let $2\theta = nx$)	**1 mark – D**
Q5 ME11-3	$-\pi \le x \le \pi$ means that $-2\pi \le 2x \le 2\pi$ So we are considering answers in the first and third quadrants for $2x$ in 2 revolutions; 1 positive, 1 negative. Hence there will be 4 solutions.	**1 mark – B**

Q6 **ME11-2**	$y = f(x)$ $\longrightarrow$ $y = f(\lvert x \rvert)$   $\longrightarrow$ $y = \sqrt{f(\lvert x \rvert)}$ $\longrightarrow$ $y = \dfrac{1}{\sqrt{f(\lvert x \rvert)}}$  	**1 mark – D**
Q7 **ME11-2**	$f(x) = x^2 - 1$ has zeroes at $x = \pm 1$ $\therefore$ the reciprocal has discontinuities at $x = \pm 1$	**1 mark – D**
Q8 **ME11-2**	$x^2 = 9 - t \quad \rightarrow \quad t = 9 - x^2$ $y = 7 - t \qquad \therefore y = 7 - (9 - x^2)$ $\qquad\qquad\qquad y = x^2 - 2$ Which is concave up parabola with vertex at (0, –2)	**1 mark – C**
Q9 **ME11-2**	Double root at $x = -3$, single roots at 0 and 2. As $x \rightarrow +\infty,\ y \rightarrow -\infty$	**1 mark – D**
Q10 **ME11-2**	$P(x) = (x-p)(x-q)Q(x) + ax + b$ $P(p) = (p-p)(p-q)Q(p) + ap + b = p^2$ $\therefore 0 + ap + b = p^2$ $\therefore \qquad b = p^2 - ap$	**1 mark – B**

Year 11	Mathematics Extension1 2024	TASK 3
	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	
ME11-5 uses concepts of permutations and combinations to solve problems involving counting or ordering		
Part / Outcome	**Solutions**	**Marking Guidelines**
ER Q11 a ME11-5	The four E's are now treated as one letter, so the question becomes how many arrangements of **PRSVR(EEEE)** No. of ways $= \frac{6!}{2!} = 144\ ways$	**2 marks** – Correct solution **1 mark** – some correct working towards correct solution
ER Q11 b ME11-5	No. of ways = all 3 even or 2 odd and 1 even $10C_3 + 10C_2 \times 10C_1 = 120 + 450 = 570$	**2 marks** – Correct solution **1 mark** – some correct working towards correct solution
ER Q11 c ME11-5	$16(n-1)! = 5n! + (n+1)!$ $16(n-1)! = 5n(n-1)! + (n+1)n(n-1)!$ $16(n-1)! - 5n(n-1)! - (n+1)n(n-1)! = 0$ $(n-1)!\,(16 - 5n - (n+1)n) = 0$ $(n-1)!\,(16 - 5n - n^2 - n) = 0$ $(n-1)!\,(-n^2 - 6n + 16) = 0$ $(n-1)!\,(n^2 + 6n - 16) = 0$ $(n-1)! = 0$ $\quad or \quad$ $(n^2 + 6n - 16) = 0$ No solution as 0!=1 $\quad$ $(n+8)(n-2) = 0$ $n = -8\ \ or\ n = 2$ Since n is a positive integer, $\therefore n = 2$	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution
ER Q11 d ME11-5	$P(Q > k \mid k = 2) = \dfrac{P(Q = 3 \cap k = 2) + P(Q = 4 \cap k = 2)}{P(k = 2)}$ $= \dfrac{\binom{4}{3} \times \binom{4}{2} \times \binom{44}{7} + \binom{4}{4} \times \binom{4}{2} \times \binom{44}{6}}{\binom{4}{2} \times \binom{48}{10}}$ $= \dfrac{53}{2162}$	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution

Year 11	Mathematics Extension 1 2024	TASK 3
Question No. 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

ME11-3 applies concepts and techniques of simplifying expressions involving compound angles in the solution of problems.

Part / Outcome	Solutions	Marking Guidelines

(a)

$$\tan 105° = \tan(60+45)°$$
$$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$$
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$
$$= \frac{3+2\sqrt{3}+1}{1-3}$$
$$= \frac{2(2+\sqrt{3})}{-2} = -2-\sqrt{3}$$

2 marks – Correct solution

1 mark – Substantially correct solution

Note: for *two* marks in this X1 question, we are expecting a higher quality solution than just writing down the values of tan60 and tan40 in an expression that can be copied off the reference sheet. Simplifying the fraction correctly was needed.
It should also be noted that this a vital skill for anyone planning to do X2 (and complex numbers)

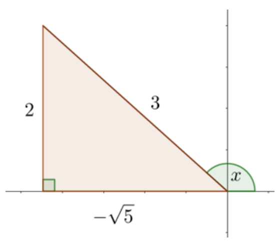
(b)

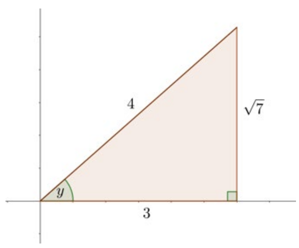
$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \frac{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$
$$= \frac{1+t^2+2t-(1-t^2)}{1+t^2+2t+1-t^2}$$
$$= \frac{2t+2t^2}{2+2t}$$
$$= \frac{2t(1+t)}{2(1+t)}$$
$$= t$$
$$= \tan\frac{\theta}{2} = RHS$$

2 marks – Correct solution

1 mark – Substantially correct solution

(c)





$\sin x = \frac{2}{3}$, $90° \le x \le 180°$ $\quad \therefore \cos x = \frac{-\sqrt{5}}{3}$ $\quad (second\ quadrant)$

$\cos y = \frac{3}{4}$, $0° \le x \le 90°$ $\quad \therefore \sin y = \frac{\sqrt{7}}{4}$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$= \frac{2}{3} \times \frac{3}{4} + \frac{-\sqrt{5}}{3} \times \frac{\sqrt{7}}{4}$$
$$= \frac{6-\sqrt{35}}{12}$$

3 marks – Correct solution

2 marks – Substantially correct solution

1 mark – significant progress towards correct solution

	Question 12 continued…	
(d)(i)	$LHS = \dfrac{\sin 2A}{1-\cos 2A}$ $= \dfrac{2\sin A\cos A}{2\sin^2 A}$ $\quad (straight\ from\ reference\ sheet)$ $= \dfrac{\cos A}{\sin A}$ $= \cot A = RHS$	**1 mark** – Correct solution
(d) (ii)	$\cot\dfrac{3\pi}{8} = \dfrac{\sin\frac{3\pi}{4}}{1-\cos\frac{3\pi}{4}}$ $= \dfrac{\frac{1}{\sqrt{2}}}{1-\left(-\frac{1}{\sqrt{2}}\right)}$ $= \dfrac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $= \dfrac{1}{\sqrt{2}+1}\times\dfrac{\sqrt{2}-1}{\sqrt{2}-1}$ $= \dfrac{\sqrt{2}-1}{2-1}$ $= -1+\sqrt{2}$ $\therefore a = -1, b = 2$	**2 marks** – Correct solution **1 mark** – Substantially correct solution **Note: the question *did* ask for the values of *a* and *b*. Many did not do this… While not the case this time, not answering the actual question that was asked runs the risk of missing out on full marks**

Question 13

Outcomes Addressed in this Question

ME 11-3 applies concepts and techniques of simplifying expressions involving compound angles in the solution of problems.

Question	Solutions	Marking Guidelines
(a)(i)	LHS $= \cos^2\left(\frac{\pi}{4} - x\right) - \sin^2\left(\frac{\pi}{4} - x\right)$ $= \cos 2\left(\frac{\pi}{4} - x\right)$ Double angle result $= \cos\left(\frac{\pi}{2} - 2x\right)$ $= \sin 2x$ Complementary angle identity	**(a)(i) 2 marks**: Correct solution illustrating trig relationships. **1 mark:** Significant relevant progress.
(ii)	$\sin 2x = \frac{\sqrt{3}}{2}$, $0 \le 2x \le 2\pi$ $2x = \frac{\pi}{3}, \pi - \frac{\pi}{3}$ $2x = \frac{\pi}{3}, \frac{2\pi}{3} \rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}$	**(ii) 2 marks:** All solutions found. **1 mark:** At least one relevant solution. **Aw 1:** All solutions plus incorrect ones from other quadrants.
(b)	$\sec^2\alpha - \tan\alpha - 1 = 0$ $\tan^2\alpha + 1 - \tan\alpha - 1 = 0$ $\tan^2\alpha - \tan\alpha = 0$ $\tan\alpha(\tan\alpha - 1) = 0$ $\tan\alpha = 0$ or $\tan\alpha = 1$, $0 \le \alpha \le 2\pi$ $\alpha = 0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$	**(b) 3 marks:** Complete solution. **2 marks:** Both cases found but not all solutions found. **1 mark:** Recognising a trig identity <u>and</u> simplifying expression.
(c)	$\sin 5x \cos x = \frac{1}{2}(\sin 6x + \sin 4x)$	**(c) 1 mark:** Correct answer from reference sheet. Ignore Subsequent Error.
(d)	$LHS = 2\cos 95° \cos 55° - 2\sin 50° \sin 10°$ $= 2 \times \frac{1}{2}\left((\cos 40° + \cos 150°) - (\cos 40° - \cos 60°)\right)$ $= \cos 150° + \cos 60°$ $= \cos 60° - \cos 30°$ $= \frac{1}{2} - \frac{\sqrt{3}}{2}$ $= \frac{1}{2}\left(1 - \sqrt{3}\right)$ $= RHS$	**(d) 2 marks:** Correct solution communicated. Comparison with related angle must be part of "show" solution. **1 mark:** Significant relevant progress made but not all concepts communicated.

Year 11	Mathematics Extension 1 2024	TASK 3
	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

ME11-1: uses algebraic and graphical concepts in the modelling and solving of problems.

Question 14 Solutions	Marking Guidelines
a)	

$$x = 4\cos\theta - 3$$

$$\cos\theta = \frac{x+3}{4} \quad ------ (1)$$

$$y = 4\sin\theta + 2$$

$$\sin\theta = \frac{y-2}{4} \quad ------ (2)$$

$(1)^2 + (2)^2$:

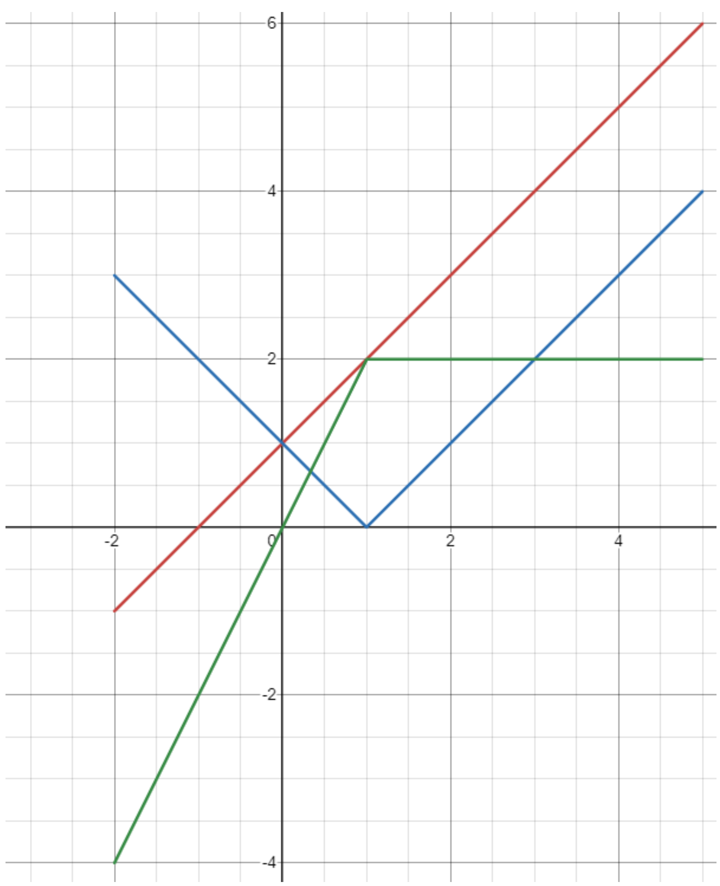
$$\left(\frac{x+3}{4}\right)^2 + \left(\frac{y-2}{4}\right)^2 = \sin^2\theta + \cos^2\theta$$

$$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{16} = 1$$

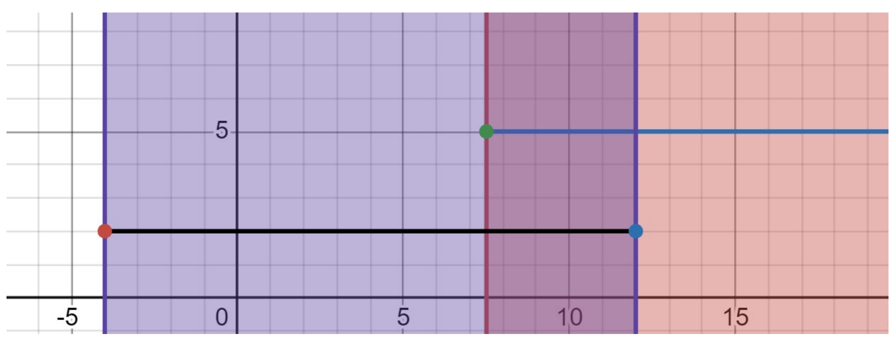
$$(x+3)^2 + (y-2)^2 = 16$$

2 marks – Correct solution
1 mark – Substantially correct solution.

b)



2 marks – Correct solution
1 mark – Substantially correct solution.

c) $$\frac{3x}{x-2} \times (x-2)^2 \;\le\; 1(x-2)^2$$ $$3x(x-2) \le (x-2)^2$$ $$3x^2 - 6x - x^2 + 4x - 4 \le 0$$ $$2x^2 - 2x - 4 \le 0$$ $$x^2 - x - 2 \le 0$$ $$(x-2)(x+1) \le 0$$ $$\therefore\; -1 \le x < 2 \qquad (x \ne 2)$$	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution
d) $$A = (x-2)(x-6) = x^2 - 8x + 12$$ $$P = 2(x-2) + 2(x-6) = 4x - 16$$ Area ≤ 60 $x^2 - 8x + 12 \le 60$ $x^2 - 8x - 48 \le 0$ $(x-12)(x+4) \le 0$ $x = 12 \quad \text{or} \quad x = -4$ $\therefore\; -4 \le x \le 12$ $P \ge 14$ $4x - 16 \ge 14$ $4x \ge 30$ $x \ge \frac{15}{2}$ $\therefore\; x \ge 7.5$  $\therefore$ The range of values x can be : $\quad 7.5 \le x \le 12$	**3 marks** – Correct solution **2 marks** – Substantially correct solution. **1 mark** – some correct working towards correct solution

Section 2 ~ Question 15 Solutions and Marking Guidelines

Outcome Addressed in this Section

ME11-2 manipulates algebraic expressions and graphical functions to solve problems.
ME11-4 applies understanding of the concept of a derivative in the solution of problems.

Part	Solutions	Marking Guidelines
(a) ME11-2 ME11-4	$P(x) = 2x^4 + mx^2 - 2$ We know that $P(-2) = 0$. $\therefore 2(-2)^4 + m(-2)^2 - 2 = 0$ $\therefore 32 + 4m - 2 = 0$ $\therefore 4m + 30 = 0$ $\therefore m = -7.5$	**Award 1** ~ correct answer
(b) (i) ME11-2 ME11-4	$\begin{array}{r} x^2 - x - 3 \\ x^2 + 0x + 4\,\overline{)\,x^4 - x^3 + x^2 - x + 1} \\ \underline{x^4 + 0x^3 + 4x^2} \\ -x^3 - 3x^2 - x \\ \underline{-x^3 + 0x^2 - 4x} \\ -3x^2 + 3x + 1 \\ \underline{-3x^2 + 0x - 12} \\ 3x + 13 \end{array}$	**Award 2** ~ correct solution **Award 1** ~ significant progress towards solution
(b) (ii) ME11-2 ME11-4	$x^4 - x^3 + x^2 - x + 1$ $= (x^2 + 4)(x^2 - x - 3) + 3x + 13$ $\therefore x^4 - x^3 + x^2 - 4x - 12 = (x^2 + 4)(x^2 - x - 3)$ $\therefore x^4 - x^3 + x^2 + ax + b$ is divisible by $(x^2 + 4)$ when $a = -4$ and $b = -12$.	**Award 2** ~ correct solution **Award 1** ~ significant progress towards solution
(c) ME11-2 ME11-4	$\alpha + \beta + \gamma = -\frac{-5}{2} = \frac{5}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-3}{2}$ $\alpha\beta\gamma = -\frac{1}{2}$ $\therefore \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{\frac{5}{2}}{-\frac{1}{2}} = -5$	**Award 2** ~ correct solution **Award 1** ~ significant progress towards solution

(d) ME11-2 ME11-4	$Q(x) = 4x^3 - 8x^2 + 5x - 1$ $Q'(x) = 12x^2 - 16x + 5 = (2x - 1)(6x - 5)$ $Q'(x) = 0$ when $x = \frac{1}{2}$ or $x = \frac{5}{6}$ $Q\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 1 = 0$ Since $Q\left(\frac{1}{2}\right) = 0$ and $Q'\left(\frac{1}{2}\right) = 0$ then $(2x - 1)$ is a double factor (i.e. $x = \frac{1}{2}$ is a double root). $\therefore\ 4x^3 - 8x^2 + 5x - 1 = (2x - 1)(2x - 1)(ax + b)$ By inspection, $a = 1$ and $b = -1$. $\therefore\ 4x^3 - 8x^2 + 5x - 1 = (2x - 1)(2x - 1)(x - 1)$ $= (2x - 1)^2(x - 1)$	**Award 3** ~ correct solution **Award 2** ~ significant progress towards solution **Award 1** ~ limited progress towards solution