International Grammar School

Name:

Teacher: Martin, Ghabbar

Year 11
Mathematics Extension 1
Assessment Task 3
2016

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 - 13, show relevant mathematical reasoning and/or calculations
- A reference sheet is provided with this paper

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

45 marks

- Attempt Questions 11-13
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 75 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

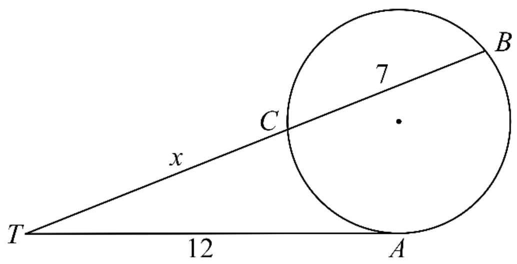
BLANK PAGE

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1–10.

---

1. Line TA is the tangent to the circle at A, and TB is a secant meeting the circle at B and C.



What is the value of x given that $TA = 12$ and $BC = 7$?

(A) 7

(B) 8

(C) 9

(D) 10

2. How many ways can 10 boys be arranged in a line if the first boy in the line is Aidan and the last boy in the line is Zac?

(A) 40,320

(B) 80,640

(C) 3,628,800

(D) 7,257,600

3. Which of the following is equivalent to the expression $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}}$?

(A) $2\cot^2 x$

(B) $2\cot x$

(C) $2\tan x$

(D) $2\tan^2 x$

4. What is the solution to the inequality $\dfrac{3}{x+4} \geq 2$?

(A) $x > -4$ and $x \leq -\dfrac{5}{2}$

(B) $x \geq -4$ and $x \leq -\dfrac{5}{2}$

(C) $x > -4$ and $x \geq -\dfrac{5}{2}$

(D) $x \geq -4$ and $x \geq -\dfrac{5}{2}$

5. Find $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{x-1}$

(A) x

(B) 0

(C) $2x$

(D) 1

6. The point $P(-3,8)$ divides the interval AB internally in the ratio k:1. If A is the point $(6,-4)$ and B is the point (0,4), what is the value of k?

(A) -1

(B) -3

(C) 1

(D) 3

7. The numberplates for motor vehicles consist of 2 letters, 2 numbers and then 2 letters such as AB01CD. How many different numberplates are possible if repetition is allowed?

(A) 135 200

(B) 270 400

(C) 37 015 056

(D) 45 697 600

8. A committee of 5 is to be chosen from 9 candidates. How many different ways can this be done?

(A) 45

(B) 120

(C) 126

(D) 15 120

9. What is the value of $\lim_{x \to -1} \frac{x^2 + 2x + 1}{x + 1}$?

(A) Undefined

(B) -1

(C) 0

(D) 1

10. What is the exact value of $\tan 15^\circ$?

(A) $2 - \sqrt{3}$

(B) $4 - \sqrt{3}$

(C) $2 + \sqrt{3}$

(D) $4 + \sqrt{3}$

End of section I

Section II

45 marks
Attempt Questions 11–13
Allow about 75 minutes for this section

Answer each question in a **separate writing booklet**. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

a) How many 5 digit numbers can be formed using the digits 2, 3, 5, 6, 8, and 9:

- **i.** If no digit can be used more than once? **1**
- **ii.** How many of the numbers formed are even numbers? **1**

b) A four-person team is chosen at random from ten women and six men.

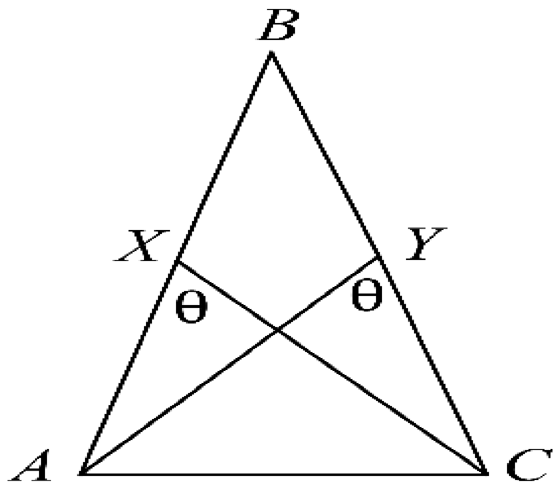
- **i.** In how many ways can this team be chosen? **1**
- **ii.** How many women only teams are possible? **1**
- **iii.** How many teams will have at least one man? **1**

c) Four men and four women meet for dinner at a restaurant. They are to be seated at a circular table.

In how many different ways can they be seated if:

- **i.** There are no restrictions? **1**
- **ii.** The men and women are to sit in alternate positions? **1**
- **iii.** The women all sit together, with two particular women always side by side. **2**

d) Two angles $\angle AXC = \theta$ and $\angle AYC = \theta$ are constructed in ΔABC. **Copy the diagram onto your writing booklet.**



i. Prove that AXYC is cyclic. **1**

ii. Hence show that $\angle BXY = \angle YCA$. **1**

e) **i.** Express $\cos x - \sin x$ in the form $A\cos(x+\alpha)$ for $0 \le \alpha \le 90^\circ$. **2**

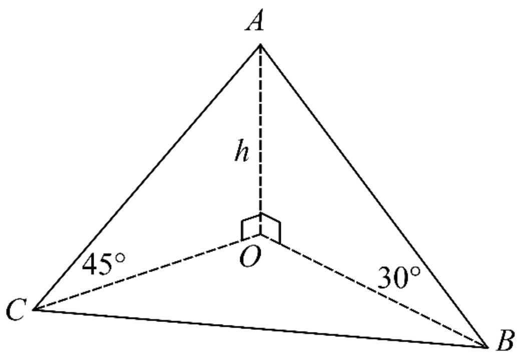
ii. Hence solve the $\cos x - \sin x = -1$ for $0^\circ \le x \le 360^\circ$. **2**

End of question 11

Question 12 (15 marks) – Start a new booklet.

a) Solve $9^x - 10.3^x + 9 = 0$ for x. **3**

b) The diagram below shows a vertical tower AO of height h metres, angle of elevation to the top of the tower from point B is 30° and from point C is 45°. Let $\angle BOC = 120^\circ$ and $\angle CAB = \theta$. **Copy the diagram onto your writing booklet.**



i. Show that $OC = h$ and $OB = \sqrt{3}h$ **2**

ii. Hence or otherwise, show that $BC^2 = (4+\sqrt{3})h^2$. **2**

c) **i.** Sketch the graph of $y = |2x-3|$ **2**

ii. Solve $|2x-3| \le |x-3|$ **2**

d) The point P divides the interval AB joining A$(-4,-3)$ and B(1, 5) externally in the ratio 3:2. What are the coordinates of P? **2**

e) Solve the inequality $\dfrac{2x+5}{x+1} < 1$ **2**

End of question 12

Question 13 (15 marks) – Start a new booklet.

a) **i.** Show that the two curves $y = x^2 + 2x - 3$ and $y = x^2 + 1$ intersect at (2,5) **1**

ii. What is the size of the acute angle, to the nearest degree, between the intersection of the tangents to the curves at (2,5)? **2**

b) Find the values of k for which the equation $4x^2 - (6+k)x + 1 = 0$ has no real zeros. **2**

c) Consider $f(x) = \dfrac{2x^2}{x^2 - 9}$

i. What are the equations of the vertical asymptotes? **1**

ii. Describe the behaviour of $y = f(x)$ as $x \to \pm\infty$ **1**

iii. Sketch $y = f(x)$, clearly showing its key features including asymptotes and x- and y-intercepts. **2**

iv. State the range of this function. **1**

d) The line $y = mx$ is inclined at 45° to the line $2x - y - 1 = 0$. **2**
What are the values of m?

e) Solve $\cos 2x - 3\cos x + 2 = 0$ in the domain $0 \le x \le 360^\circ$ **3**

End of Exam

2016 Year 11 Assessment Task 3 **Student Name: . . . .**

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Section I:

1. $TA^2 = TC \times TB$

$12^2 = x \times (x+7)$

$144 = x^2 + 7x$

$x^2 + 7x - 144 = 0$

$(x+16)(x-9) = 0$

$x = -16,\ x = 9$

$\therefore x = 9 > 0.$ C

2. $1 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$ A

3. $\sqrt{\dfrac{4+4\cos 2x}{1-\cos 2x}} = \sqrt{\dfrac{4(1+\cos 2x)}{1-\cos 2x}}$

$= 2\sqrt{\dfrac{1+(2\cos^2 x - 1)}{1-(1+2\sin^2 x)}}$

$= 2\sqrt{\dfrac{\cancel{1}+2\cos^2 x \cancel{-1}}{\cancel{1-1}+2\sin^2 x}}$

$= 2\sqrt{\cot^2 x} = 2\cot x$ B

4. $\dfrac{3}{x+4} \geqslant 2 \quad \times (x+4)^2$

$3(x+4) \geqslant 2(x+4)^2$

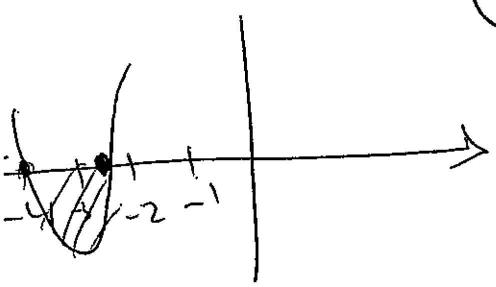
$2(x+4)^2 - 3(x+4) \leqslant 0$

$(x+4)[2(x+4)-3] \leqslant 0$

$(x+4)(2x+8-3) \leqslant 0$

$(x+4)(2x+5) \leqslant 0$

$-4 \leqslant x \leqslant -2\frac{1}{2}$ A



5. $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{x-1}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{2x^2}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}}$

$= \lim\limits_{x\to\infty} \dfrac{2x}{1}$ C

$= 2x.$

6. $A(6,-4),\ B(0,4),\ P(-3,8)$

$m:n = k:1$

$-3 = \dfrac{k \times 0 + 1 \times 6}{k+1}$

$-3k - 3 = 6$

$-3k = 9 \rightarrow \boxed{k = -3}$ B

7. $26^2 \times 10^2 \times 26^2 = 45697600$ D

8. ${}^9C_5 = 126$ C

9. $\lim\limits_{x\to -1} \dfrac{x^2+2x+1}{x+1}$

$= \lim\limits_{x\to -1} \dfrac{(x+1)\cancel{(x+1)}}{\cancel{x+1}}$

$= -1+1$

$= 0$ C

10. $\tan 15^\circ = \dfrac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ}$

$= \dfrac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{3-2\sqrt{3}+1}{3-1}$

$= \dfrac{4-2\sqrt{3}}{2}$

$= 2-\sqrt{3}$ A

Question 11:

a) 2, 3, 5, 6, 8 and 9.

i) $6 \times 5 \times 4 \times 3 \times 2 = 720$ (1)

or

$^6P_5 = 720$ 5 digit numbers.

ii. Half the numbers will be even since there's an equal number of odd and even digits.

$720 \div 2 = 360$ (1)

or

$^5P_4 \times 3 = 360$ since there are 3 numbers that are even 2, 6 and 8.

b) i. $^{16}C_4 = 1820$ (1)

ii. $^{10}C_4 = 210$ (1)

iii. At least one man: All combinations − all women

$= {}^{16}C_4 - {}^{10}C_4$

$= 1610$ (1)

* carry thru mark given for (i) − (ii) even if (i) or (ii) incorrect

or

$^6C_1 \times {}^{10}C_3 + {}^6C_2 \times {}^{10}C_2 + {}^6C_3 \times {}^{10}C_1 + {}^6C_4 \times {}^{10}C_0 = 1610$

c) i. Four men and 4 women at a circular table:

No restrictions: $7! = 5040$ (1)

ii. 1st position is fixed men: $3!\,4! = 144$. (1)

iii. 4 men and one group of women = 5 groups

$\therefore 4!$

4 women = $3!$

2 women can sit together in $2!$ ways

2 for correct
1 for dealing with women seating.

$\therefore$ total number of arrangements $= 4!\,3!\,2!$

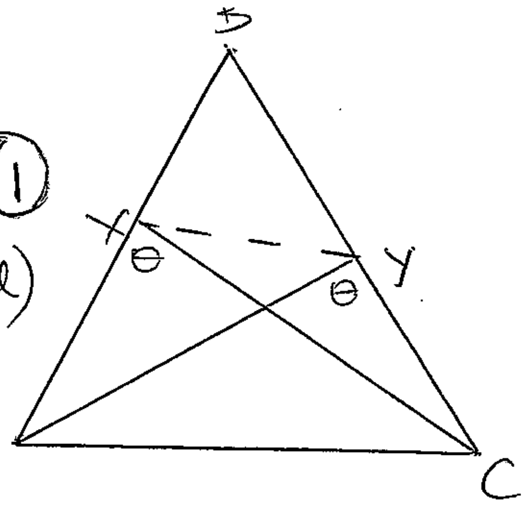
$= 288$

carry thru mark given if (ii) answered $4! \times 4!$ and (iii) $4! \times 4! \times 2!$

d) i. $\angle AXC = \angle AYC$

$= \theta$ (given)

$\therefore$ AXYC is cyclic (converse of $\angle$s in the same segment, AC, are equal) (1)



ii. Since AXYC is cyclic,

$\angle BXY = \angle YCA$ (external $\angle$ in a cyclic quadrilateral = opposite interior $\angle$). (1)

e) i. $\cos x - \sin x = A\cos(x+\alpha)$, $0 \leq \alpha \leq 90°$

$A = \sqrt{a^2+b^2}$

$A = \sqrt{1^2+1^2}$

$A = \sqrt{2}$

$\tan\alpha = \frac{b}{a}$

$\tan\alpha = 1$

$\therefore \alpha = 45°$

$\therefore \cos x - \sin x = \sqrt{2}\cos(x+45°)$

2 mark $\sqrt{2}\cos(x+45°)$
1 mark either $\sqrt{2}$ or $+45°$.

ii. $\cos x - \sin x = -1$, $0° \leq x \leq 360°$

$\sqrt{2}\cos(x+45°) = -1$

$\cos(x+45°) = -\frac{1}{\sqrt{2}}$ 2nd and 3rd quadrants.

$\therefore x+45° = 180°-45°, 180°+45°$, $45° \leq x+45° \leq 360°+45°$

$x+45° = 135°, 225°$ (1)

$x = 90°, 180°$ (1) for $0° \leq x \leq 360°$

2 mark for correct answer.

1 mark some correct progress

Question 12:

a) $9^x - 10.3^x + 9 = 0$

Let $u = 3^x$

then $9^x - 10.3^x + 9 = 0$ is equivalent to:

$(3^2)^x - 10.3^x + 9 = 0$

$(3^x)^2 - 10.3^x + 9 = 0$

i.e: $u^2 - 10u + 9 = 0$ - 1

1 for correct u equation

1 for each correct x-value

$(u-9)(u-1) = 0$

$u = 9$ or $u = 1$

$3^x = 9$ or $3^x = 1$

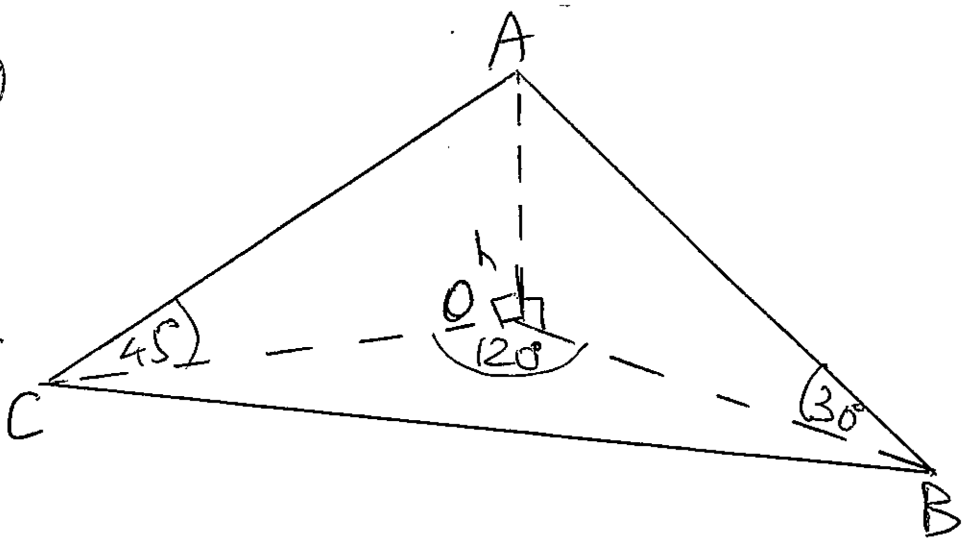
$x = 2$ or $x = 0$

1 1

b) i. $\tan 45^\circ = \frac{h}{OC}$ 1

$1 = \frac{h}{OC}$

$\therefore \boxed{OC = h}$ 1



$\tan 30^\circ = \frac{h}{OB}$

$\frac{1}{\sqrt{3}} = \frac{h}{OB}$

$\boxed{OB = \sqrt{3}h}$ 1

ii. $BC^2 = OC^2 + OB^2 - 2 \times OC \times OB \times \cos 120^\circ$

$BC^2 = h^2 + (\sqrt{3}h)^2 - \cancel{2} \times h \times \sqrt{3}h \times -\frac{1}{\cancel{2}}$ 1 mark

$BC^2 = h^2(1 + 3 + \sqrt{3})$

$BC^2 = (4 + \sqrt{3})h^2$ 1 mark

c.) $y = |2x - 3|$

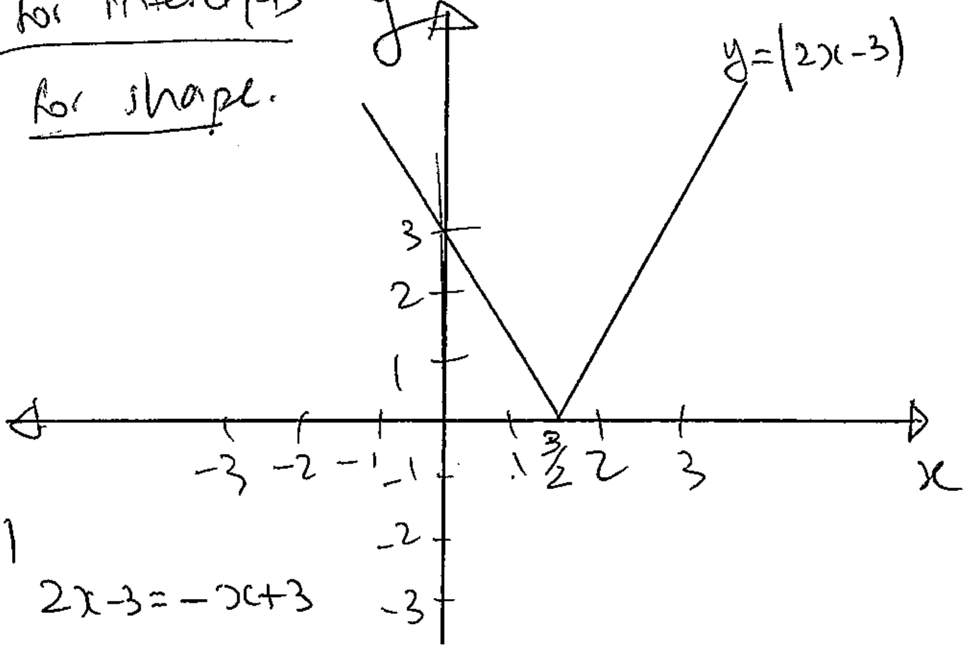
1 for intercepts

1 for shape.

$y = 2x - 3$

or

$y = -2x + 3$



ii. $|2x-3| \leq |x-3|$

solve: $|2x-3| = |x-3|$

$2x - 3 = x - 3$ or $2x - 3 = -x + 3$

$\boxed{x = 0}$ or $3x = 6$

$\boxed{x = 2}$

Test: $x = -1$, $x = 1$, $x = 3$

$x = -1$

1 for working

1 for result.

$|2 \times -1 - 3| \leq |-1 - 3|$

$|-5| = |-4|$ False.

$\boxed{x = 1}$

$|2 \times 1 - 3| \leq |1 - 3|$

$|-1| \leq |-2|$ True

$x = 3$

$|2 \times 3 - 3| \leq |3 - 3|$

$|3| \leq 0$ False.

$\therefore$ solution: $0 \leq x \leq 2$

d) $A(-4,-3)$, $B(1,5)$

$m:n$
$-3:2$.

$x = \frac{-3\times 1 + 2\times -4}{-3+2}$

$x = \frac{-11}{-1}$

$x = 11$

$y = \frac{-3\times 5 + 2\times -3}{-3+2}$

$y = \frac{-15-6}{-1}$

$y = 21$ 1 for each coordinate.

$\therefore \underline{P(11, 21)}$. 1 mark for significant progress towards answer

e) $\frac{2x+5}{x+1} < 1$ $\times(x+1)^2$, $x \neq -1$

$(2x+5)(x+1) < (x+1)^2$

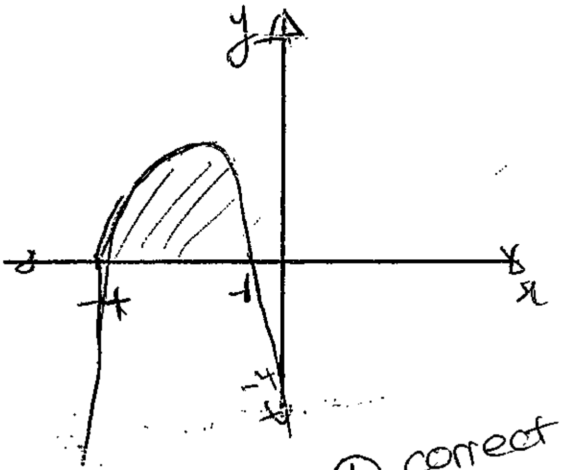
$(x+1)^2 - (2x+5)(x+1) > 0$

$(x+1)[(x+1) - (2x+5)] > 0$

$(x+1)(x+1-2x-5) > 0$

$(x+1)(-x-4) > 0$

$\boxed{-4 < x < -1}$



① correct progress having noted $x \neq -1$.

Or

Solve $\frac{2x+5}{x+1} = 1$ then test x-values:

$2x+5 = x+1$

$x = -4$

Test: $x = -5$, $x = -2$, $x = 0$

$\underline{x = -5}$:

$\frac{2\times -5+5}{-5+1} < 1$

$\frac{-5}{-4} < 1$ False.

$\underline{x = -2}$

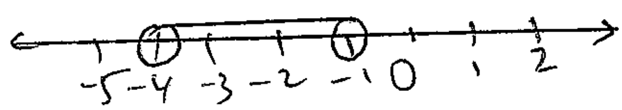
$\frac{2\times -2+5}{-2+1} < 1$

$\frac{1}{-2} < 1$ True.

$\underline{x = 0}$

$\frac{2\times 0+5}{1} < 1$

$5 < 1$ False.



$\therefore -4 < x < -1$

Question 13:

a) i. $y = x^2 + 2x - 3$ and $y = x^2 + 1$ solving simultaneously.

$\cancel{x^2} + 2x - 3 = \cancel{x^2} + 1$

$2x = 4$

$x = 2$

$y = x^2 + 1 \rightarrow y = 2^2 + 1 \rightarrow y = 5$

1 any correct working.

$\therefore (2, 5)$

ii. $y = x^2 + 2x - 3$, $y = x^2 + 1$

$y' = 2x + 2$, $y' = 2x$

1 for gradients

1 for result.

$\therefore m_1 = 2 \times 2 + 2$

$\boxed{m_1 = 6}$

$\therefore m_2 = 2 \times 2$

$\boxed{m_2 = 4}$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\tan\theta = \left| \frac{6-4}{1+6\times4} \right|$

$\tan\theta = \left| \frac{2}{25} \right|$

$\therefore \theta = 4.5739\ldots$

$\therefore \theta \doteqdot 5^\circ$

b) $4x^2 - (6+k)x + 1 = 0$

No real zeros $\rightarrow \Delta < 0$

$b^2 - 4ac < 0$

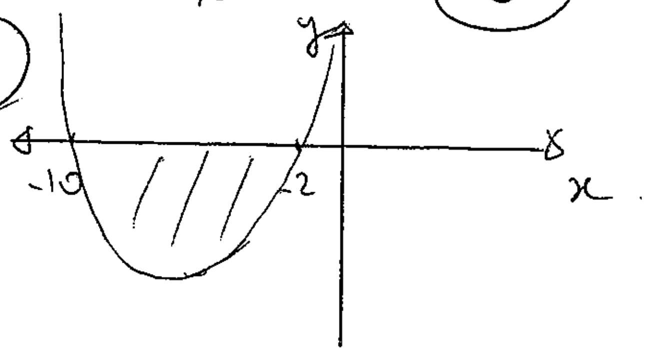
$[-(6+k)]^2 - 4 \times 4 \times 1 < 0$ 1

$36 + 12k + k^2 - 16 < 0$

$k^2 + 12k + 20 < 0$

$(k+2)(k+10) < 0$

$\therefore -10 < k < -2$ 1



c) i. $x \neq \pm 3$ (1) $\quad f(x) = \dfrac{2x^2}{x^2-9}$

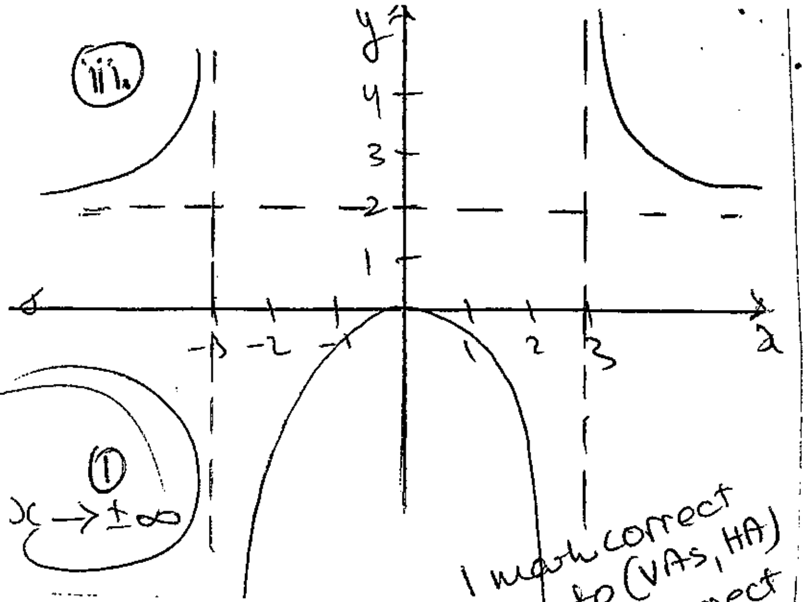
$x = 3$ or $x = -3$

ii. $\lim_{x \to \pm\infty} \dfrac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$

$\lim_{x \to \pm\infty} \dfrac{2}{1-0}$

$= 2$, y approaches 2 as $x \to \pm\infty$ (1)

iii.



1 mark correct info (VAs, HA)
1 mark correct graph.

iv. Range: $y \leq 0$, $y > 2$

d) $y = mx$ $\qquad 2x - y - 1 = 0$

$y = 2x - 1$

$\therefore m_1 = 2$.

$\tan 45° = \left| \dfrac{m_1 - m_1}{1 + m_1 m_2} \right|$

$1 = \left| \dfrac{2-m}{1+2m} \right|$

$\therefore \quad 1 = \dfrac{2-m}{1+2m} \quad$ or $\quad -1 = \dfrac{2-m}{1+2m}$

$1 + 2m = 2 - m \qquad -1 - 2m = 2 - m$

$3m = 1 \qquad -m = 3$

$\therefore m = \frac{1}{3}$ (1) $\qquad \therefore m = -3$ (1)

1 mark may be awarded for correct progress, eg: correct substitution or one correct solution.

e) $\cos 2x - 3\cos x + 2 = 0$

$2\cos^2 x - 1 - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$2\cos x - 1 = 0$, $\cos x - 1 = 0$

$2\cos x = 1$ or $\cos x = 1$

$\cos x = \frac{1}{2}$

$x = 60°, 360° - 60°$ or $x = 0°, 360°$

$\therefore x = 0°, 60°, 300°, 360°$

1 for correct substitution for $\cos 2x$

1 for correct $\cos x$ values

1 for solution.