International Grammar School

Year 11
Mathematics Extension 1
Assessment Task 3
Yearly Examination
2012

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10

Section II

45 marks

- Attempt Questions 11, 12 & 13
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet and writing sheet to be handed in.
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section A

10 marks
Attempt Questions 1 - 10

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

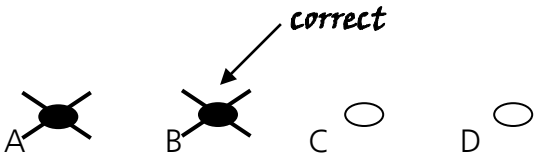
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ○ B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.



Section I

10 marks
Attempt Questions 1 – 10
Use the multiple-choice answer sheet for Questions 1-10

1 How many different nine letter combinations can be made using the letters forming the word **AUSTRALIA**?

(A) 720

(B) 60 480

(C) 362 874

(D) 362 880

2 What is the acute angle between the lines $2x-y-7=0$ and $3x-5y-2=0$?

(A) 4°24`

(B) 85°36`

(C) 57°32`

(D) 32° 28`

3 Which of the following is equal to $\dfrac{\sin(90-\theta)}{\cos(180+\theta)}$?

(A) -1

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{2}$

(D) 1

4 State the largest possible domain of the function:

$$f(x)=\sqrt{x+3}-\sqrt{x-3}$$

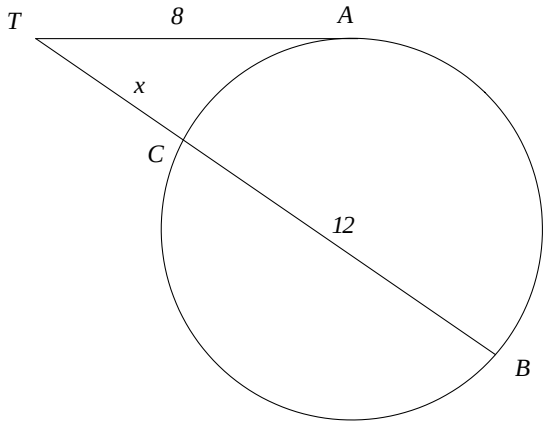
(A) $x\le -3,\ x\ge 3$

(B) $x\ge 3$

(C) $-3\le x\le 3$

(D) $x\ge -3,\ x\ge 3$

5 The line *TA* is a tangent to the circle at *A* and *TB* is a secant meeting the circle at *B* and *C*.



Given that $TA=8$, $CB=12$ and $TC=x$, what is the value of *x*?

(A) 16

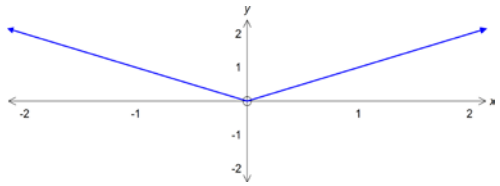
(B) $4\frac{3}{8}$

(C) 4

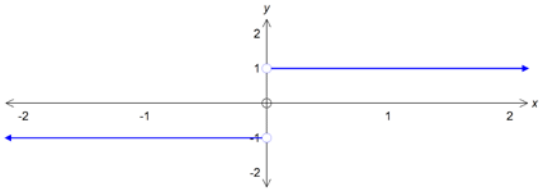
(D) $6\frac{2}{3}$

6 The sketch of the graph of $f(x)=\frac{|x|}{x},\ x\neq 0$ is:

(A)



(B)



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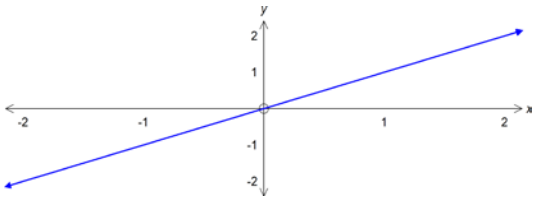
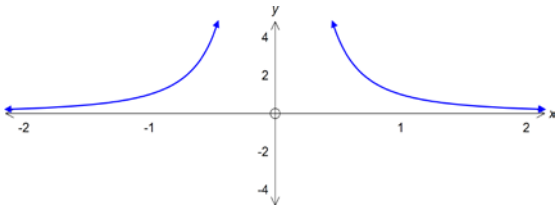


(D)



7 Given that $\sin\alpha=\frac{1}{4}$, what is the value of $\cos 2\alpha$?

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{7}{8}$

(D) $\frac{2\sqrt{15}}{16}$

8 In how many ways can the letters of the word RHOMBUS be arranged so that the vowels are together?

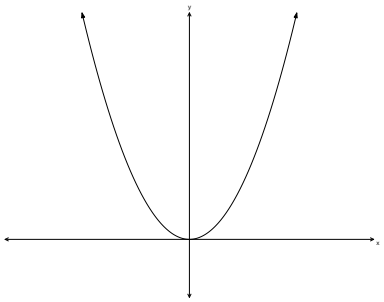
(A) 720

(B) 5040

(C) 1440

(D) 360

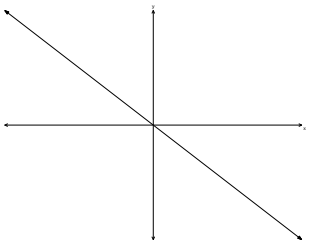
9



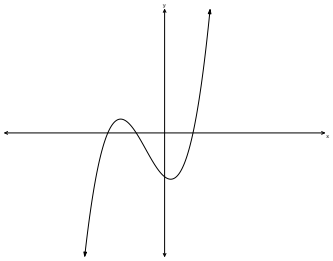
$y = f'(x)$

Given the graph of $y = f'(x)$ above, which graph best represents $y = f(x)$?

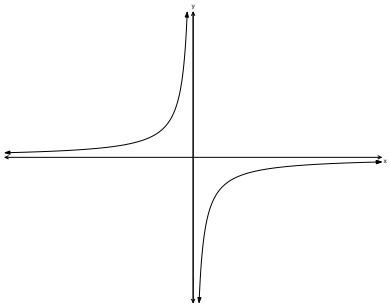
(A)



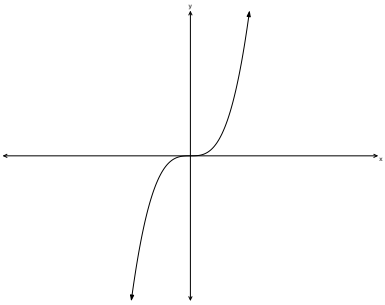
(B)



(C)



(D)



10 Evaluate $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

(A) 0

(B) undefined

(C) 4

(D) 12

END OF SECTION I

Section II

45 marks
Attempt Questions 11 – 13
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 11 (15 Marks) **Marks**

a) Solve for x:

$$16^{4-x} = \frac{1}{8^x}$$

2

b) Find the coordinates of the point P which divides the interval AB joining $A(1,-3)$ and $B(2,-2)$ externally in the ratio $2:3$. **2**

c) How many different committees of 8 people with equal numbers of men and women can be chosen if there are 6 women and 10 men? **2**

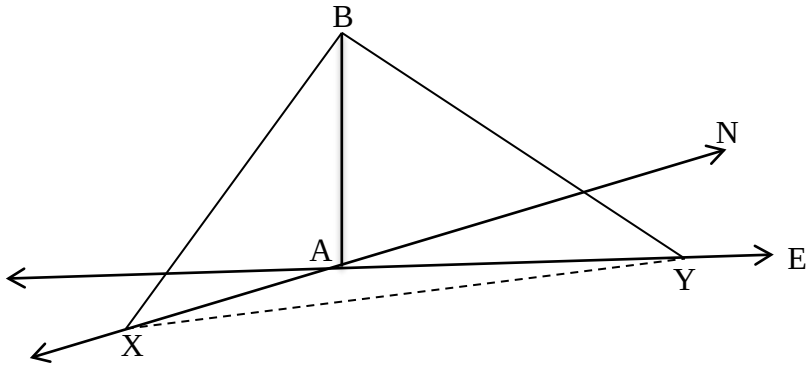
d) i) Find the x coordinate of the point of intersection of $y = x^2 - 2x$ and $y = x^2 - 7x + 10$. **1**

ii) Hence find the acute angle between these two curves at that point. Give your answer to the nearest degree. **2**

e) Solve for x: $\dfrac{x^2-4}{x} > 0$. **3**

Question 11 continues on the next page.

f)



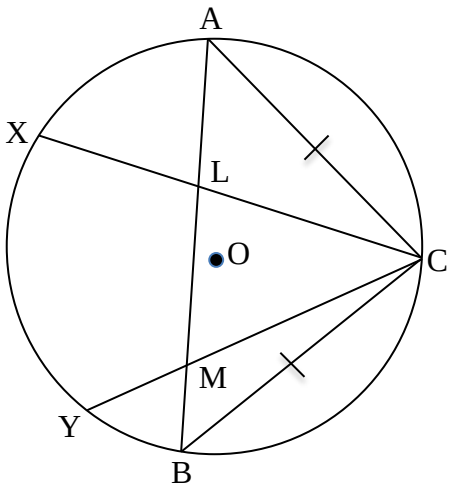
A pole AB is 4 metres high and on level ground. Point X is due south of the pole and the angel of elevation from X to the top of the pole is 45^o. Another point Y is due east of the pole and the angle of elevation from Y to the top of the pole is 30^o.

i) Find the exact value for AX and AY. **1**

ii) Find the distance XY. **2**

Question 12 (15 Marks) **Marks**

a)



In the diagram A, B and C are three points on the circle. Chords CX and CY cut AB at L and M respectively. $AC = CB, \angle CAB = \theta$ and $\angle ACX = \alpha$.

Copy or trace the diagram onto your paper.

i) State why $\angle CLB = \theta + \alpha$. **1**

ii) Explain why $\angle AYC = \theta$ and $\angle AYX = \alpha$. **2**

iii) Prove that $XYML$ is a cyclic quadrilateral. **1**

b) Find the derivative of the following functions, giving your answer in fully factorised form where appropriate.

i) $y = \dfrac{3x^2 - 2x + 1}{\sqrt{x}}$ **2**

ii) $y = \dfrac{x^2}{1 - x^3}$ **2**

iii) $y = (x+1)\sqrt{4x-1}$. For this question you must give your answer in fully factorised form. **3**

Question 12 continues on the next page.

c) The tangent to $y=\frac{a}{x+2}$ at $x=2$ has a gradient of $\frac{1}{4}$. **2**
Find the value of a.

d) One of the roots of $x^2+bx+c=0$ is 3 times the other root. **2**
Show that $3b^2-16c=0$

Question 13 (15 Marks) **Marks**

a) If $t = \tan\frac{\theta}{2}$ show that: $\frac{\cos\theta}{1-\sin\theta} = \frac{1+t}{1-t}$. **3**

b) i) Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0^o \le \alpha \le 90^o$. **2**

ii) Hence or otherwise solve $\sqrt{3}\sin x - \cos x = 1$ for $0^0 \le x \le 360^o$. **2**

c) i) Show that $\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$ **2**

ii) Hence or otherwise, show that the exact value of $\tan 22\frac{1}{2}^o = \sqrt{2}-1$. **2**

d) i) Find any values of a and b such that $x^4 + 4 \equiv \left(x^2 + a\right)^2 - \left(bx\right)^2$. **2**

ii) Hence express $x^4 + 4$ as a product of two quadratic factors. **2**

END OF EXAMINATION

Preliminary Yearly Examination – Extension 1 Mathematics 2012
Multiple Choice Answer Sheet

Name _____

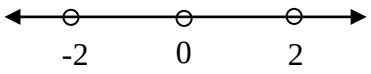
Completely fill the response oval representing the most correct answer.

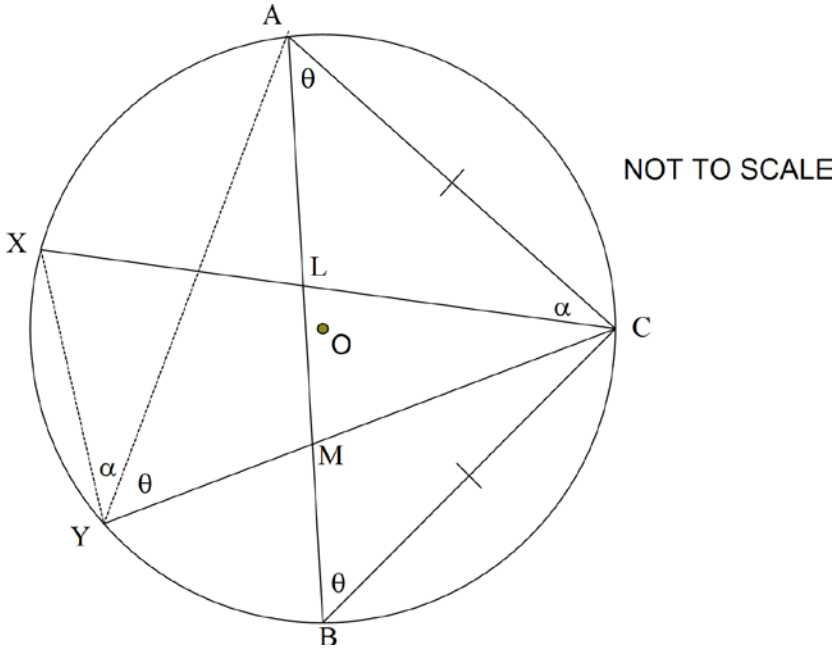
1. A ⬭ B ⬭ C ⬭ D ⬭
2. A ⬭ B ⬭ C ⬭ D ⬭
3. A ⬭ B ⬭ C ⬭ D ⬭
4. A ⬭ B ⬭ C ⬭ D ⬭
5. A ⬭ B ⬭ C ⬭ D ⬭
6. A ⬭ B ⬭ C ⬭ D ⬭
7. A ⬭ B ⬭ C ⬭ D ⬭
8. A ⬭ B ⬭ C ⬭ D ⬭
9. A ⬭ B ⬭ C ⬭ D ⬭
10. A ⬭ B ⬭ C ⬭ D ⬭

MULTIPLE CHOICE

1. B
2. D
3. A
4. B
5. C
6. B
7. C
8. C
9. D
10. D

Question 11	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment
(a)	$16^{4-x} = \dfrac{1}{8^x}$ $\left(2^4\right)^{4-x} = 2^{-3x}$ $2^{16-4x} = 2^{-3x}$ $16 - 4x = -3x$ $x = 16$	**2** **1** **1**	
(b)	$A(1,-3)$ and $B(2,-2)$, Externally $2:-3$ $x = \dfrac{mx_2 + nx_1}{m+n}$ $\quad$ $y = \dfrac{my_2 + ny_1}{m+n}$ $= \dfrac{2\times 2 + -3\times 1}{2+-3}$ $\quad$ $= \dfrac{2\times -2 + -3\times -3}{2+-3}$ $= -1$ $\quad$ $= -5$ The coordinates of P are $(-1,-5)$	**2**	**1** Correct use of formula **1** correct coordinates
(c)	${}^6\mathbf{C}_4 \times {}^{10}\mathbf{C}_4 = 3150$	**2**	**1** mark for correct combinations **1** mark correct numerical answer
(d)	i) For intersection of $y = x^2 - 2x$ and $y = x^2 - 7x + 10$: $x^2 - 2x = x^2 - 7x + 10$ $5x = 10$ $x = 2$ ii) For $y = x^2 - 2x$, $y' = 2x - 2$ then $m_1 = 2$ For $y = x^2 - 7x + 10$, $y' = 2x - 7$ then $m_2 = -3$ $\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \dfrac{2-(-3)}{1+2\times(-3)} \right\rvert$ $= \left\lvert \dfrac{5}{-5} \right\rvert$ $= 1$ $\theta = 45^o$	**1** **2** 	 **1** mark correct **1** correct substitution into correct formula **1** correct answer

Question 11	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment
(e)	$\frac{x^2-4}{x} > 0$ $\frac{x^2(x^2-4)}{x} > 0 \times x^2$ $x(x-2)(x+2) > 0$ $-2 < x < 0,\ x > 2$ OR $\frac{x^2-4}{x} > 0, x \neq 0$ $(x+2)(x-2) = 0$ $x = \pm 2$  Test: $x = 1, \frac{1-4}{1} = -3 < 0$ $x = -1, \frac{1-4}{-1} = 3 > 0$ So $-2 < x < 0, x > 2$	**3**	**1** mark any correct working **1** mark each for correct inequalities
(f)(i)	In ΔABX $\tan 45^o = \frac{4}{AX}$ $AX = \frac{4}{\tan 45^o}$ $= 4$ In ΔABE $\tan 30^o = \frac{4}{AY}$ $AY = \frac{4}{\tan 30^o}$ $= \frac{4}{\frac{1}{\sqrt{3}}} = 4\sqrt{3}$	**1**	1 mark correct
(ii)	Pythagoras theorem $XY^2 = BX^2 + BY^2$ $= 4^2 + (4\sqrt{3})^2$ $= 64$ $XY = 8$ cm	**2**	1 mark recognises Pythagoras' theorem (or may for some reason use cosine rule) 1 mark correct answer

Question 12	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment
(a)			
(i)	The exterior angle of a triangle (Δ ALC) is equal to the sum of the opposite interior angles.		**1**
(ii)	$\angle ABC = \angle BAC = \theta$ (*base angles of isos* ΔACB) $\angle AYC = \angle ABC = \theta$ (*angles standing on the same arc are equal*) $\angle AYX = \angle ACX = \alpha$ (*angles standing on the same arc are equal*)	**2**	**1** mark each
(iii)	$\angle XYM = \alpha + \theta$ (*all shown above*) $= \angle CLB$ $= \alpha + \theta$ $\therefore$ *YMLX is a cyclic quadrilateral.* (*exterior angle of cyclic quad is equal to opp interior angle)*		**1**

Question 12	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment

(b)i) **2**

$$y = \frac{3x^2 - 2x + 1}{\sqrt{x}} = 3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

1 mark correct differentiation

$$y' = 3 \times \frac{3}{2} \times x^{\frac{1}{2}} - 2 \times \frac{1}{2} \times x^{-\frac{1}{2}} + -\frac{1}{2} x^{-\frac{3}{2}}$$

$$= \frac{9}{2} x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{3}{2}} \quad OR$$

$$= \frac{9}{2}\sqrt{x} - \frac{1}{\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$

1 mark correct simplification (negative indices acceptable)

OR (*and this is much messier*)

$$u = 3x^2 - 2x + 1 \qquad v = x^{\frac{1}{2}}$$

$$u' = 6x - 2 \qquad v' = \frac{1}{2} x^{-\frac{1}{2}}$$

$$y' = \frac{x^{\frac{1}{2}}(6x-2) - \frac{1}{2} x^{-\frac{1}{2}}(3x^2 - 2x + 1)}{\left(x^{\frac{1}{2}}\right)^2}$$

1 mark correct use of quotient rule

$$= \frac{x^{-\frac{1}{2}}\left(2x(6x-2) - (3x^2 - 2x + 1)\right)}{2x}$$

$$= \frac{12x^2 - 4x - 3x^2 + 2x - 1}{2x\sqrt{x}}$$

I mark to this factorisation

$$= \frac{9x^2 - 2x - 1}{2x\sqrt{x}}$$

ii) **2**

$$y = \frac{x^2}{1 - x^3}$$

$$u = x^2 \qquad v = 1 - x^3$$

$$u' = 2x \qquad v' = -3x^2$$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1-x^3) \times 2x - x^2 \times -3x^2}{\left(1 - x^3\right)^2}$$

1 mark correct use of quotient rule

$$= \frac{2x - 2x^4 + 3x^4}{\left(1 - x^3\right)^2}$$

$$= \frac{x^4 + 2x}{\left(1 - x^3\right)^2}$$

1 mark correct factorised form

$$= \frac{x(x^3 + 2)}{\left(1 - x^3\right)^2}$$

b)iii)	$y=(x+1)\sqrt{4x-1}$ $u=(x+1) \qquad v=(4x-1)^{\frac{1}{2}}$ $u'=1 \qquad v'=\frac{1}{2}\times 4(4x-1)^{-\frac{1}{2}}$ $=2(4x-1)^{-\frac{1}{2}}$ $y'=uv'+vu'$ $=(x+1)\times 2(4x-1)^{-\frac{1}{2}}+(4x-1)^{\frac{1}{2}}\times 1$ $=(4x-1)^{-\frac{1}{2}}[2(x+1)+(4x-1)]$ $=\frac{(2x+2+4x-1)}{\sqrt{4x-1}}$ $=\frac{6x+1}{\sqrt{4x-1}}$	**3**	**1** mark correct use of product rule **1** mark correct progress in factorisation **1** mark correct fully factorised
c)	$y=\frac{a}{x+2}=a(x+2)^{-1}$ $y'=a\times -1(x+2)^{-2}$ $=\frac{-a}{(x+2)^2}$ $x=2,\ m=\frac{1}{4}.$ $\frac{1}{4}=\frac{-a}{(x+2)^2}=\frac{-a}{(2+2)^2}$ $=\frac{-a}{16}$ $16=-4a$ $a=-4$	**2**	1 mark for expression equating correct derivative to given gradient 1 mark correct
d)	$x^2+bx+c=0$ Roots are $\alpha, 3\alpha$ Sum: $\alpha+3\alpha=-b$ $4\alpha=-b$ Product: $\alpha\times 3\alpha=c$ $3\alpha^2=c$ From sum, $\alpha=-\frac{b}{4}$ $3\alpha^2=3(-\frac{b}{4})^2$ So from product: $\frac{3b^2}{16}=c$ $3b^2-16=0$	**2**	1 mark correct expression for both sum and product of roots 1 mark correct derivation of result

Question 13	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment

(a) **3**

$$\frac{\cos\theta}{1-\sin\theta}-\frac{1-t}{1+t}$$

$$LHS=\frac{\dfrac{1-t^2}{1+t^2}}{1-\dfrac{2t}{1+t^2}} \quad u\sin g\ \cos\theta=\frac{1-t^2}{1+t^2}\ \ and\ \ \sin\theta=\frac{2t}{1+t^2}\ \ where\ \tan\frac{\theta}{2}=t$$

$$=\frac{1-t^2}{1+t^2-2t}$$

$$=\frac{1-t^2}{(1-t)^2}$$

$$=\frac{(1+t)(1-t)}{(1-t)^2}$$

$$=\frac{(1+t)}{(1-t)}$$

$$=RHS$$

1 for correct substitution of correct t results

1 for correct simplification

1 for correct simplification to reach result

(b) **2**

$$\sqrt{3}\sin x-\cos x=R\sin(x-\alpha)$$

$$=R\sin x\cos\alpha-R\cos x\sin\alpha$$

Equating coefficients:

$$R\cos\alpha=\sqrt{3} \qquad R\sin\alpha=1$$

For R:

$$R^2\cos^2\alpha+R^2\sin^2\alpha=(\sqrt{3})^2+1$$

$$R^2(\cos^2\alpha+\sin^2\alpha)=4$$

$$R^2=4$$

$$R=2,\quad R>0$$

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$$

For α : $$=\frac{1}{\sqrt{3}}$$

$$\alpha=30^o \qquad 0^o\le\alpha\le 90^o$$

So: $\sqrt{3}\sin x-\cos x=2\sin(x-30^o)$

May use long or short method

1 mark each correct R and α .

Question 13	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment
b)ii)	$\sqrt{3}\sin x - \cos x = 1 \quad 0^o \le x \le 360^o$ $2\sin(x-30^o) = 1 \quad$ *above* $-30^o \le x \le 330^o$ $\sin(x-30^o) = \frac{1}{2} \quad$ *Quadrants* 1 & 2 $(x-30^o) = 30^o, 150^o$ $x = 60^o, 180^o$	**2**	1 mark 1 mark BOTH solutions
(c)(i)	$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}$ $= \frac{2\sin^2\theta}{2\cos^2\theta}$ $= \tan^2\theta$	**2**	**1** **1** Any correct substitution
(ii)	$\tan^2 22\frac{1}{2}^\circ = \frac{1-\cos 45^\circ}{1+\cos 45^\circ}$ $= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $= \frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}$ $= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$ $\tan^2 22\frac{1}{2}^\circ = \frac{(\sqrt{2}-1)^2}{2-1}$ $= (\sqrt{2}-1)^2$ $\therefore \quad \tan 22\frac{1}{2}^\circ = \sqrt{2}-1$	**2**	**1** using result above correctly **1** Or alternate method eg forming quadratic equation

Question 13	Preliminary Mathematics Extension 1	2012	
Part	Solution	Marks	Comment
(d)(i)	$x^4+4\equiv\left(x^2+a\right)^2-(bx)^2$ $\equiv x^4+\left(2a-b^2\right)x^2+a^2$ *equating coefficients* $2a-b^2=0\Rightarrow b^2=2a$ (since *coefficient of* x^2 *is* 0) $a^2=4\Rightarrow a=\pm2$ (since 4 *is the constant term*) *but* $b^2=2a$, *ie* $a=2$ since $b^2>0$ *so* $b^2=4\Rightarrow b=\pm2$	**2**	1 for correct expansion and grouping of coefficients according to powers of x **1** for a & b
(ii)	$x^4+4\equiv\left(x^2+2\right)^2-(2x)^2$ $=\left\{\left(x^2+2\right)+2x\right\}\left\{\left(x^2+2\right)-2x\right\}$ $=\left(x^2+2x+2\right)\left(x^2-2x+2\right)$	**2**	**1** correct factorisation into difference of squares **1** correct result