International Grammar School

Year 11
Mathematics Extension 1
Assessment Task 3
Yearly Examination
2013

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 50

Section I

5 marks

- Attempt Questions 1 – 5

Section II

45 marks

- Attempt Questions 6, 7 & 8
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet and writing sheet to be handed in.
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

5 marks
Attempt Questions 1 – 5
Use the multiple-choice answer sheet for Questions 1-5

1 How many solutions are there for $\cos\theta = -1$ in the domain $0^o \le \theta \le 720^o$?

(A) 1

(B) 5

(C) 2

(D) 4

2. Which of the following is an expression for $\dfrac{1}{(n-1)!} + \dfrac{1}{n!}$ **?**

(A) $\dfrac{n+1}{n!}$

(B) $\dfrac{1}{(n-1)!}$

(C) $\dfrac{n+1}{n(n-1)!}$

(D) $\dfrac{2n-1}{n!(n-1)!}$

3. The acute angle between the lines $2x + y - 3 = 0$ and $x - 3y + 2 = 0$ is closest to

(A) 18°

(B) 45°

(C) 67°

(D) 82°

4. How many different arrangements of the letters of the word EXERCISE are possible?

(A) 720

(B) 6720

(C) 20160

(D) 40320

5. Which of the following is NOT an expression for $\cos 2\theta$?

(A) $\cos^2\theta - \sin^2\theta$

(B) $1 - 2\sin^2\theta$

(C) $\cos^2\theta + \sin^2\theta$

(D) $2\cos^2\theta - 1$

END OF SECTION I

Section II

45 marks
Attempt Questions 6 – 8
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 6 (15 Marks) **Marks**

(a) Solve $\frac{2x}{6-x} \geq 1$ **2**

(b) Find the coordinates of the point P which divides the interval AB with end points $A(2,3)$ and $B(5,-7)$ in the ratio $4:9$. **2**

(c) Find the number of ways in which 2 brothers and 4 sisters can line up in a queue:

(i) Without restriction **1**

(ii) So that 2 brothers are NOT next to each other **2**

(d) By expressing $\tan 15^o$ as $\tan(60^o - 45^\circ)$, show that, as an exact value: **2**

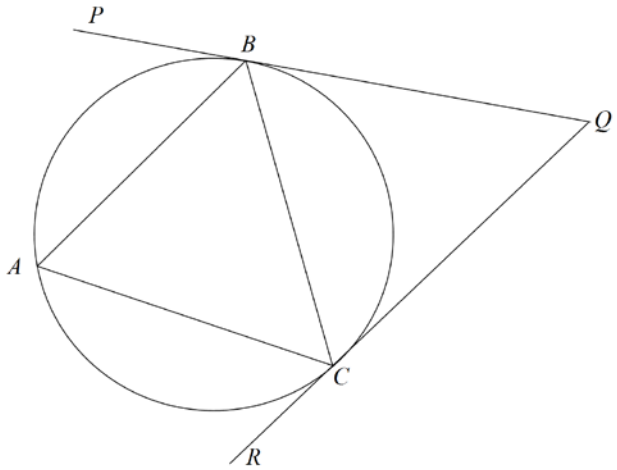
$$\tan 15^o = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

(e) (i) Show that $\sin 3x = 3\sin x - 4\sin^3 x$ **2**

(ii) Hence, solve $3\sin x - 4\sin^3 x = \frac{1}{2}$ for $0^o \leq x \leq 360^o$. **2**

Question 6 continues on the next page.

(f) In the diagram below PQ and RQ are tangents to the circle touching the circle at B and C respectively. $\angle BAC = 65^\circ$.



Not to scale

Copy or trace the diagram into your answer book.

Show that $\angle BQC = 50^\circ$ **2**

END OF QUESTION 6

Question 7 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The curves $y=\sqrt{x}$ and $y=\frac{x^2}{4}-2$ meet at the point $(4,2)$. Calculate the acute angle between the tangents to curves at this point. **3**

(b) By using $t=\tan\frac{\theta}{2}$, show that $\sin\theta=\frac{2t}{1+t^2}$ and $\cos\theta=\frac{1-t^2}{1+t^2}$ **3**

(c) (i) Express $\sin x+\sqrt{3}\cos x$ in the form of $R\sin(x+\alpha)$ where $R>0$ and $0\le\alpha\le 90°$. **2**

(ii) Hence or otherwise, find the general solution of $\sin x+\sqrt{3}\cos x=\sqrt{3}$. **2**

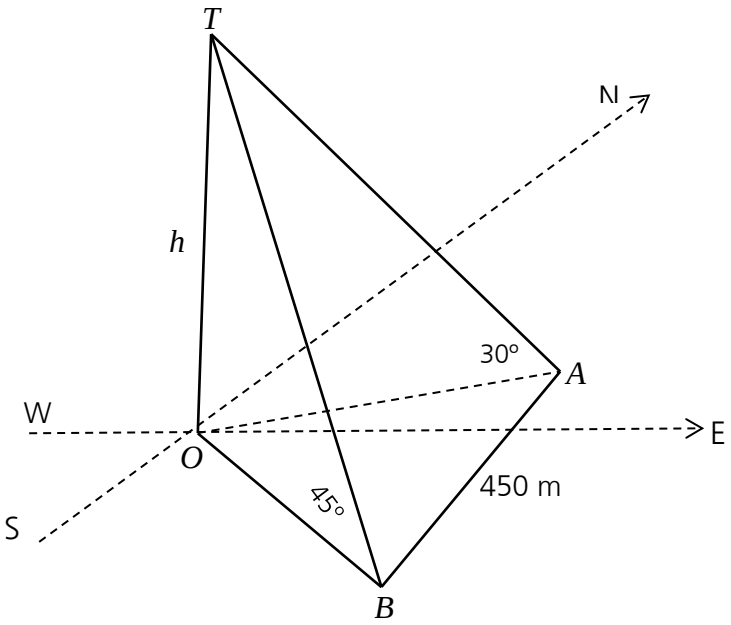
(d) Show that $\tan^2 x-\tan^2 y=\frac{\cos^2 y-\cos^2 x}{\cos^2 x\cos^2 y}$ **2**

(e) Solve $3^{2x}-8\times 3^x-9=0$. **3**

END OF QUESTION 7

Question 8 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a)



A tower stands on level ground. An observation point A bears NE from the base of the tower. Another point of observation B, bears SE from the base of the tower A and B are 450 metres apart. The angle of elevation of the tower from A and B respectively are 30° and 45°.

(i) Show that $OB = \dfrac{h}{\tan 45^o}$ and $OA = \dfrac{h}{\tan 30^o}$. **1**

(ii) By using Pythagoras' theorem on ΔOAB , find the value of h. **3**

(b) Let $f(x) = \dfrac{x}{x^2 - 1}$.

(i) For what values of x is $f(x)$ undefined? **1**

(ii) Show that $f'(x) < 0$ for all values of x for which the function is defined. **2**

(iii) Hence, sketch $y = f(x)$. Show all intercepts and asymptotes clearly. **2**

(iv) Find the equation of the tangent to the curve at $x = 2$.
Express your answer in general form. **2**

Question 8 continues on the next page.

(c) Consider the circle $x^2 + y^2 - 2x - 14y + 25 = 0$.

(i) Show that if the line $y = mx$ intersects the circle in two distinct points, then $(1+7m)^2 - 25(1+m^2) > 0$. **2**

(ii) For what values of m is the line $y = mx$ a tangent to the circle? **2**

END OF EXAMINATION

2013 Year 11 Yearly Examination **Student Name: . . . . . .**

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭

2013 Year 11 Yearly Examination **Student Name:** SOLS

Mathematics Extension 1

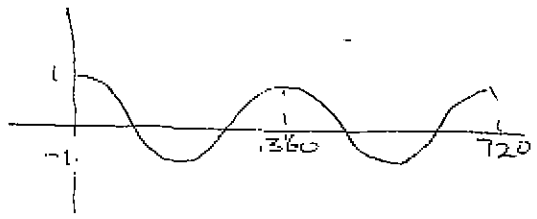
Section I Multiple-Choice Answer Sheet

1	A ○	B ○	C ●	D ○
2	A ●	B ○	C ○	D ○
3	A ○	B ○	C ○	D ●
4	A ○	B ●	C ○	D ○
5	A ○	B ○	C ●	D ○

2013 Prelim E1 Yearly AT3.

MC sols:

1.



sols $\cos\theta = -1$

(C) 2.

2. $\frac{1}{(n-1)!} + \frac{1}{n!} = \frac{n+1}{n!}$ (A).

3. $2x + y - 3 = 0$ $\quad x - 3y + 2 = 0$

$y = 3 - 2x$ $\quad y = \frac{1}{3}(x+2)$

$m_1 = -2$. $\quad m_2 = \frac{1}{3}$

$$\tan\theta = \left| \frac{-2 - \frac{1}{3}}{1 + (-2)(+\frac{1}{3})} \right|$$

$$= \left| \frac{-7}{1} \right|$$

$$= 7$$

$\theta = 81^\circ 52'$ (D)

4, EXERCISE $n = 8$ 3E

$\# = \frac{8!}{3!} = 6720$

(B)

5. (C)

Question 6

a) $\dfrac{2x}{6-x} \geq 1 \qquad x \neq 6$

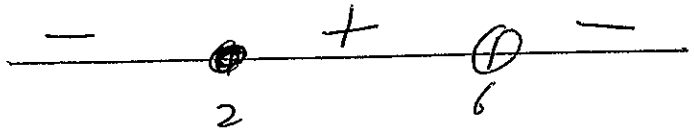
$\left(\dfrac{2x}{6-x}\right)(6-x)^2 \geq (1)(6-x)^2$

$2x(6-x) \geq (6-x)^2$

$2x(6-x) - (6-x)^2 \geq 0$

$(6-x)[2x-(6-x)] \geq 0$

$(6-x)(3x-6) \geq 0$



$\therefore$ Solution is $2 \leq x < 6$

✓ (for any reasonable correct progress)

$x \in \mathbb{R}$ ✓ Both ineq. correct!

b) $A(2,3) \quad B(5,-7) \quad m:n = 4:9$

$x = \dfrac{mx_2 + nx_1}{m+n} = \dfrac{(4)(5)+(9)(2)}{13} = \dfrac{38}{13}$

$y = \dfrac{my_2 + ny_1}{m+n} = \dfrac{(4)(-7)+(9)(3)}{4+9} = -\dfrac{1}{13}$

$\therefore P\left(\dfrac{38}{13}, -\dfrac{1}{13}\right)$

✓✓ 1 mark each x and y values

c) (i) Without restriction –

no. of ways = 6! or 720

✓ correct

(ii) With 2 brothers next to each other –

no. ways = 2! × 5! = 240

✓ 1 mark 2! × 5!

∴ 2 brothers not next to each other –

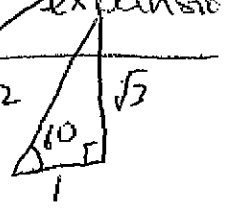
no. of ways = 720 − 240

= 480

✓ 1 correct

d) $\tan(60-45) = \dfrac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$

✓ correct expansion



$= \dfrac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}$

✓

$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

e) (i) $\sin 3x = \sin(2x + x)$

correct expansion ✓

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cdot \cos x + \sin x(1 - 2\sin^2 x)$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$= RHS$

* could use $\sin(4x - x)$.

e) $3\sin x - 4\sin^3 x = \frac{1}{2}$

$\sin 3x = \frac{1}{2}$

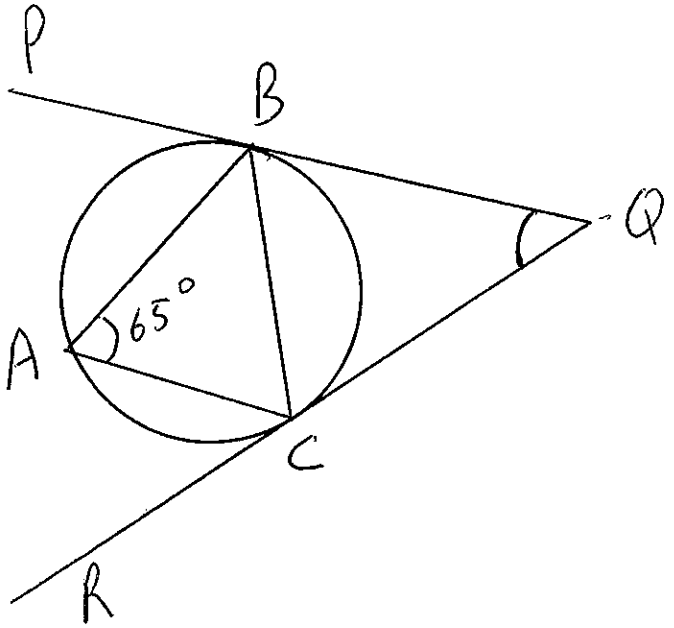
1 mark for ✓ 3x (change in domain)

1 mark ALL sol'ns ✓

$3x = 30°, 150°, 390°, 510°, 750°, 870°$

$\therefore x = 10°, 50°, 130°, 170°, 250°, 290°$

f)



✓ $\angle BCQ = \angle BAC = 65°$ (∠s in alternate segment)

✓ $BQ = QC$ (equal tangents from external point.

$\therefore \triangle BQC$ is isosceles

$\therefore \angle CBQ = \angle QCB = 65°$

$\therefore \angle BQC = 180 - 65 - 65$ (∠ sum of △)

$= 50°$

OR 2 marks any correct proof

Question 7

a) $y = \sqrt{x} = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$x = 4 \Rightarrow m_1 = \frac{1}{2(\sqrt{4})} = \frac{1}{4}$

✓ finding m_1, m_2.

$y = \frac{x^2}{4} - 2 = \frac{1}{4}x^2 - 2$

$\frac{dy}{dx} = \frac{1}{2}x$

$x = 4 \Rightarrow m_2 = \frac{1}{2}(4) = 2$ ✓

correct use of formula

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2 - \frac{1}{4}}{1 + (2)(\frac{1}{4})}\right| = \frac{7}{6}$

$\therefore \theta = \tan^{-1}(\frac{7}{6})$

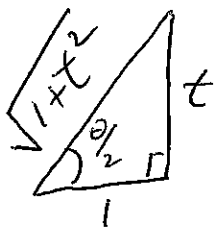
$= 49.3987^\circ$

$= 49.40^\circ$ or $49^\circ 24'$

✓ correct

Question 7

b) $\tan\left(\frac{\theta}{2}\right) = t = \frac{t}{1}$



$$\sin\left(\frac{\theta}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{1+t^2}}$$

✓ 1 mark both $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$.

$$\sin\theta = \sin 2\left(\frac{\theta}{2}\right) = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$= 2\left(\frac{t}{\sqrt{1+t^2}}\right)\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{2t}{1+t^2}$$ ✓ $\sin\theta$

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and $\cos\theta = \cos 2\left(\frac{\theta}{2}\right) = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$

$$= 1 - 2\left(\frac{t}{\sqrt{1+t^2}}\right)^2$$

$$= 1 - \frac{2t^2}{1+t^2}$$

$$= \frac{1+t^2-2t^2}{1+t^2}$$

$$= \frac{1-t^2}{1+t^2}$$ ✓ $\cos\theta$

Question 7

c) (i) $\sqrt{3}\cos x + \sin x = R\sin(x+\alpha)$

$R^2 = (\sqrt{3})^2 + 1^2 = 4$

$R = 2$

$\therefore \sqrt{3}\cos x + \sin x = 2\sin(x+\alpha)$

$\sqrt{3}\cos x + \sin x = 2\sin x\cos\alpha + 2\cos x\sin\alpha$

$\therefore \sqrt{3}\cos x = 2\cos x\sin\alpha$ ——— (1)

$\sin\alpha = \sqrt{3}/2$

and $\sin x = 2\sin x\cos\alpha$

$\cos\alpha = 1/2$ ——— (2)

From (1) and (2) α is in 1st quadrant

$\alpha = \cos^{-1}(\frac{1}{2}) = 60^\circ$ ✓

$\therefore \sqrt{3}\cos x + \sin x = 2\sin(x+60)$ ✓

1 mark
R=2
1 mark
$\alpha = 60^\circ$

(ii) $\sqrt{3}\cos x + \sin x = 2\sin(x+60) = \sqrt{3}$

$\sin(x+60) = \frac{\sqrt{3}}{2}$ ✓

$x+60 = 60^\circ, 120^\circ, \cancel{420^\circ}, \cancel{480^\circ}, \ldots$

$x+60 = 60^\circ + 2\pi n$
$= 120^\circ + 2\pi n$ $\Big\}\ n \in \mathbb{I}$

$x = 2\pi n, 60^\circ + 2\pi n$

$= 180n + (-1)^n 60$

$\therefore x = 180n + (-1)^n 60 - 60$ ✓

OR $x = 360n$ and

$x = 360n \pm 60$

Question 7

d) Show $\tan^2 x - \tan^2 y = \dfrac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$

$$RHS = \frac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$$

$$= \frac{\cos^2 y}{\cos^2 x \cos^2 y} - \frac{\cos^2 x}{\cos^2 x \cos^2 y}$$

✓ correct progress

$$= \frac{1}{\cos^2 x} - \frac{1}{\cos^2 y}$$

$$= \sec^2 x - \sec^2 y$$

$$= (\tan^2 x + 1) - (\tan^2 y + 1)$$

$$= \tan^2 x + 1 - \tan^2 y - 1$$

$$= \tan^2 x - \tan^2 y$$

$$= LHS$$

✓ obtaining result

e) $3^{2x} - 8 \cdot 3^x - 9 = 0$

Let $u = 3^x$

$\therefore u^2 - 8u - 9 = 0$

✓ correct quadratic

$(u-9)(u+1) = 0$

$u = 9$ or $u = -1$

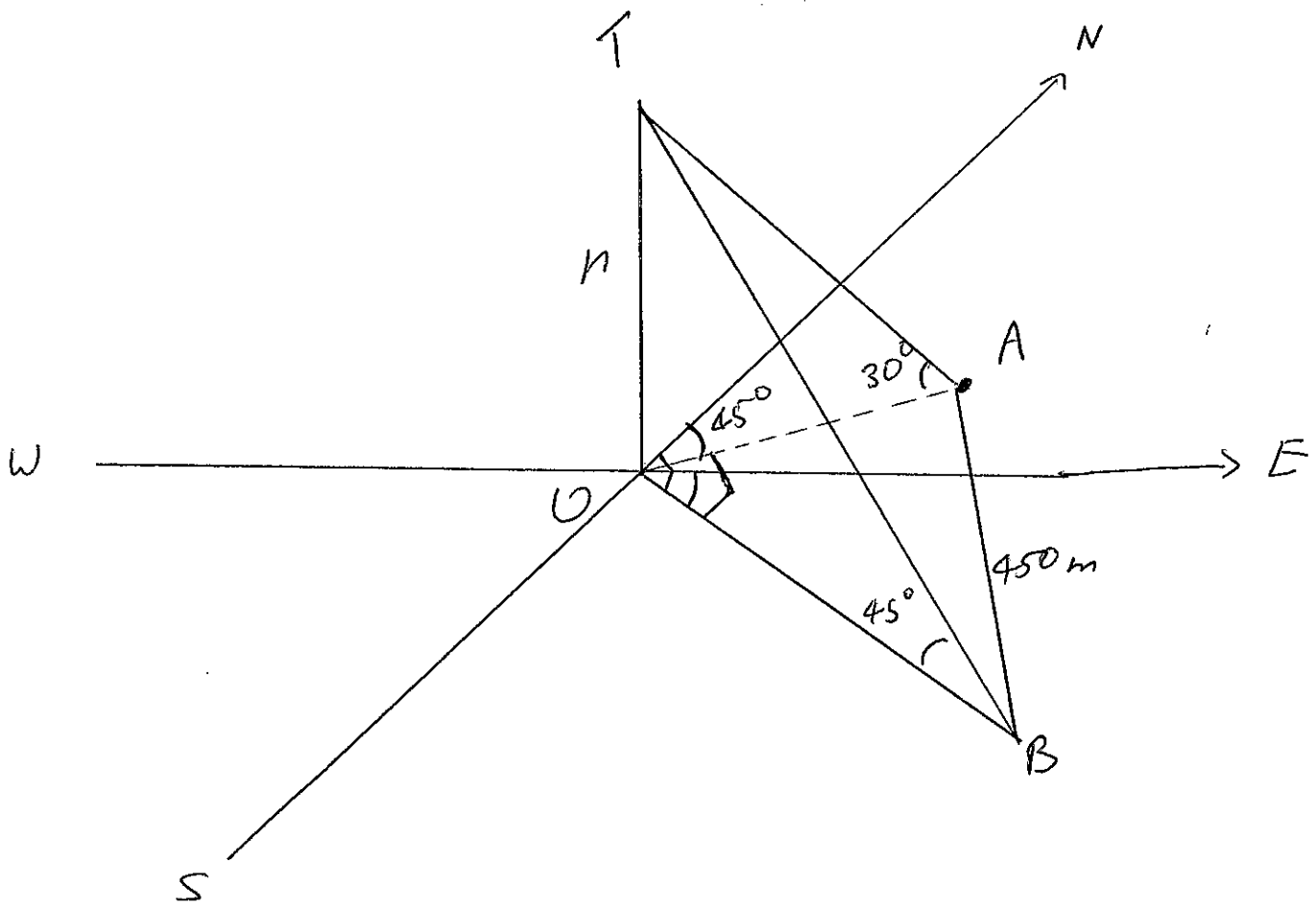
✓ quadratic sol'n's

ie; $3^x = 9$ | $3^x = -1$

$\therefore x = 2$ | x not defined

$\therefore x = 2$ is the solution. ✓ for $x=2$ ONLY!

Question 8



(i) $\tan 45 = \frac{h}{OB} \Rightarrow OB = \frac{h}{\tan 45}$

$\tan 30 = \frac{h}{OA} \Rightarrow OA = \frac{h}{\tan 30}$

✓ (for both correct)

(ii) $OA^2 + OB^2 = 450^2$

$\left(\frac{h}{\tan 45}\right)^2 + \left(\frac{h}{\tan 30}\right)^2 = 202500$

✓ correct use Pythag.

$h^2 + \frac{h^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 202500 \quad \left(\tan 30 = \frac{1}{\sqrt{3}}\right)$

✓ correct subs.

$h^2 + 3h^2 = 202500$

$4h^2 = 202500$

$h = 225$ m

✓ correct

Question 8

b) $f(x) = \dfrac{x}{x^2-1}$ $\quad x \neq \pm 1$

(i) $x = \pm 1$ $f(x)$ undefined

✓ 1 mark BOTH

(ii) $f(x) = \dfrac{x}{x^2-1}$ using $f'\left(\frac{u}{v}\right) = \dfrac{vu' - uv'}{v^2}$

$$f'(x) = \frac{(1)(x^2-1) - (x)(2x)}{(x^2-1)^2}$$

✓ correct use Quotient Rule

$$= \frac{x^2 - 1 - 2x^2}{(x^2-1)^2}$$

$$= \frac{-x^2 - 1}{(x^2-1)^2}$$

$$\therefore f'(x) = -\frac{(x^2+1)}{(x^2-1)^2}$$

Since $-(x^2+1)$ is < 0 for all x

and $(x^2-1)^2 > 0$ for all x

✓ correct explanation

$$\therefore f'(x) = \frac{-(x^2+1)}{(x^2-1)^2} < 0 \text{ for all } x$$

Question 8

(iii) $f(x) = \dfrac{x}{x^2-1}$

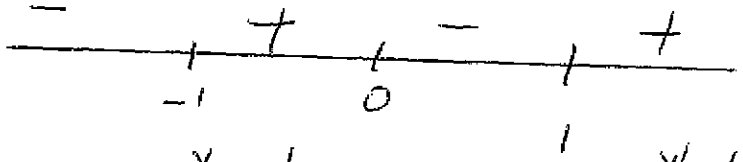
$x = 0 \Rightarrow y = 0$ $(0,0)$ is y-intercept

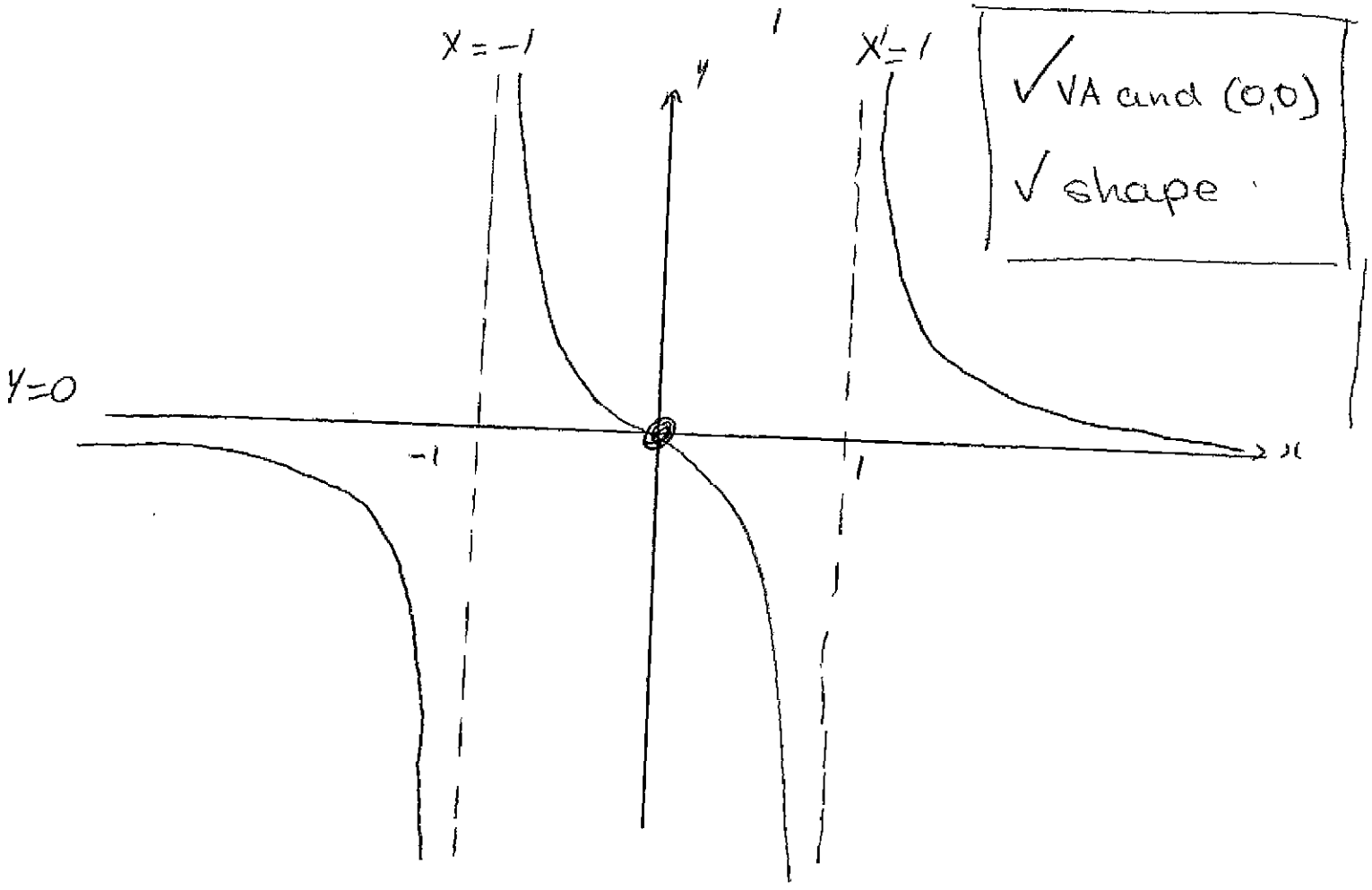
$x = 1$ and $x = -1$ are vertical asymptotes.

horizontal asymptote

$$y = \lim_{x \to \infty} \frac{x}{x^2-1} = 0$$

$y = 0$ is an asymptote





✓ VA and (0,0)

✓ shape

Question 8

(iv) $x = 2 \Rightarrow f(x) = \frac{2}{2^2 - 1} = \frac{2}{3}$

Tangent at $(2, \frac{2}{3})$

$x = 2 \Rightarrow m = \frac{dy}{dx} = \frac{-(2^2+1)}{(2^2-1)^2} = \frac{-5}{9}$

$\therefore$ equation of tangent:

$y - \frac{2}{3} = -\frac{5}{9}(x - 2)$

$9y - 6 = -5x + 10$

$5x + 9y - 16 = 0$

✓ correct use of formula.

✓ must be in general form.

(c) Solving $x^2 + y^2 - 2x - 14y + 25 = 0$ and $y = mx$ simultaneously:

$x^2 + (mx)^2 - 2x - 14mx + 25 = 0$

$(1+m^2)x^2 - x(2 + 14m) + 25 = 0$

✓ correct sub for pt of intersection.

For distinct roots $b^2 - 4ac > 0$

$(2+14m)^2 - (4)(25)(1+m^2) > 0$

$4(1+7m)^2 - 4(25)(1+m^2) > 0$

÷ by 4 both sides:

$\therefore (1+7m)^2 - 25(1+m^2) > 0$

✓ correct expression

Question 8

(c) (ii) For $y = mx$ to be a tangent

$$b^2 - 4ac = 0$$

$$\therefore (1 + 7m)^2 - 25(1 + m^2) = 0$$

$$1 + 14m + 49m^2 - 25 - 25m^2 = 0$$

for setting $\Delta = 0$

$$24m^2 + 14m - 24 = 0$$

$$12m^2 + 7m - 12 = 0$$

$$12m^2 + 16m - 9m - 12 = 0$$

$$4m(3m + 4) - 3(3m + 4) = 0$$

$$(3m + 4)(4m - 3) = 0$$

$$\therefore m = -\frac{4}{3} \text{ or } m = \frac{3}{4}$$

✓ Both correct values.