International Grammar School

Year 11
Mathematics Extension 1
Yearly Examination
2011

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 60

- Attempt Questions 1 – 5
- All questions are of equal value
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and “N/A” on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Total Marks – 60
Attempt Questions 1–5
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Simplify $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$ 2

(b) i) Expand $\sin(A - B)$ 1

ii) Hence, or otherwise write the exact value of $\sin 15^o$ 2

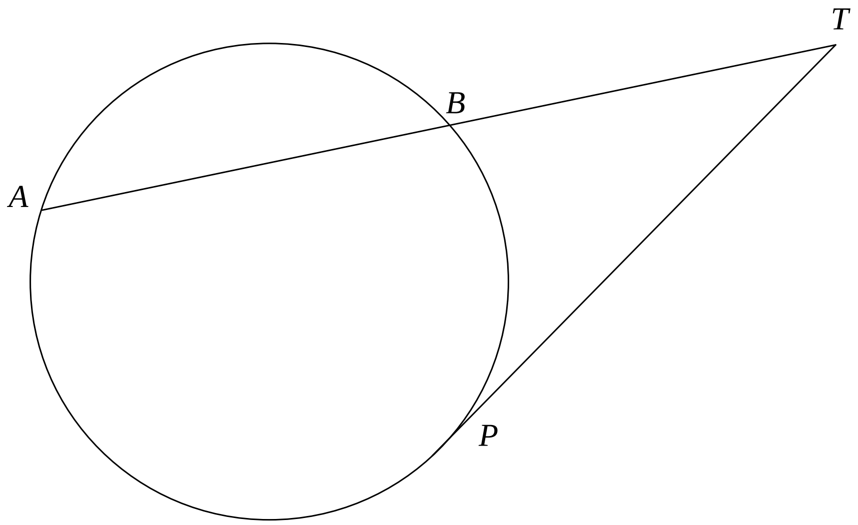
(c) Simplify $(n + 1)! - n!$ 2

(d) Solve for x where $\dfrac{x+2}{x} \geq 0$ 3

(e) Write the general solution to the equation $\tan x = 1$ 2

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram PT is a tangent to the circle at P.
AB = $5cm$ and PT = $8cm$. Find the length of the interval BT to the nearest tenth of a centimetre. 3

(b) Find the coordinates of the point P which divides the interval AB, where $A = (-2, 1)$ and $B = (1, 7)$, internally in the ratio $2:3$ 2

(c) Find $\lim\limits_{x\to\infty} \dfrac{3x+2x^3}{5x^2-1}$ 2

(d) Find the numbers of ways in which the letters of the word **FUNCTIONS** can be arranged in a line. 2

(e) i) Sketch $y=|x+5|$ and $y=2x$ on the same number plane 1

ii) Find the x coordinate(s) of the point(s) of intersection of the two functions $y=|x+5|$ and $y=2x$. 1

ii) Hence, or otherwise solve $2x \le |x+5|$ 1

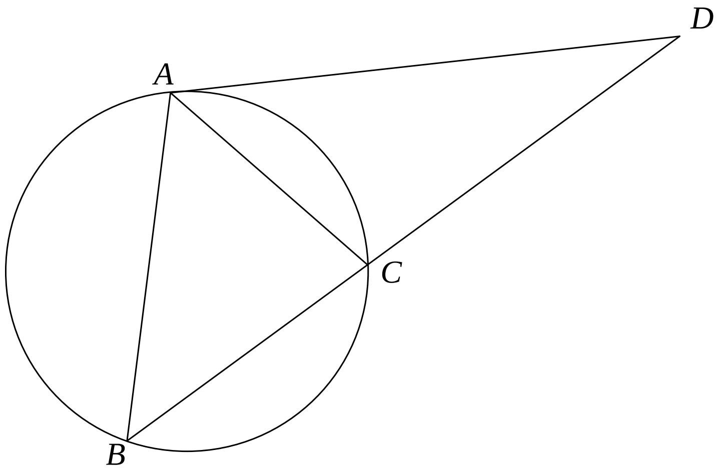
Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) Show that the equation $\cos 2x + 3\cos x + 2 = 0$ is equivalent to the equation $2\cos^2 x + 3\cos x + 1 = 0$ 2

ii) Hence, or otherwise solve the equation $\cos 2x + 3\cos x + 2 = 0$ for $0^o \le x \le 360^o$ 2

(b) Find the domain of the function $f(x) = \sqrt{4 - x^2}$ 2

(c)



ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D. $\angle CAB = 60^o$, $\angle CDA = 30^0$ and $\angle ABC = x^o$. Copy the diagram, showing the given information, then find the value of x. 2

(d) In a game of Lotto four numerals are chosen from the numerals **1 2 3 4 5 6 7 8 9** without repetition. Order is not important. Find the number of ways in which this can be done so that

i) **No** odd numerals are chosen 2

ii) **At least** one odd numeral is chosen 2

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) Show that $\cos x - \sqrt{3}\sin x \equiv 2\cos(x+60)^{o}$ 2

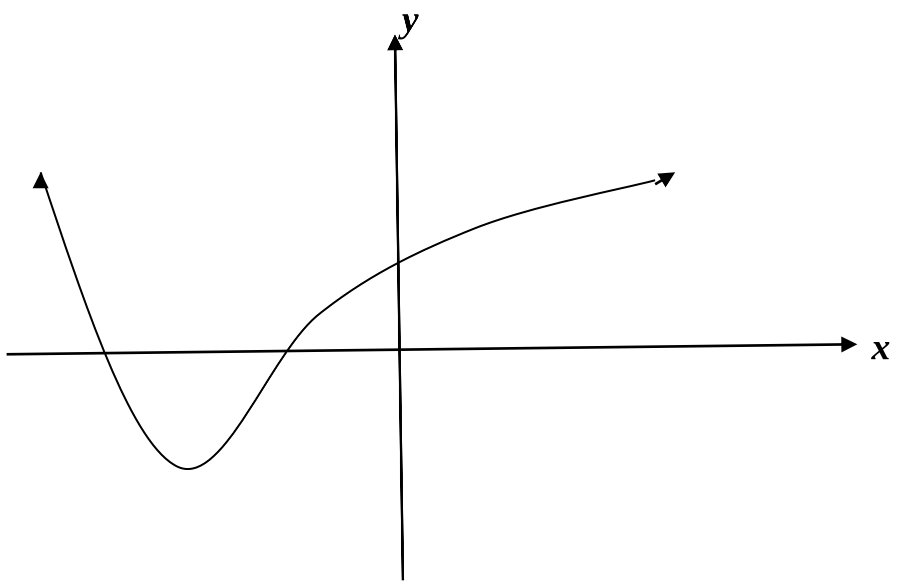
ii) Hence solve $\cos x - \sqrt{3}\sin x = 2$ over $0^{o} \le x \le 360^{o}$ 2

(b) i) Show that the acute angle θ between the lines $x - y = 0$ and $x - 2y = 0$ is such that $\tan\theta = \frac{1}{3}$. 2

ii) Find the equation of the other line $y = mx$ which is also inclined at the same acute angle θ to the line $x - 2y = 0$. 2

(c) Show that the line $y = x$ is a tangent to the circle $(x-2)^2 + y^2 = 2$ 2

(d) The graph shown is the function $y = f(x)$. On the answer sheet provided Sketch the gradient function $y = f'(x)$. 2

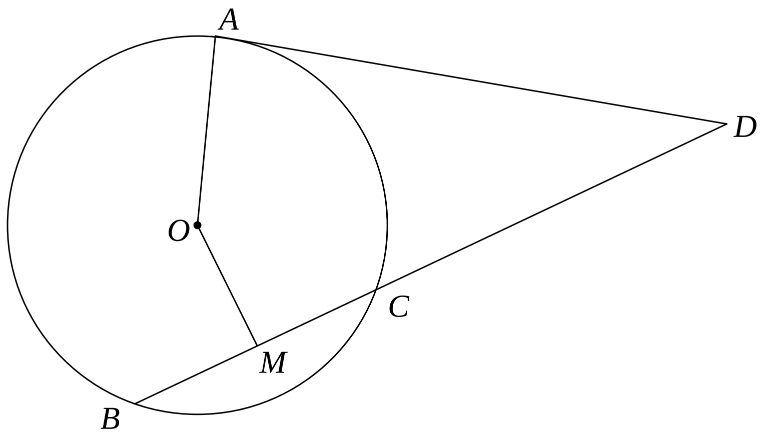


Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) If $y = x\sqrt{x+3}$ find the gradient function $\dfrac{dy}{dx}$ 2

ii) Find the coordinates of the point, P on the curve $y = x\sqrt{x+3}$ where the tangent is parallel to the x axis. 2

(b)



A, B and C are points on a circle with centre O. The tangent to the circle at A meets BC produced at D. M is the midpoint of BC.

Copy the diagram into your answer book.

i) Show that $AOMD$ is a cyclic quadrilateral. 3

ii) Give a reason why OD is a diameter of the circle through A, O, M and D. 1

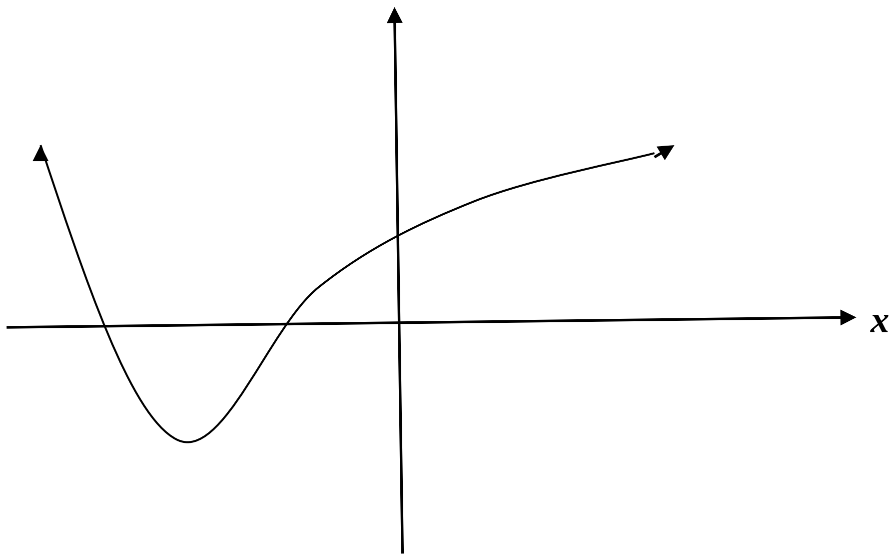
(c) If $t = \tan\dfrac{x}{2}$ show that $\dfrac{1-\cos x}{\sin x} = t$ 2

(d) 7 boys and 2 girls are to be seated around a circular table. In how many ways can this be done so that the girls are always seated together? 2

End of examination

Answer sheet for Question 4 (d) **Name……………………………….**

The graph shown is the function $y = f(x)$. On the same number plane below sketch the gradient function $y = f'(x)$. 2



Section I:

1. $TA^2 = TC \times TB$

$12^2 = x \times (x+7)$

$144 = x^2 + 7x$

$x^2 + 7x - 144 = 0$

$(x+16)(x-9) = 0$

$x = -16, \; x = 9$

$\therefore x = 9 > 0.$ (C)

2. $1 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$ (A)

3. $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}} = \sqrt{\dfrac{4(1+\cos 2x)}{1-\cos 2x}}$

$= 2\sqrt{\dfrac{1+(2\cos^2 x - 1)}{1-(1+2\sin^2 x)}}$

$= 2\sqrt{\dfrac{\cancel{1}+2\cos^2 x \cancel{-1}}{\cancel{1}\cancel{-1}+2\sin^2 x}}$

$= 2\sqrt{\cot^2 x} = 2\cot x$ (B)

4. $\dfrac{3}{x+4} \geq 2 \quad \times(x+4)^2$

$3(x+4) \geq 2(x+4)^2$

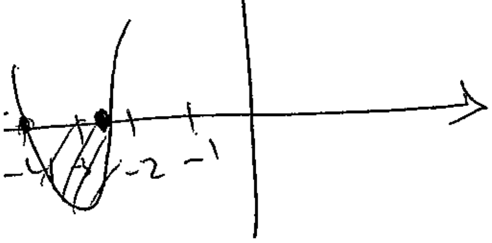
$2(x+4)^2 - 3(x+4) \leq 0$

$(x+4)[2(x+4)-3] \leq 0$

$(x+4)(2x+8-3) \leq 0$

$(x+4)(2x+5) \leq 0$

$-4 \leq x \leq -2\frac{1}{2}$ (A)



5. $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{x-1}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{2x^2}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{1}$ (C)

$= 2x.$

6. $A(6,-4), B(0,4), P(-3,8)$

$m:n = k:1$

$-3 = \dfrac{k \times 0 + 1 \times 6}{k+1}$

$-3k - 3 = 6$

$-3k = 9 \rightarrow \boxed{k = -3}$ (B)

7. $26^2 \times 10^2 \times 26^2 = 45697600$ (D)

8. ${}^9C_5 = 126$ (C)

9. $\lim\limits_{x\to -1} \dfrac{x^2+2x+1}{x+1}$

$= \lim\limits_{x\to -1} \dfrac{(x+1)\cancel{(x+1)}}{\cancel{x+1}}$

$= -1+1$

$= 0$ (C)

10. $\tan 15° = \dfrac{\tan 45° - \tan 30°}{1+\tan 45° \tan 30°}$

$= \dfrac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{3-2\sqrt{3}+1}{3-1}$

$= \dfrac{4-2\sqrt{3}}{2}$

$= 2-\sqrt{3}$ (A)

Question 11:

a) 2, 3, 5, 6, 8 and 9.

i) $6 \times 5 \times 4 \times 3 \times 2 = 720$ 1

or

$^6P_5 = 720$ 5 digit numbers.

ii. Half the numbers will be even since there's an equal number of odd and even digits.

$720 \div 2 = 360$ 1

or

$^5P_4 \times 3 = 360$ since there are 3 numbers that are even 2, 6 and 8.

b) i. $^{16}C_4 = 1820$ 1

ii. $^{10}C_4 = 210$ 1

iii. At least one man: All combinations − all women

$= {}^{16}C_4 - {}^{10}C_4$

$= 1610$ 1

* carry thru mark given for (i) − (ii) even if (i) or (ii) incorrect

or

$^6C_1 \times {}^{10}C_3 + {}^6C_2 \times {}^{10}C_2 + {}^6C_3 \times {}^{10}C_1 + {}^6C_4 \times {}^{10}C_0 = 1610$

c) i. Four men and 4 women at a circular table:

No restrictions: $7! = 5040$ 1

ii. 1st position is fixed men: $3!4! = 144$. 1

iii. 4 men and one group of women = 5 groups

$\therefore 4!$

4 women = $3!$

2 women can sit together in $2!$ ways

2 for correct
1 for dealing with women seating.

$\therefore$ total number of arrangements $= 4!\,3!\,2!$

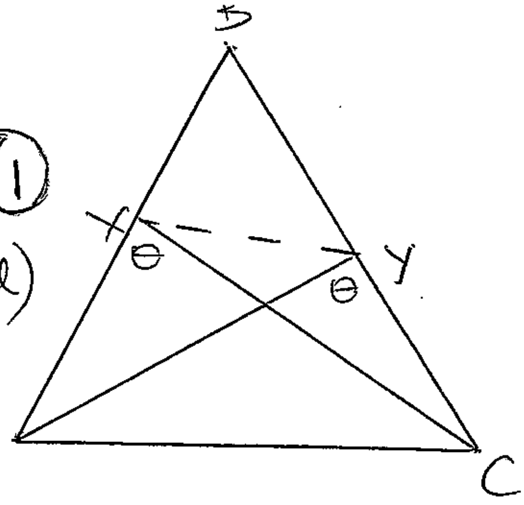
$= 288$

carry thru mark given if (ii) answered $4! \times 4!$ and (iii) $4! \times 4! \times 2!$

d) i. $\angle AXC = \angle AYC$

$= \theta$ (given)

$\therefore$ AXYC is cyclic (converse of $\angle$s in the same segment, AC, are equal) 1



ii. Since AXYC is cyclic,

$\angle BXY = \angle YCA$ (external $\angle$ in a cyclic quadrilateral = opposite interior $\angle$). 1

e) i. $\cos x - \sin x = A\cos(x+\alpha)$, $0 \le \alpha \le 90^\circ$

$A = \sqrt{a^2+b^2}$

$A = \sqrt{1^2+1^2}$

$A = \sqrt{2}$

$\tan\alpha = \frac{b}{a}$

$\tan\alpha = 1$

$\therefore \alpha = 45^\circ$

$\therefore \cos x - \sin x = \sqrt{2}\cos(x+45^\circ)$

2 mark $\sqrt{2}\cos(x+45^\circ)$
1 mark either $\sqrt{2}$ or $+45^\circ$.

ii. $\cos x - \sin x = -1$, $0^\circ \le x \le 360^\circ$

$\sqrt{2}\cos(x+45^\circ) = -1$

$\cos(x+45^\circ) = -\frac{1}{\sqrt{2}}$ 2nd and 3rd quadrants.

$\therefore x+45^\circ = 180^\circ - 45^\circ, 180^\circ + 45^\circ$, $45^\circ \le x+45^\circ \le 360^\circ + 45^\circ$

$x + 45^\circ = 135^\circ, 225^\circ$ 1

$x = 90^\circ, 180^\circ$ 1 for $0^\circ \le x \le 360^\circ$

$\boxed{\text{2 mark for correct answer.}}$

1 mark some correct progress

Question 12:

a) $9^x - 10.3^x + 9 = 0$

Let $u = 3^x$

then $9^x - 10.3^x + 9 = 0$ is equivalent to:

1 for correct u equation

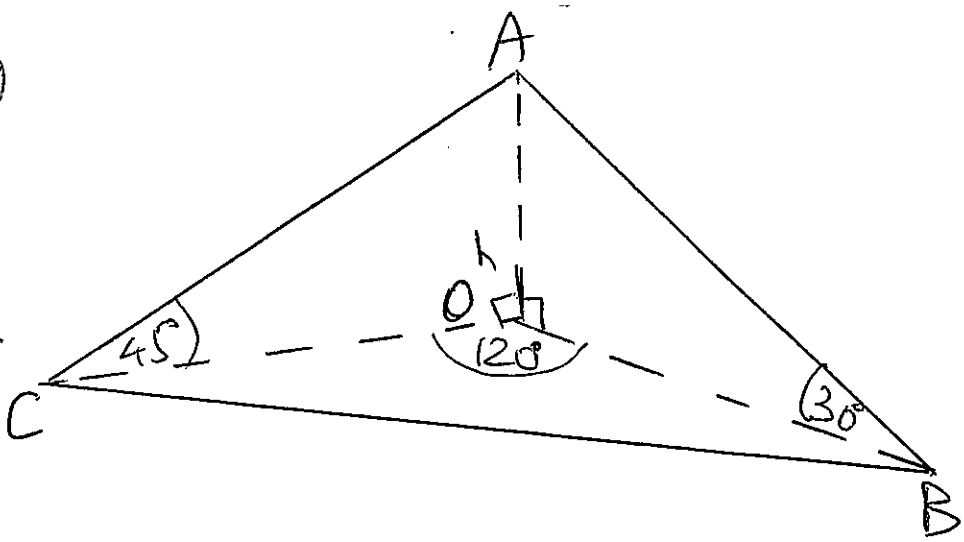
1 for each correct x-value

$$(3^2)^x - 10.3^x + 9 = 0$$
$$(3^x)^2 - 10.3^x + 9 = 0$$

i.e: $u^2 - 10u + 9 = 0$ - (1)

$$(u-9)(u-1) = 0$$
$$u = 9 \quad \text{or} \quad u = 1$$
$$3^x = 9 \quad \text{or} \quad 3^x = 1$$
$$x = 2 \quad \text{or} \quad x = 0$$
(1) (1)

b) i. $\tan 45^\circ = \frac{h}{OC}$ (1)



$$1 = \frac{h}{OC}$$
$$\therefore \boxed{OC = h} \text{ (1)}$$
$$\tan 30^\circ = \frac{h}{OB}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{OB}$$
$$\boxed{OB = \sqrt{3}h} \text{ (1)}$$

ii. $BC^2 = OC^2 + OB^2 - 2 \times OC \times OB \times \cos 120^\circ$ 1 mark

$$BC^2 = h^2 + (\sqrt{3}h)^2 - \cancel{2} \times h \times \sqrt{3}h \times -\frac{1}{\cancel{2}}$$
$$BC^2 = h^2(1 + 3 + \sqrt{3})$$
$$BC^2 = (4+\sqrt{3})h^2$$
1 mark

c.) $y = |2x-3|$

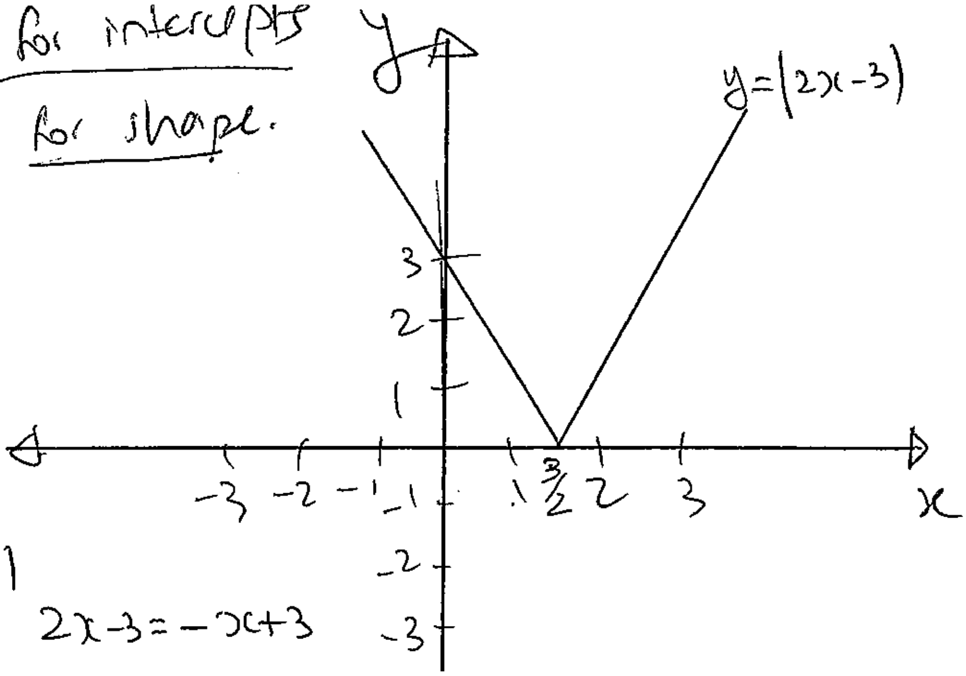
$y = 2x - 3$

or

$y = -2x + 3$

1 for intercepts

1 for shape.



ii. $|2x-3| \leq |x-3|$

solve: $|2x-3| = |x-3|$

$2x - 3 = x - 3$ or $2x - 3 = -x + 3$

$\boxed{x = 0}$ or $3x = 6$

$\boxed{x = 2}$

Test: $x = -1$, $x = 1$, $x = 3$

$x = -1$

$|2 \times -1 - 3| \leq |-1-3|$

$|-5| = |-4|$ False.

1 for working

1 for result.

$\boxed{x = 1}$

$|2 \times 1 - 3| \leq |1 - 3|$

$|-1| \leq |-2|$ True

$x = 3$

$|2 \times 3 - 3| \leq |3 - 3|$

$|3| \leq 0$ False.

$\therefore$ solution: $0 \leq x \leq 2$

d) $A(-4,-3)$, $B(1,5)$

$m:n$
$-3:2$.

$$x = \frac{-3\times 1 + 2\times -4}{-3+2}$$

$$x = \frac{-11}{-1}$$

$$x = 11$$

$$y = \frac{-3\times 5 + 2\times -3}{-3+2}$$

$$y = \frac{-15-6}{-1}$$

$$y = 21$$

1 for each coordinate.

$\therefore P(11, 21)$.

1 mark for significant progress towards answer

e) $\frac{2x+5}{x+1} < 1$, $x \neq -1$

$\times (x+1)^2$ $\times (x+1)^2$

$(2x+5)(x+1) < (x+1)^2$

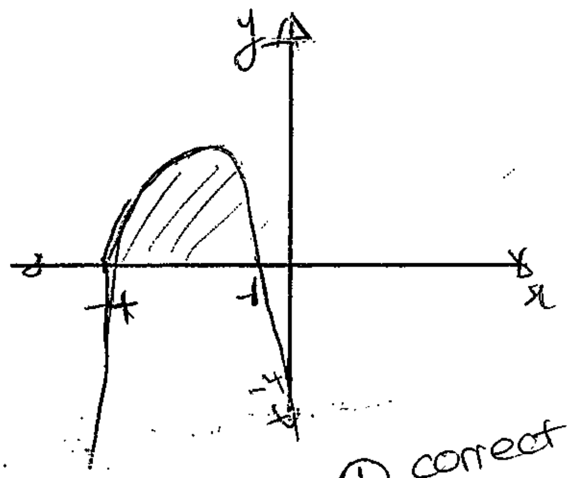
$(x+1)^2 - (2x+5)(x+1) > 0$

$(x+1)[(x+1) - (2x+5)] > 0$

$(x+1)(x+1-2x-5) > 0$

$(x+1)(-x-4) > 0$

$\boxed{-4 < x < -1}$



① correct progress having noted $x \neq -1$.

Or

Solve $\frac{2x+5}{x+1} = 1$ then test x-values:

$2x+5 = x+1$

$x = -4$

Test: $x = -5$, $x = -2$, $x = 0$

$x = -5$:

$\frac{2\times -5+5}{-5+1} < 1$

$\frac{-5}{-4} < 1$ False.

$x = -2$

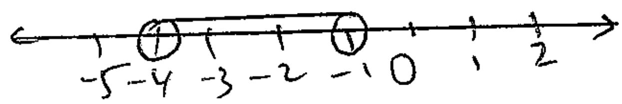
$\frac{2\times -2+5}{-2+1} < 1$

$\frac{1}{-2} < 1$ True.

$x = 0$

$\frac{2\times 0+5}{1} < 1$

$5 < 1$ False.



$\therefore -4 < x < -1$

Question 13:

a) i. $y = x^2 + 2x - 3$ and $y = x^2 + 1$ solving simultaneously.

$\cancel{x^2} + 2x - 3 = \cancel{x^2} + 1$

$2x = 4$

$x = 2$

$y = x^2 + 1 \rightarrow y = 2^2 + 1 \rightarrow y = 5$

1 any correct working.

$\therefore (2, 5)$

ii. $y = x^2 + 2x - 3$, $y = x^2 + 1$

$y' = 2x + 2$, $y' = 2x$

$\therefore m_1 = 2 \times 2 + 2$, $\therefore m_2 = 2 \times 2$

$\boxed{m_1 = 6}$, $\boxed{m_2 = 4}$

1 for gradients

1 for result.

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan\theta = \left| \frac{6 - 4}{1 + 6 \times 4} \right|$

$\tan\theta = \left| \frac{2}{25} \right|$

$\therefore \theta = 4.5739\ldots$

$\therefore \theta \doteqdot 5^\circ$

b) $4x^2 - (6+k)x + 1 = 0$

No real zeros $\rightarrow \Delta < 0$

$b^2 - 4ac < 0$

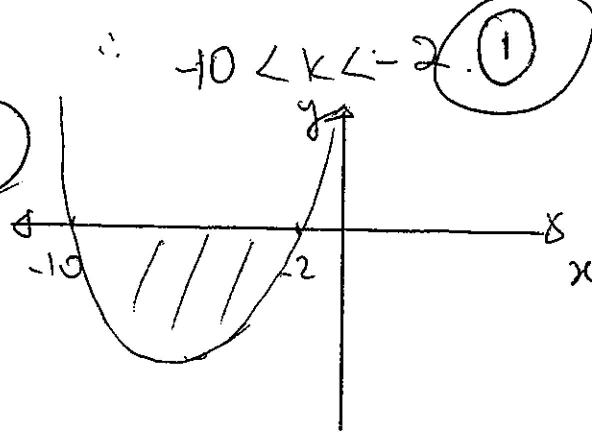
$[-(6+k)]^2 - 4 \times 4 \times 1 < 0$ ①

$36 + 12k + k^2 - 16 < 0$

$k^2 + 12k + 20 < 0$

$(k+2)(k+10) < 0$

$\therefore -10 < k < -2$ ①



c) i. $x \neq \pm 3$ (1) $f(x) = \frac{2x^2}{x^2-9}$

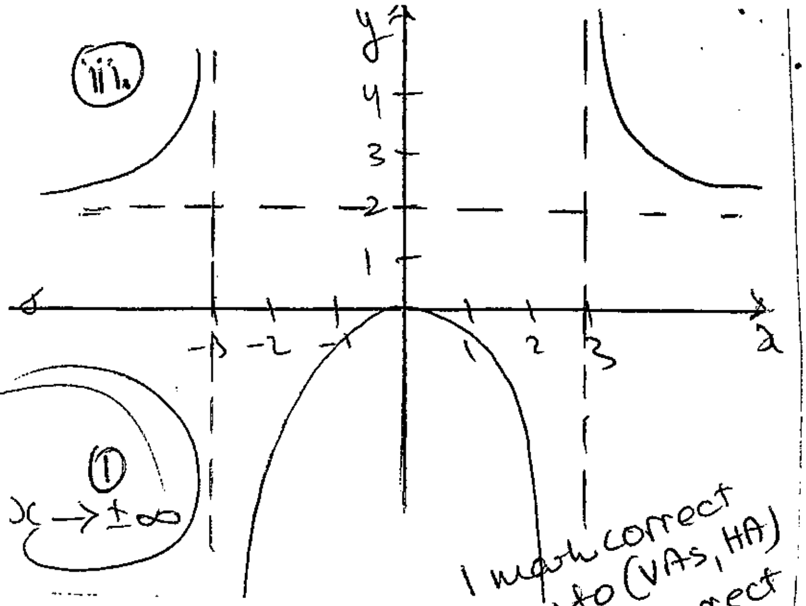
$x=3$ or $x=-3$

ii. $\lim_{x \to \pm\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$

$\lim_{x \to \pm\infty} \frac{2}{1-0}$

$=2$, y approaches 2 as $x \to \pm\infty$ (1)

iii.



1 mark correct info (VAs, HA)

1 mark correct graph.

iv. Range: $y \leq 0$, $y > 2$

d) $y = mx$ $\quad 2x - y - 1 = 0$

$y = 2x - 1$

$\therefore m_1 = 2.$

$\tan 45^\circ = \left| \frac{m_1 - m_1}{1 + m_1 m_2} \right|$

$1 = \left| \frac{2-m}{1+2m} \right|$

$\therefore \quad 1 = \frac{2-m}{1+2m} \quad$ or $\quad -1 = \frac{2-m}{1+2m}$

$1 + 2m = 2 - m \qquad -1 - 2m = 2 - m$

$3m = 1 \qquad -m = 3$

$\therefore m = \frac{1}{3}$ (1) $\qquad \therefore m = -3$ (1)

1 mark may be awarded for correct progress, eg: correct substitution or one correct solution.

e) $\cos 2x - 3\cos x + 2 = 0$

$2\cos^2 x - 1 - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0, \quad \cos x - 1 = 0$

$2\cos x = 1$ or $\cos x = 1$

$\cos x = \frac{1}{2}$

$x = 60^\circ, 360^\circ - 60^\circ$ or $x = 0^\circ, 360^\circ$

$\therefore x = 0^\circ, 60^\circ, 300^\circ, 360^\circ$

1 for correct substitution for $\cos 2x$

1 for correct $\cos x$ values

1 for solution.