International Grammar School

Year 11
Mathematics Extension 1
Assessment Task 3
2015

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes + 5 minutes reading
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 - 13, show relevant mathematical reasoning and/or calculations

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

45 marks

- Attempt Questions 11-13
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 75 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the acute angle between the lines $y = 2x - 3$ and $3x + 5y - 1 = 0$, correct to the nearest degree?

(A) $32°$

(B) $50°$

(C) $82°$

(D) $86°$

2 What are the coordinates of the point that divides the interval joining $(-7,5)$ and $(-1,-7)$ internally in the ratio 1:3?

(A) $(-10,8)$

(B) $(-10,11)$

(C) $(2,8)$

(D) $(-5.5,2)$

3 Five children have a race. In how many ways can the first three positions be filled if Joanna always comes first?

(A) 120

(B) 20

(C) 12

(D) 10

4 How many ten letter words can be formed from the letters in WARRAGAMBA if each letter can be used only once.

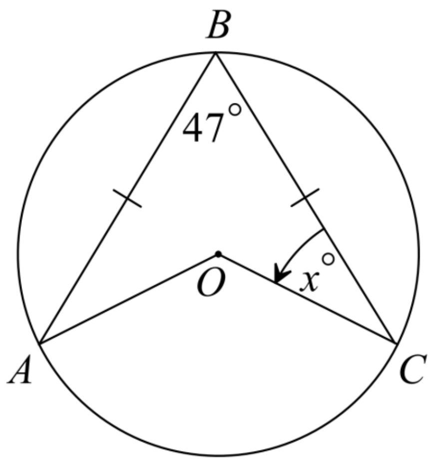
(A) 3628800

(B) 907200

(C) 75600

(D) 15120

5 What is the value of x in the diagram below?



(A) 21.5°

(B) 23.5°

(C) 43°

(D) 66.5°

6 What is the solution to the inequality $\frac{3}{x-2} \le 4$?

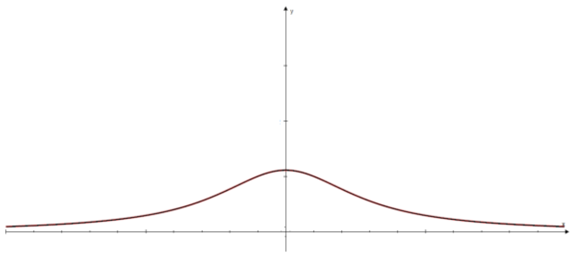
(A) $x < -2$ and $x \ge -\frac{11}{4}$

(B) $x > -2$ and $x \le -\frac{11}{4}$

(C) $x < 2$ and $x \ge \frac{11}{4}$

(D) $x > 2$ and $x \le \frac{11}{4}$

7 Which one of the following equations is represented by the curve shown below?



(A) $y = \frac{1}{x^2 + 9}$

(B) $y = \frac{(x+3)}{(x+3)(x-3)}$

(C) $y = \frac{1}{x^2 - 9}$

(D) $y = \frac{1}{\left(x^2 - 9\right)^2}$

8 There are 38 girls and 47 boys in a sporting club. In how many ways can a committee of 3 girls and 4 boys be chosen from this club?

(A) ${}^{38}\mathrm{P}_3 \, {}^{47}\mathrm{P}_4$

(B) ${}^{38}\mathrm{P}_3 + {}^{47}\mathrm{P}_4$

(C) ${}^{38}\mathrm{C}_3 \, {}^{47}\mathrm{C}_4$

(D) ${}^{38}\mathrm{C}_3 + {}^{47}\mathrm{C}_4$

9 ${}^{n}C_{k}$ is represented by which one of the following?

(A) ${}^{n}P_{k}\,n!$

(B) $\dfrac{{}^{n}P_{k}}{n!}$

(C) ${}^{n}P_{k}\,k!$

(D) $\dfrac{{}^{n}P_{k}}{k!}$

10 How many four digit numbers greater than 3000 can be formed with the digits 2, 3, 4, 5 and 6 if no digit is used more than once in a number?

(A) 96

(B) 120

(C) 196

(D) 216

End of Section I

Section II

45 marks
Attempt Questions 11–13
Allow about 75 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) (i) Show that $\frac{1+\cos 2A}{\sin 2A} = \cot A$. **3**

(ii) Hence, or otherwise, find the exact value of $\cot 22\frac{1}{2}^{0}$. **2**

(b) (i) Express $\cos\theta + \sin\theta$ in the form $R\sin(\theta+\alpha)$ where $R>0$ and α is acute. **2**

(ii) Hence, or otherwise, solve $\cos\theta + \sin\theta - 1 = 0$ where $0^{\circ} \le \theta \le 180^{\circ}$ **3**

(c) α and β are acute angles such that $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{4}{5}$.

(i) Express $\cos\alpha$ and $\cos\beta$ as fractions. **1**

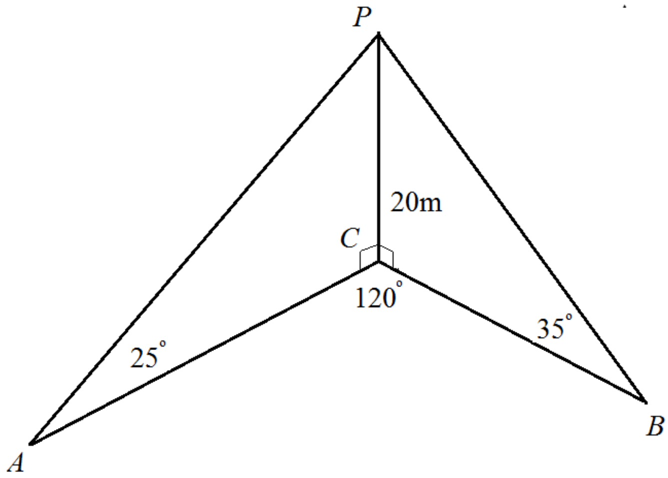
(ii) Hence, evaluate $\sin(\alpha+\beta)$ **1**

(d) What is the exact value of $\tan 75^{\circ}$? **3**
Express your answer with a rational denominator.

End of Question 11

Question 12 (15 marks) Start a NEW writing booklet **Marks**

(a) The angles of elevation to P, the top of a tower PC, are $25°$ and $35°$ from A and B respectively. A and B subtend an angle of $120°$ at C the foot of PC. The height of the tower, PC, is 20m



(i) Find AC and CB in terms of $\tan 65°$ and $\tan 55°$ respectively. **2**

(ii) Calculate the distance AB correct to the nearest metre. **3**

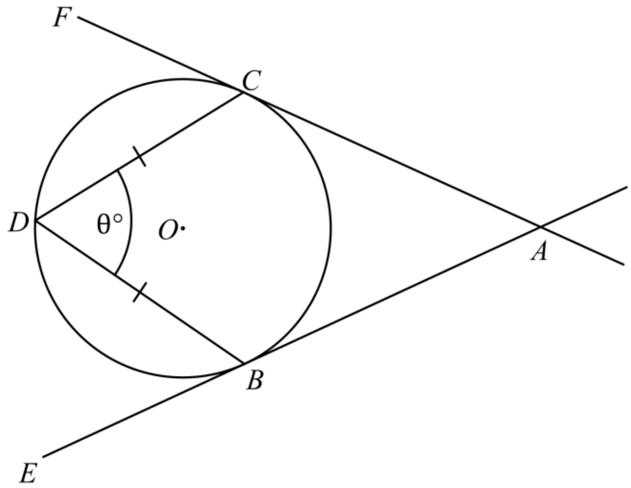
(b) By using the substitution $t = \tan\frac{\theta}{2}$, or otherwise, show that $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$. **2**

(c) Show that $\frac{dy}{dx} = \frac{x\left(3x^2-5\right)}{\sqrt{\left(3x^2-1\right)^3}}$ when $y = \frac{x^2+1}{\sqrt{3x^2-1}}$. **2**

Question 12 continues over the page.

Question 12 (continued) **Marks**

(d) Two equal chords, CD and BD are drawn in a circle, centre O, such that they subtend an angle of $\theta°$. Tangents are drawn from C and B such that they intercept at A. AB is produced to E and AC is produced to F. This is shown in the diagram below.



Copy or trace the diagram into your writing booklet.

(i) Prove that $\angle DBE = 90° - \dfrac{\theta}{2}$. **2**

(ii) Given that ΔABC is equilateral, prove $\angle COB = 120°$. **2**

(iii) Given that AC = 20 cm, show that the radius of the circle is $\dfrac{20\sqrt{3}}{3}$ cm. **2**

End of Question 12

Question 13 (15 marks) Start a NEW writing booklet **Marks**

(a) The functions $y = x^2$ and $y = (x-2)^2$ intersect at the point (1, 1). Find the acute angle between these two curves at their point of intersection to the nearest degree. **3**

(b) Consider the equation $y = \dfrac{x}{x^2 - 4}$.

(i) Find the domain of this function. **1**

(ii) Find any horizontal asymptotes. **1**

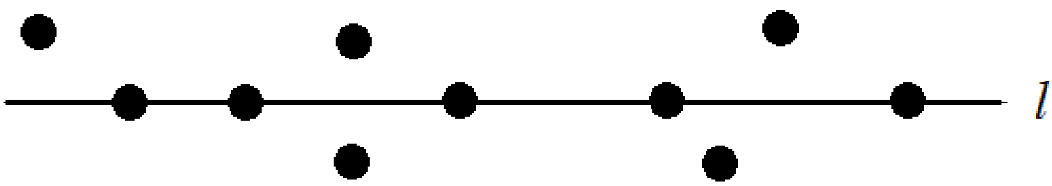
(iii) Find any x or y intercepts. **1**

(iv) Sketch this function showing the information above. **3**

(c) In how many ways can eight people be seated around a table if three people must be seated together? **2**

(d) Simplify $\dfrac{N!}{(N-1)!} \times \dfrac{(N-1)!}{(N+1)!}$ **1**

(e) The figure above shows 10 points lying in a plane, five of which are on the line l.



(i) How many sets of 3 points can be chosen from the 5 points lying on l? **1**

(ii) How many different triangles can be formed using the 10 points as vertices? **2**

End of Assessment Task

2015 Year 11 Assessment Task 3 **Student Name: . . . .**

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

2015 **Year 11 Assessment Task 3** **Student Name:** Solutions....

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

1	A ○	B ○	C ○	D ●
2	A ○	B ○	C ○	D ●
3	A ○	B ○	C ●	D ○
4	A ○	B ○	C ●	D ○
5	A ○	B ●	C ○	D ○
6	A ○	B ○	C ●	D ○
7	A ●	B ○	C ○	D ○
8	A ○	B ○	C ●	D ○
9	A ○	B ○	C ○	D ●
10	A ●	B ○	C ○	D ○

1. $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ $\qquad m_1 = 2$, $m_2 = -\frac{3}{5}$

$= \left|\dfrac{2 + \frac{3}{5}}{1 + 2 \times -\frac{3}{5}}\right|$

$\theta = 85.6^\circ$

D.

2. $\left(\dfrac{n \times x_2 + m \times x_1}{m+n}, \dfrac{n \times y_2 + n y_1}{m+n}\right)$

$= \left(\dfrac{3 \times -7 + -1}{4}, \dfrac{3 \times 5 + -7}{4}\right)$

$= (-5.5, 2)$

D.

3. $\underline{1} \times \underline{4} \times \underline{3} = 12$

C.

4. $\dfrac{10!}{4!3!} = 75600$

C

5. $x = \frac{47}{2}$

$x = 23.5°$

B

6. $\frac{3}{x-2} \leqslant 4$

$3(x-2) \leqslant 4(x-2)^2$

$3x - 6 \leqslant 4x^2 - 16x + 16$

$4x^2 - 19x + 22 \geqslant 0$

$(4x-11)(x-2) \geqslant 0$

$x < 2 \quad x \geqslant \frac{11}{4}$

C.

7. A

8. C.

9. D.

10. $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 120$

B.

11

(a)(i) $\dfrac{1+\cos 2A}{\sin 2A}$

$= \dfrac{1+\cos^2 A - \sin^2 A}{2\sin A\cos A}$ ① sin double angle ① cos double angle

$= \dfrac{\cos A}{\sin A}$ ① simplification

$= \cot A$

(ii) $\cos 22.5^\circ = \dfrac{1+\cos 45^\circ}{\sin 45}$

$= \dfrac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$ ① Correct substitution

$= \dfrac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2}+1$ ① Correct answer.

(b) (i) $r = \sqrt{1^2+1^2}$ $\tan\alpha = 1$

$r = \sqrt{2}$ $\alpha = 45^\circ$

$\cos\theta + \sin\theta = \sqrt{2}\sin(\theta + 45^\circ)$

(ii) $\cos\theta + \sin\theta - 1 = 0$

$\sqrt{2}\sin(\theta + 45°) = 1$ ① setting up equation

$\sin(\theta + 45) = \frac{1}{\sqrt{2}}$

$\theta + 45° = 45$ $\theta + 45 = 90°$ ① one solution or

$\theta = 0$ $\theta = 45$ ② both solutions

(c) (i) $\cos\alpha = \frac{15}{17}$ $\cos\beta = \frac{3}{5}$ ① both

(ii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \frac{8}{17} \times \frac{3}{5} + \frac{15}{17} \times \frac{4}{5}$

$= \frac{84}{85}$ ① both

(d) $\tan 75° = \tan(30° + 45°)$

$= \frac{\tan 30 + \tan 45}{1 - \tan 30 \times \tan 45}$ ① correct expression

$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$ ① correct evaluation

$= \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{2}$ $= 2 + \sqrt{3}$ ① correct answer

12

a) (i) $\tan 65^\circ = \frac{AC}{PC}$

$AC = 20 \tan 65^\circ$ ✓

$CD = 20 \tan 55^\circ$ ✓

(ii) $AB^2 = AC^2 + CD^2 - 2 \times AC \times CB \cos \angle ACB$ ✓

① Correct formula

$= (20 \tan 65)^2 + (20 \tan 55)^2 - 2 \times (20 \tan 65)(20 \tan 55) \cos 120^\circ$ ✓

① Correct evaluation

$= 3880.475978$

$= 62\,m$ (nearest metre) ✓ ① Correct answer.

(b) $\sin\theta = \dfrac{2t}{1+t^2}$ $\quad \cos\theta = \dfrac{1-t^2}{1+t^2}$ ✓ ① Correct t-formulas or similarly significant progress,

$$\frac{1-\cos\theta}{\sin\theta} = \left(1 - \frac{1-t^2}{1+t^2}\right) \div \frac{2t}{1+t^2}$$

$$= \frac{1+t^2-1+2t^2}{1+t^2} \times \frac{1+t^2}{2t}$$

$$= \frac{2t^2}{2t}$$ ✓ ① Correct simplification

$$= t$$

$$= \tan\frac{\theta}{2}$$

(c) $y = \dfrac{x^2+1}{\sqrt{3x^2-1}}$ $\quad u = x^2+1$ $\quad V = (3x^2+1)^{\frac{1}{2}}$

$u' = 2x$ $\quad v' = 3x(3x^2+1)^{-\frac{1}{2}}$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{2x(3x^2+1)^{\frac{1}{2}} - 3x(x^2+1)(3x^2+1)^{-\frac{1}{2}}}{3x^2-1}$$ ✓ ① Correct quotient.

$$= \frac{2x(3x^2+1) - 3x(x^2+1)}{(3x^2-1)^{\frac{3}{2}}}$$ ✓

$$= \frac{6x^2+2x-3x^2-3x}{(3x^2-1)^{\frac{3}{2}}}$$ ① Correct simplification

$$= \frac{3x^2-5x}{(3x^2-1)^{\frac{3}{2}}} = \frac{x(3x^2-5)}{(3x^2-1)^{3/2}},$$

12.

(a) (i) $\angle PBE = \angle BCP$ ($\angle$ in alternate segment)

$\angle BCP = \angle CBP$ (base $\angle$'s in an isosceles Δ)

$2\angle BCP = 180° - \angle BPC$ (angle sum of Δ)

$\angle BCP = 90° - \frac{\theta}{2}$

$\therefore x = 90 - \frac{\theta}{2}$

② correct solution with reasons OR
① Partially correct & reasoned solution.

(ii) $\angle EBP = \angle PBC$ (from above)

$\angle ABC = 60°$ ($\angle$s in equilateral Δ)

$\angle EBP + \angle PBC + \angle ABC = 180°$ ($\angle$'s on a straight line)

$\angle EBP = 60$

$60° = 90° - \frac{\theta}{2}$

$\theta = 60$

$\angle COB = 2\theta$ (angle at centre is twice angle at circumference)

$\angle COB = 120°$

② Correct reasoned solution OR
① Partially correct reasoned solution.

(iii) $AC = 20$ (given)

$CB = 20$ (equilateral Δ)

$OC = OB$

$\angle OCB = 30°$ (base $\angle$ in isosceles Δ, $\angle$ sum of Δ)

$$\frac{OB}{\sin 30} = \frac{20}{\sin 120°}$$

$$OB = \frac{20 \sin 30°}{\sin 120°}$$

$$= 10 \times \frac{2}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$$

OR

$$20^2 = 2r^2 - 2r^2 \cos 120°$$
$$= 2r^2 + 2r^2 \cdot \frac{1}{2}$$
$$= 3r^2$$

$$r^2 = \frac{20^2}{3}$$

$$r = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \quad (r>0)$$

② Complete solution OR
① Partial solution with significant progress.

(c) 8 people, 3 together $\Rightarrow$ 6 'people' and 3 can be arranged in 3! ways.

$$(6-1)! \times 3! = 5! \times 3!$$
$$= 720.$$

(d) $\dfrac{N!}{(N-1)!} \times \dfrac{(N-1)!}{(N+1)!}$

$$= \frac{\cancel{N!}}{(N+1)\cancel{!}}$$

$$= \frac{1}{N+1}$$

(e) (i) ${}^{5}C_{3} = 10.$

(ii) We can choose any set of 3 points to make a triangle as long as not all 3 points are on the line.

$${}^{10}C_{3} - {}^{5}C_{3}$$

$$= 120 - 10$$

$$= 110 \text{ triangles.}$$

13.

(a) $y = x^2$ $\qquad$ $y = (x-2)^2$

$y' = 2x$ $\qquad$ $y' = 2(x-2)$

$m_1 = 2$ $\qquad$ $m_2 = -2$ ✓ $\qquad$ ① Finds gradients

$\tan\theta = \left|\dfrac{2+2}{1-4}\right|$ ✓ $\qquad$ ① Applies formula

$\theta = 53^{\circ}$ ✓ $\qquad$ ① Correct answer

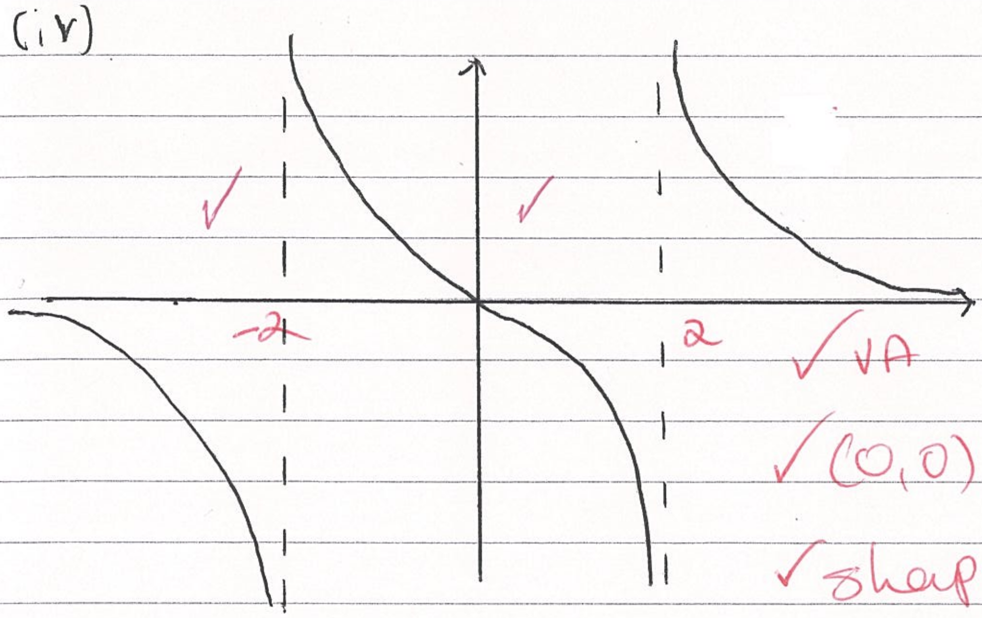
(b)

(i) $x \neq \pm 2$ ✓

(ii) $y = 0$ ✓

(iii) $(0, 0)$ ✓

(iv)



✓ VA

✓ (0,0)

✓ shape

2015 **Year 11 Assessment Task 3** **Student Name:** Solutions

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

	A	B	C	D
1	○	○	○	●
2	○	○	○	●
3	○	○	●	○
4	○	○	●	○
5	○	●	○	○
6	○	○	●	○
7	●	○	○	○
8	○	○	●	○
9	○	○	○	●
10	○	●	○	○

1. $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ $\quad m_1 = 2$, $m_2 = -\frac{3}{5}$

$= \left|\dfrac{2 + \frac{3}{5}}{1 + 2 \times -\frac{3}{5}}\right|$

$\theta = 85.6°$

D.

2. $\left(\dfrac{n \times x_2 + m \times x_1}{m+n}, \dfrac{n \times y_2 + n y_1}{m+n}\right)$

$= \left(\dfrac{3 \times -7 + -1}{4}, \dfrac{3 \times 5 + -7}{4}\right)$

$= (-5.5, 2)$

D.

3. $\underline{1} \times \underline{4} \times \underline{3} = 12$

C.

4. $\dfrac{10!}{4!3!} = 75600$

C

5. $x = \frac{47}{2}$

$x = 23.5^\circ$

B

6. $\frac{3}{x-2} \leqslant 4$

$3(x-2) \leqslant 4(x-2)^2$

$3x - 6 \leqslant 4x^2 - 16x + 16$

$4x^2 - 19x + 22 \geqslant 0$

$(4x-11)(x-2) \geqslant 0$

$x < 2 \quad x \geqslant \frac{11}{4}$

C.

7. A

8. C.

9. D.

10. $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 120$

B.

11.

(a) (i) $\dfrac{1+\cos 2A}{\sin 2A}$

$= \dfrac{1+\cos^2 A - \sin^2 A}{2\sin A\cos A}$ ✓

① sin double angle

① cos double angle

$= \dfrac{1+2\cos^2 A - 1}{2\sin A\cos A}$ ✓

$= \dfrac{\cos A}{\sin A}$ ✓

① simplification

$= \cot A$

(ii) $\cos 22.5° = \dfrac{1+\cos 45}{\sin 45}$

$= \dfrac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}$ ✓

①correct substitution

$= \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$= 2\sqrt{2}+3.$ ✓

①correct answer.

(b) (i) $r = \sqrt{1^2+1^2}$ $\quad \tan\alpha = 1$

$r = \sqrt{2}$ ✓ $\quad \alpha = 45°$ ✓

$\cos\theta + \sin\theta = \sqrt{2}\sin(\theta + 45)$

(ii) $\cos\theta + \sin\theta - 1 = 0$

$\sqrt{2}\sin(\theta + 45^\circ) = 1$ ✓

$\sin(\theta + 45^\circ) = \frac{1}{\sqrt{2}}$

$\theta + 45^\circ = 45^\circ$ $\qquad$ $\theta + 45^\circ = 135^\circ$

$\theta = 0$ ✓ $\qquad$ $\theta = 90^\circ$ ✓

c)

(i) $\cos\alpha = \frac{15}{17}$ $\qquad$ $\cos\beta = \frac{3}{5}$ ✓

(ii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \frac{8}{17} \times \frac{3}{5} + \frac{15}{17} \times \frac{4}{5}$

$= \frac{84}{85}$ ✓

d) $\tan 75^\circ = \tan(30^\circ + 45^\circ)$

$= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \times \tan 45^\circ}$ ✓ ① Correct expression

$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$ ✓ ① Correct evaluation

$= \frac{1+\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$

$= \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{(1+\sqrt{3})}{1+\sqrt{3}} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$ ✓

① Correct answer.

12

a) (i) $\tan 65^\circ = \frac{AC}{PC}$

$AC = 20 \tan 65^\circ$

$CD = 20 \tan 55^\circ$

(ii) $AB^2 = AC^2 + CD^2 - 2 \times AC \times CB \cos \angle ACB$

① Correct formula

$= (20 \tan 65)^2 + (20 \tan 55)^2 - 2 \times (20 \tan 65)(20 \tan 55) \cos 120^\circ$

① Correct evaluation

$= 3880.475978$

$= 62$ m (nearest metre) ① Correct answer

(b) $\sin\theta = \dfrac{2t}{1+t^2}$ $\qquad \cos\theta = \dfrac{1-t^2}{1+t^2}$ ✓ (1) Correct t-formulas or similarly significant progress.

$$\frac{1-\cos\theta}{\sin\theta} = \left(1 - \frac{1-t^2}{1+t^2}\right) \div \frac{2t}{1+t^2}$$

$$= \frac{1+t^2-1+2t^2}{1+t^2} \times \frac{1+t^2}{2t}$$

$$= \frac{2t^2}{2t}$$ ✓ (1) Correct simplification

$$= t$$

$$= \tan\frac{\theta}{2}.$$

(c) $y = \dfrac{x^2+1}{\sqrt{3x^2-1}}$, $\qquad u = x^2+1$, $u' = 2x$ $\qquad v = (3x^2+1)^{\frac{1}{2}}$, $v' = 3x(3x^2+1)^{-\frac{1}{2}}$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{2x(3x^2+1)^{\frac{1}{2}} - 3x(x^2+1)(3x^2+1)^{-\frac{1}{2}}}{3x^2-1}$$ ✓ (1) Correct quotient.

$$= \frac{2x(3x^2+1) - 3x(x^2+1)}{(3x^2-1)^{\frac{3}{2}}}$$ ✓

$$= \frac{6x^2+2x-3x^2-3x}{(3x^2-1)^{\frac{3}{2}}}$$ (1) Correct simplification

$$= \frac{3x^2-5x}{(3x^2-1)^{\frac{3}{2}}} = \frac{x(3x^2-5)}{(3x^2-1)^{3/2}}.$$

12.

(d) (i)

$\angle DBE = \angle BCD$ ($\angle$ in alternate segment)

$\angle BCD = \angle CBD$ (base $\angle$'s in an isosceles Δ)

$2\angle BCD = 180^\circ - \angle BDC$ (angle sum of Δ)

$\angle BCD = 90^\circ - \frac{\theta}{2}$

② Correct solution with reasons

$\therefore x = 90^\circ - \frac{\theta}{2}$

or

① Partially correct & reasoned solution

(ii) $\angle EBD = \angle DBC$ (from above)

$\angle ABC = 60^\circ$ ($\angle$ in equilateral Δ)

$\angle EBD + \angle DBC + \angle ABC = 180^\circ$ (angles on a straight line)

$\angle EBD = 60^\circ$

② Correct reasoned solution

$60^\circ = 90^\circ - \frac{\theta}{2}$

or

$\theta = 60^\circ$

① Partially correct reasoned solution.

$\angle COB = 2\theta$ (angle at centre is twice $\angle$ at circumference

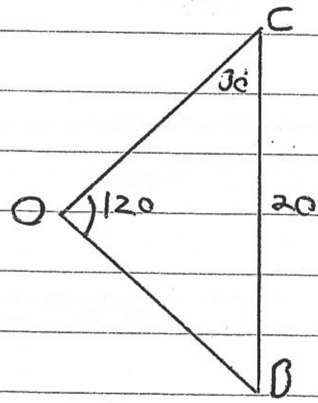
$\angle COB = 120^\circ$

(iii) $AC = 20$ (given)

$CB = 20$ (equilateral Δ)

$OC = OB$ (radii)

$\angle OCB = 30^\circ$ (base $\angle$ in isosceles Δ, $\angle$ sum of Δ)



$$\frac{OB}{\sin 30^\circ} = \frac{20}{\sin 120^\circ}$$

③ Complete solution

or

$$OB = \frac{20 \sin 30^\circ}{\sin 120^\circ}$$

① Partial solution with significant progress.

$$= 10 \times \frac{2}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{ cm}, \text{ as required,}$$

13.

(a) $y = x^2$ $\quad$ $y = (x-2)^2$

$y' = 2x$ $\quad$ $y' = 2(x-2)$

$m_1 = 2$ $\quad$ $M_2 = -2$ ✓ $\quad$ ① Finds gradients

$\tan\theta = \left|\frac{2+2}{1-4}\right|$ ✓ $\quad$ ① Applies formula

$\theta = 53^\circ$ ✓ $\quad$ ① Correct answer

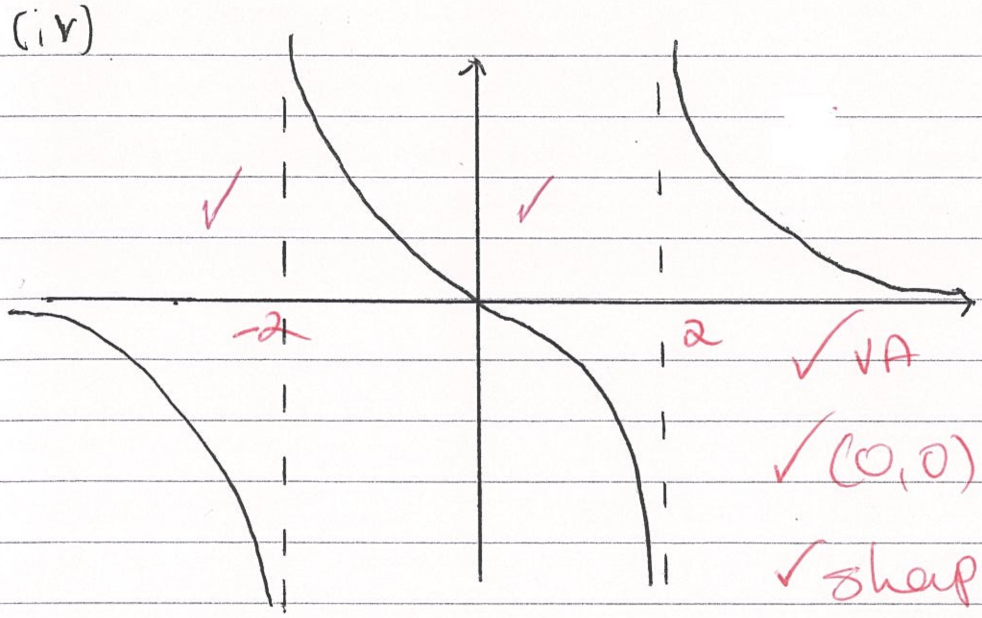
(b)

(i) $x \neq \pm 2$ ✓

(ii) $y = 0$ ✓

(iii) $(0, 0)$ ✓

(iv)



✓ VA

✓ (0,0)

✓ shape

(c) 8 people, 3 together $\Rightarrow$ 6 'people' and 3 can be arranged in 3! ways.

$$(6-1)! \times 3! = 5! \times 3!$$
$$= 720.$$

(d) $$\frac{N!}{(N-1)!} \times \frac{(N-1)!}{(N+1)!}$$

$$= \frac{\cancel{N!}}{(N+1)\cancel{!}}$$

$$= \frac{1}{N+1}$$

(e) (i) ${}^{5}C_{3} = 10.$

(ii) We can choose any set of 3 points to make a triangle as long as not all 3 points are on the line.

$${}^{10}C_{3} - {}^{5}C_{3}$$

$$= 120 - 10$$

$= 110$ triangles.