International Grammar School

Year 11
Mathematics Extension 1
Assessment Task 3
2014

General Instructions

- Reading time – 5 minutes
- Working time – $1\frac{1}{2}$ hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 - 13, show relevant mathematical reasoning and/or calculations

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 10 minutes for this section.

Section II

45 marks

- Attempt Questions 11-13
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 80 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

10 marks
Attempt Questions 1–10

Use the multiple-choice answer sheet for Questions1–10.

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1 Which of the following is the equivalent to $\cos(a - b)$

(A) $\sin a \cos b - \sin b \cos a$

(B) $\sin a \cos b + \sin b \cos a$

(C) $\cos a \cos b - \sin a \sin b$

(D) $\cos a \cos b + \sin a \sin b$

2 How many arrangements of the letters of the word PROBABILITY are possible?

(A) 362 880

(B) 9 979 200

(C) 19 958 400

(D) 39 916 800

3 If θ is an acute angle where $\tan\theta = \frac{a}{b}$, find exact value of $\sin 2\theta$.

(A) $\frac{ab}{a^2 + b^2}$

(B) $\frac{2ab}{a^2 + b^2}$

(C) ab

(D) $2ab$

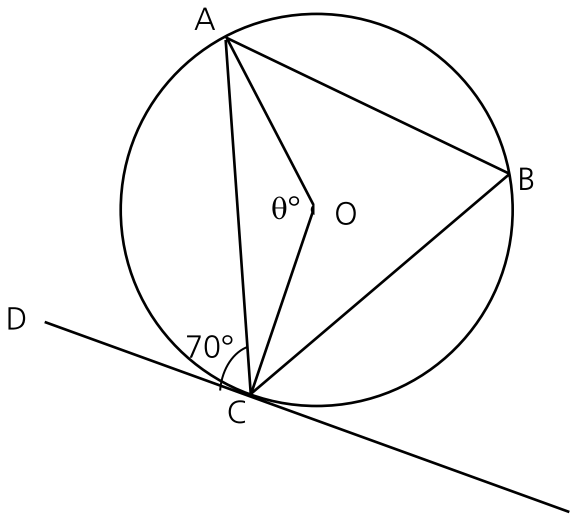
4 The diagram shows a circle with centre O. A, B, C are points on the circle. The line DC is a tangent to the circle at C and $\angle ACD = 70°$. Find $\angle\theta^{\circ}$.

(A) 70°

(B) 35°

(C) 140°

(D) 110°



5 The point $P(x, y)$ divides the interval joining the points A(3, –7) and B(–17, 3) in the ratio $2:3$. Find the coordinates of P.

(A) (-5, -3)

(B) (-9, -1)

(C) (-25, -15)

(D) (-45, -5)

6 Which of the following represents the gradient function of $\dfrac{1}{\sqrt{(2x-3)^3}}$?

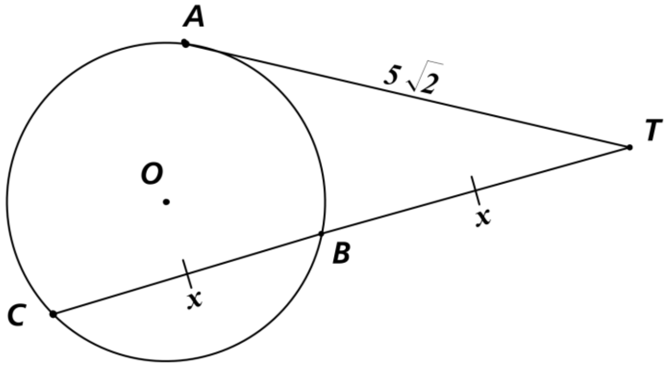
(A) $\dfrac{-3}{\sqrt[5]{(2x-3)^2}}$

(B) $\dfrac{-3}{\sqrt{(3-2x)^5}}$

(C) $\dfrac{3}{\sqrt[5]{(3-2x)^2}}$

(D) $\dfrac{-3}{\sqrt{(2x-3)^5}}$

7 AT is a tangent to a circle centre O. CT cuts the circle at B. If BC= BT = x and $AT = 5\sqrt{2}$, which of the following is the value of x ?



(A) $5\sqrt{2}$

(B) $4\sqrt{2}$

(C) 5

(D) $\sqrt{\dfrac{5\sqrt{2}}{2}}$

8 Evaluate a and b given that $a + b\sqrt{5} = \dfrac{1}{3+\sqrt{5}}$

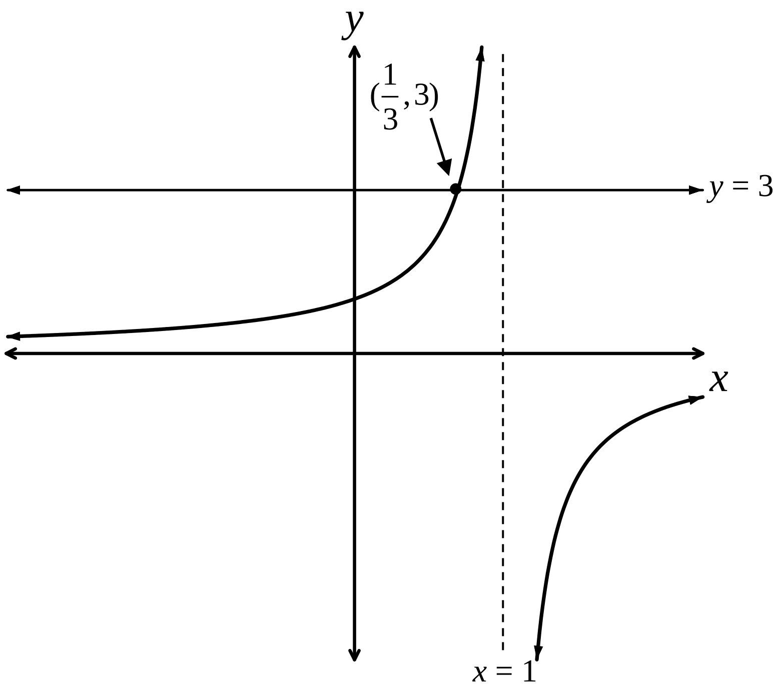
(A) $a = \dfrac{3}{4},\ b = \dfrac{1}{4}$

(B) $a = \dfrac{3}{4},\ b = -\dfrac{1}{4}$

(C) $a = \dfrac{3}{4}, b = \dfrac{\sqrt{5}}{4}$

(D) $a = \dfrac{3}{4}, b = -\dfrac{\sqrt{5}}{4}$

9 The graphs of $y = \dfrac{2}{1-x}$ and $y = 3$ are shown on the number plane below.



The solution to $\dfrac{2}{1-x} \geq 3$ is:

(A) $\dfrac{1}{3} \leq x \leq 1$

(B) $\dfrac{1}{3} \leq x < 1$

(C) $x \leq \dfrac{1}{3}$

(D) $x \geq \dfrac{1}{3}$

10 How many solutions are there to the equation $(2\sin x-1)(\sin x+1)=0$, for $0\leq x\leq 180^{\circ}$.

(A) 0

(B) 1

(C) 2

(D) 3

Section II

45 marks
Attempt Questions 11–13

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and / or calculations.

Question 11 (15 marks) **Marks**

(a) Solve the inequality $\frac{1}{x-3} \le 4$. 3

(b) Sketch the lines with equations $3x+4y=12$ and $2x-4y+3=0$. Hence or otherwise find the acute angle between the lines, leaving your answer to the nearest degree. 3

(c)

(i) Rationalise $\frac{1}{1+\sqrt{2}}$ 1

(ii) Hence, show that $1+\cfrac{1}{2+\cfrac{1}{1+\sqrt{2}}}=\sqrt{2}$ 2

(d) Solve $4\sin^2 2\theta = 3$ for $0^\circ \le \theta \le 360^\circ$. 3

(e) 4 couples go to the movie theatre.

(i) In how many ways can the 4 couples sit together if each couple sit together? 1

(ii) Patrick and Lara have an argument outside and refuse to sit next to each other. In how many ways can the group sit if Patrick and Lara are separated? 2

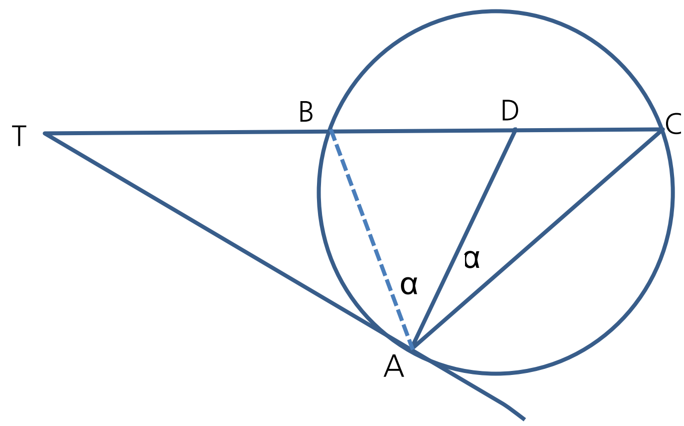
Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate the following:

(i) $y=\dfrac{x^2+1}{\sqrt{x}}$ 2

(ii) $f(x)=(3x+1)(x+2)^3$ 2

(b) In the diagram TA is a tangent to the circle at A. $\angle BAD=\angle DAC=\alpha$



Copy the diagram into your writing booklet.

(i) Explain why $\angle BAT=\angle ACD$ 1

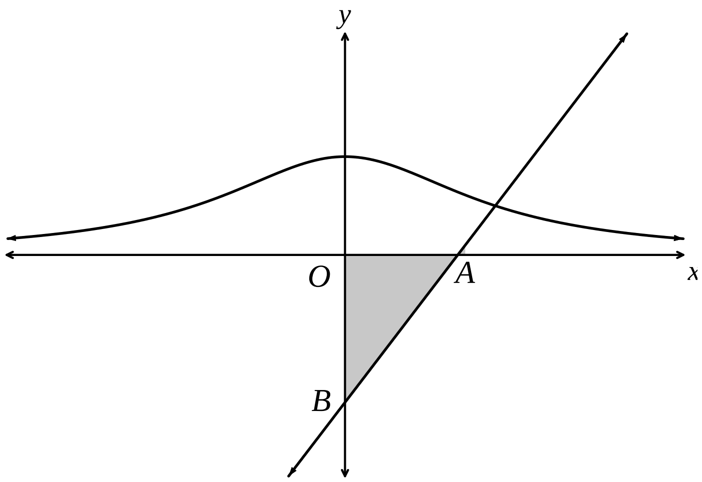
(ii) Hence, show that $\mathrm{TA}=\mathrm{TD}$. 2

(c) Prove that $1+\dfrac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$. 3

(d)

(i) Show that the equation of the normal to the curve of $y=\dfrac{1}{1+x^2}$ at $x=1$ is $4x-2y-3=0$ 3

(ii) The normal intersects the x and y axis at A and B respectively.



Find the area of ΔAOB. 2

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $\sin\theta+\sqrt{3}\cos\theta$ can be written in the form $r\sin(\theta+\alpha)$ where $r>0$ and $0<\alpha<90^{\circ}$. 3

(ii) Hence or otherwise, solve the equation $\sin\theta+\sqrt{3}\cos\theta=\sqrt{2}$ in the domain $0°\le\theta\le 360°$. 2

(b) Given a function $f(x)=\dfrac{x^2+1}{x^2-1}$,

i) Show that $f(x)=1+\dfrac{2}{x^2-1}$ 1

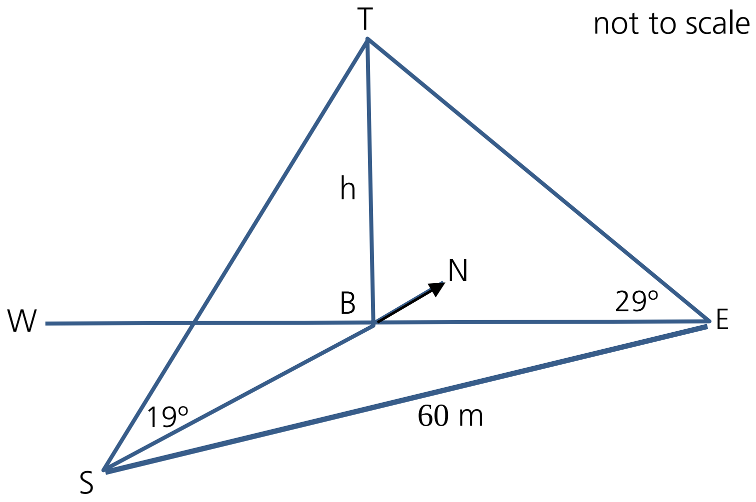
ii) State the domain of $f(x)$ 1

iii) Describe the behaviour of graph of $f(x)$ as x approaches $\pm\infty$. 1

iv) Show that $f(x)$ is an even function 1

v) Sketch the graph of $y=f(x)$. Label clearly the asymptotes and intercepts. 2

(c) From a point S due south of the base B of a building, the angle of elevation to the top of the building is 19° and from a point E, due east of the building, it is 29°. The distance from S to E is 60 metres.



i) Show that $BS = h\tan 71^\circ$ and $BE = h\tan 61^\circ$ 1

ii) Show that $h^2 = \dfrac{3600}{\tan^2 71 + \tan^2 61}$ 2

iii) Find h to the nearest metre. 1

END OF EXAMINATION

Student Name: . . . .

2014 Year 11 Yearly Examination

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

SOLUTIONS TO AT3 2014

Section I

1. D
2. B
3. B
4. C
5. A
6. D
7. C
8. B
9. B
10. C

Question 11

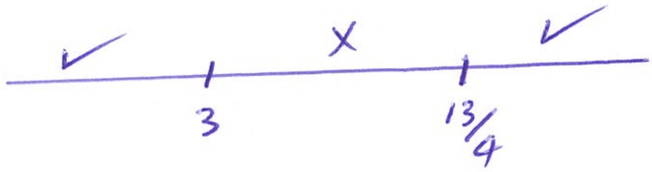
a) $\dfrac{1}{x-3} \leq 4 \qquad x \neq 3$

$x-3 \leq 4(x-3)^2 \qquad \times$ bs by $(x-3)^2$

$(x-3) - 4(x-3)^2 \leq 0$

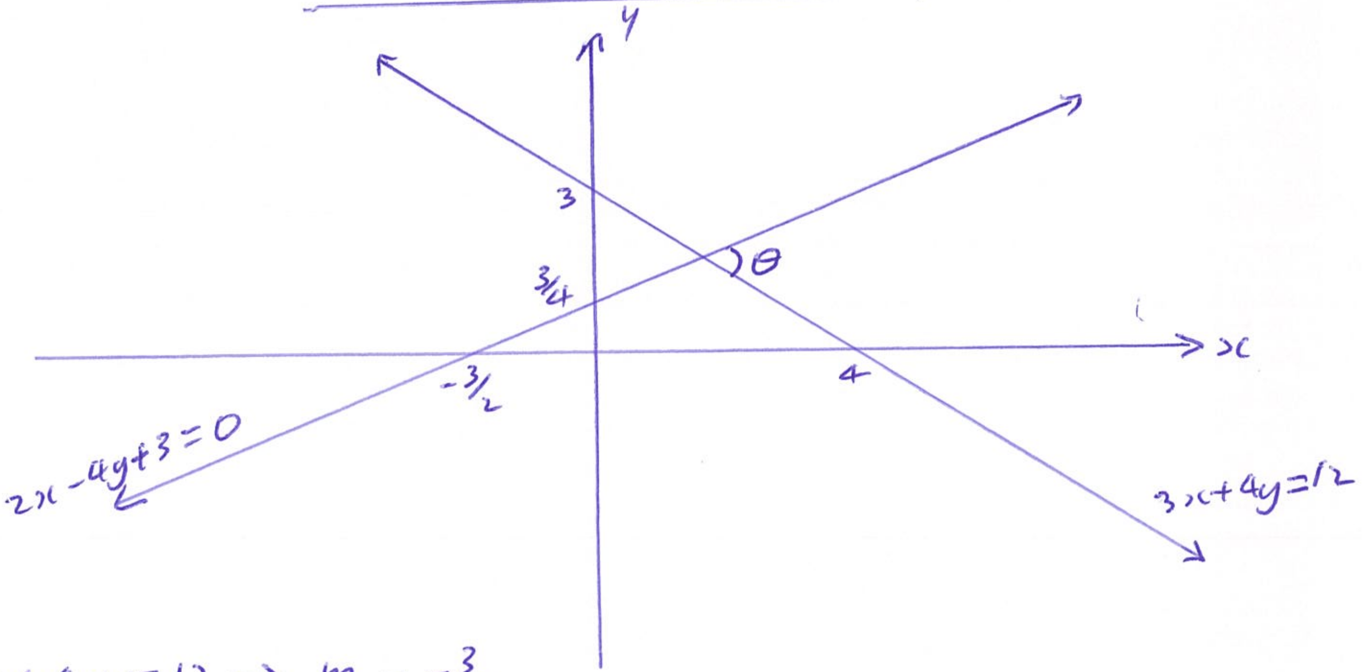
$(x-3)(1-4x+12) \leq 0$

$(x-3)(13-4x) \leq 0$



$\therefore x < 3 \quad , \quad x \geq \dfrac{13}{4}$

b)



$3x+4y=12 \Rightarrow m_1 = -\frac{3}{4}$

$2x-4y+3=0 \Rightarrow m_2 = \frac{1}{2}$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1+m_1 m_2}\right| = \left|\dfrac{-\frac{3}{4}-\frac{1}{2}}{1+(-\frac{3}{4})(\frac{1}{2})}\right| = 2$

$\therefore \theta = 63^\circ$ (to nearest degree) c)

(i) $\dfrac{1}{1+\sqrt{2}} = \dfrac{1}{1+\sqrt{2}} \times \dfrac{1-\sqrt{2}}{1-\sqrt{2}}$

$= \dfrac{1-\sqrt{2}}{1-2}$

$= -(1-\sqrt{2})$

$= \underline{\sqrt{2}-1}$

(ii) $LHS = 1 + \dfrac{1}{2+\dfrac{1}{1+\sqrt{2}}}$

$= 1 + \dfrac{1}{2+\sqrt{2}-1}$ using (i) result.

$= 1 + \dfrac{1}{1+\sqrt{2}}$

$= 1 + \sqrt{2} - 1$ using (i) result again

$= \sqrt{2}$

$= RHS.$

d) $4\sin^2 2\theta = 3 \quad 0 \leq \theta \leq 360°$

$\sin^2 2\theta = \frac{3}{4}$

$\sin 2\theta = \pm \frac{\sqrt{3}}{2}$

$2\theta = 60°, 120°, 240°, 300°, 420°, 480°, 600°, 660°$

$\therefore \theta = 30°, 60°, 120°, 150°, 210°, \cancel{480°}\ 240°, 300°, 330°$

(e) (i) No. of ways $= {}^4P_2 \times 2^4$

$= \underline{384}$

(ii) If Patrick and Lara were not to sit together then:

No. of ways $= {}^8P_8 - ({}^7P_7 \times 2)$

number of ways 8 can sit in a row.

number of ways Patrick and Lara can sit together

$= \underline{30240}$

Question 12

a) $y = \dfrac{x^2+1}{\sqrt{x}} \Rightarrow \begin{array}{l} u = x^2+1 \Rightarrow u' = 2x \\ v = \sqrt{x} \Rightarrow v' = \dfrac{1}{2\sqrt{x}} \end{array}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(\sqrt{x})(2x) - (x^2+1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$$

$$= \frac{2x\sqrt{x} - \frac{x^2+1}{2\sqrt{x}}}{x} = \frac{4x^2 - x^2 + 1}{2x\sqrt{x}}$$

$$= \frac{3x^2+1}{2x\sqrt{x}}$$

(ii) $f(x) = (3x+1)(x+2)^3$

Let $u = 3x+1 \Rightarrow u' = 3$

$v = (x+2)^3 \Rightarrow v' = 3(x+2)^2$

$$\therefore f'(x) = uv' + vu'$$

$$= (3x+1)3(x+2)^2 + (x+2)^3(3)$$

$$= 3(x+2)^2(4x+3)$$

b) Refer to diagram in question paper

(i) $\angle BAT = \angle ACD$

$\because$ Angle between tangent and chord equals angle in alternate segment.

(ii) Let $\angle BAT = \angle ACD = \theta$

$\therefore \angle TDA = \alpha + \theta$ (exterior $\angle$ of $\triangle ACD$)

$\angle TAD = \angle TAB + \angle BAD$

$= \theta + \alpha$

$\therefore \angle TDA = \angle TAD \Rightarrow \triangle TAD$ is isosceles base $\angle$s equal

$\therefore TA = TD$

(isosceles $\triangle$)

c) Prove $1 + \dfrac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

$$LHS = \frac{1 + \operatorname{cosec}\theta + \cot^2\theta}{1+\operatorname{cosec}\theta}$$

$$= \frac{\operatorname{cosec}^2 + \operatorname{cosec}\theta}{1+\operatorname{cosec}\theta} \quad (1+\cot^2\theta = \operatorname{cosec}^2\theta)$$

$$= \frac{\operatorname{cosec}\theta\,\cancel{(1+\operatorname{cosec}\theta)}}{\cancel{(1+\operatorname{cosec}\theta)}}$$

$$= \operatorname{cosec}\theta$$

$$= RHS$$

d) (i) $y = \frac{1}{1+x^2}$

$y = (1+x^2)^{-1}$

$\therefore m_T = \frac{dy}{dx} = (-1)(2x)(1+x^2)^{-2}$

$= \frac{-2x}{(1+x^2)^2}$

$x = 1 \Rightarrow m_T = \frac{-2}{4}$

$m_T = -\frac{1}{2}$

$\therefore \underline{m_n = 2}$ also $x=1 \Rightarrow y = \frac{1}{1+1^2} = \frac{1}{2}$

$\therefore$ equation of normal : $m_n = 2$ $(1, \frac{1}{2})$

$y - \frac{1}{2} = 2(x-1)$

$y - \frac{1}{2} = 2x - 2$

$2y - 1 = 4x - 4$

$\underline{4x - 2y - 3 = 0}$ as required.

(ii) $4x - 2y - 3 = 0$

x-intercept : $y = 0 \Rightarrow x = \frac{3}{4}$ $\therefore A(\frac{3}{4}, 0)$

y-intercept : $x = 0 \Rightarrow y = -\frac{3}{2}$ $\therefore B(0, -\frac{3}{2})$

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times \frac{3}{4} \times \frac{3}{2}$

$= \underline{\frac{9}{16} u^2}$

Question 13

a)(i) $\sin\theta + \sqrt{3}\cos\theta = r\sin(\theta+\alpha)$

$= r\sin\theta\cos\alpha + r\cos\theta\sin\alpha$

$\therefore r\cancel{\sin\theta}\cos\alpha = \cancel{\sin\theta}$

$r\cos\alpha = 1$ ——— (1)

and $r\cos\theta\sin\alpha = \sqrt{3}\cos\theta$

$r\sin\alpha = \sqrt{3}$ ——— (2)

$(1)^2 + (2)^2$ gives $r^2 = 4$

$r = 2$

and $(2) \div (1)$ gives

$\tan\alpha = \sqrt{3}$ (α is in 1st quadrant as $\sin\alpha$ and $\cos\alpha$ are positive)

$\therefore \alpha = 60°$

$\therefore \sin\theta + \sqrt{3}\cos\theta = 2\sin(\theta + 60°)$

(ii) $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$

$\therefore 2\sin(\theta+60) = \sqrt{2}$

$\sin(\theta+60) = \frac{\sqrt{2}}{2}$

$\theta + 60 = 45°, 135°, 405°$

$\therefore \theta = -15°, 75°, 345°$

$\therefore \theta = 75°, 345°$ $(0 \leq \theta \leq 360°)$

b) $f(x) = \dfrac{x^2+1}{x^2-1}$

(i) $1 + \dfrac{2}{x^2-1} = \dfrac{x^2-1+2}{x^2-1} = \dfrac{x^2+1}{x^2-1} = f(x)$

It can also be shown by long division.

(ii) domain of $f(x)$: $x \in \mathbb{R}$, $x \neq \pm 1$

(iii) $f(x) = \dfrac{x^2+1}{x^2-1} = 1 + \dfrac{2}{x^2-1}$

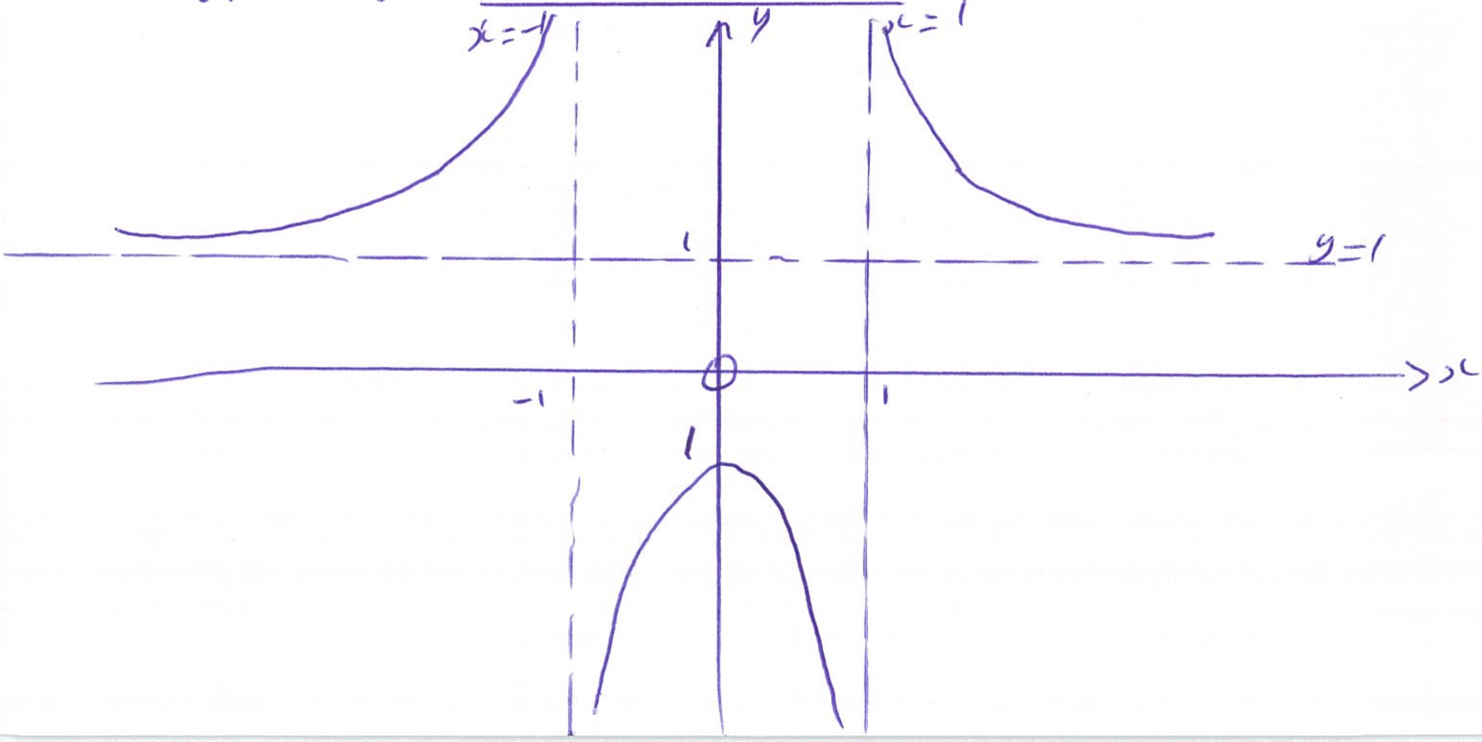
As $x \to \pm\infty$, $\dfrac{2}{x^2-1} \to 0$ $\therefore f(x) \to 1$

$\therefore f(x) \to 1$ as $x \to \pm\infty$

(iv) $f(x) = \dfrac{x^2+1}{x^2-1}$

$f(-x) = \dfrac{(-x)^2+1}{(-x)^2-1} = \dfrac{x^2+1}{x^2-1} = f(x)$

Since $f(x) = f(-x)$, $f(x)$ is even function.



(c) Refer to diagram on question paper

(i) $\angle STB = 180 - 90 - 19$ ($\angle$ Sum of Δ)

$= 79°$

$\therefore \text{Tan } 79 = \dfrac{BS}{h}$

$BS = h \text{ Tan } 79$

and $\angle ETB = 180 - 90 - 29$ ($\angle$ Sum of Δ)

$= 61°$

$\therefore \text{Tan } 61 = \dfrac{BE}{h}$

$\underline{BE = h \text{ Tan } 61}$

(ii) In ΔBSE $SE^2 = BS^2 + BE^2$

$\therefore 60^2 = h^2 \tan^2 79 + h^2 \text{Tan}^2 61$

$3600 = h^2(\tan^2 79 + \tan 61)$

$\therefore h^2 = \dfrac{3600}{\text{Tan}^2 79 + \text{Tan}^2 61}$ as required.

(iii) $h^2 = \dfrac{3600}{\text{Tan}^2 79 + \text{Tan}^2 61}$

$= 121.126377\ldots$

$\therefore h = 11.0057$

$\underline{h = 11 \text{ m}}$ (to nearest m).