International Grammar School

Name: _____

Teacher: _____

Year 11
Mathematics Extension 1
Assessment Task 3
2017

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 13, show relevant mathematical reasoning and/or calculations

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

45 marks

- Attempt Questions 11 – 13
- Allow about 75 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following quadrilaterals are always concyclic?

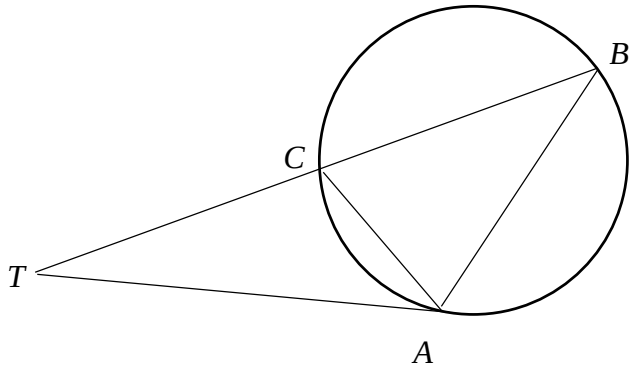
(A) Parallelogram and Rhombus

(B) Trapezium and Rectangle

(C) Square and Rhombus

(D) Square and Rectangle

2 Line *TA* is the tangent to the circle at *A*, and *TB* is a secant meeting the circle at *B* and *C*.



Which of the following is correct?

(A) $\angle TAC = \angle ABC$ (Alternate Segment Theorem)

(B) $\angle TAC = \angle ACB$ (Alternate Segment Theorem)

(C) $\angle ACB = \angle ABC$ (Angles in the same segment are equal)

(D) $\angle TAC = 2\angle ABC$ (Angles at the centre are twice the angle at the circumference, subtended by the same arc)

3 12 students are studying Spanish and another 15 students are studying French. How many ways can a student council be formed that comprises of 3 students who study Spanish and 4 who study French?

(A) ${}^{12}P_3 + {}^{15}P_4$

(B) ${}^{12}P_3 \times {}^{15}P_4$

(C) ${}^{12}C_3 + {}^{15}C_4$

(D) ${}^{12}C_3 \times {}^{15}C_4$

4 The point $P(x, y)$ divides the line joining the points $A(5, -2)$ and $B(-1, -5)$ **externally** in the ratio $1:2$.

What are the coordinates of *P*?

(A) $(3, -1)$

(B) $(1, -2)$

(C) $(-7, -8)$

(D) $(11, 1)$

5 Which of the following is equal to $\sec\theta$ in terms of t where $t = \tan\frac{\theta}{2}$.

(A) $\frac{2t}{1-t^2}$

(B) $\frac{1-t^2}{2t}$

(C) $\frac{1+t^2}{1-t^2}$

(D) $\frac{1+t^2}{2t}$

6 Find the exact value of $\sin 75°$.

(A) $\dfrac{1+\sqrt{3}}{2\sqrt{2}}$

(B) $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$

(C) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(D) $\cos 25°$

7 Solve $\sin^2 x = \dfrac{3}{4}$ for $0° \le x \le 360°$.

(A) $x = 30°, 150°$

(B) $x = 60°, 120°$

(C) $x = 30°, 150°, 210°, 330°$

(D) $x = 60°, 120°, 240°, 300°$

8 Find $\lim\limits_{x\to\infty} \dfrac{2x^3}{4x^3-1}$

(A) $\dfrac{1}{2}$

(B) 2

(C) 0

(D) ∞

9 Four couples sit at a round table. In how many ways can they be arranged if each couple sits together?

(A) 6

(B) 8

(C) 96

(D) 630

10 Which of the following equates to $\frac{1-\cos 2x}{\sin 2x}$?

(A) $\cot 2x$

(B) $\cot x$

(C) $\tan x$

(D) $\cos x$

Section II

45 marks
Attempt Questions 11–13
Allow about 75 minutes for this section

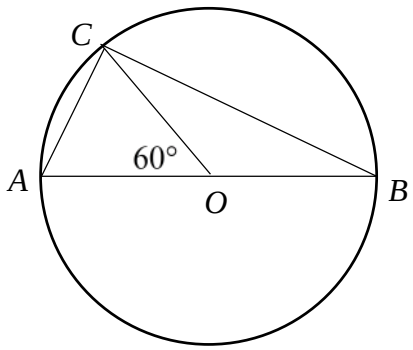
Answer each question in a **separate writing booklet**. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Solve $\dfrac{4}{x-3} \geq -2$ **3**

(b) In the diagram below, AB is a diameter, passing through the centre O and $\angle AOC = 60°$



(i) Find the size of $\angle ABC$, give a reason for your answer **2**

(ii) Find the size of $\angle ACB$, give a reason for your answer **2**

Question 11 continues on page 8

(c) In how many ways can the letters of the word EXTENSION be arranged in a line? **2**

(d) A class consists of 13 students. At the end of a lesson, 1 student is to be chosen to clean the board and another student to put up the chairs. **1**

In how many different ways can the two student be chosen?

(e) A country's numberplates consist of 2 upper case letters, 3 digits and then another upper case letter such as PQ707E.

(i) How many different numberplates are possible given letters and digits may be used more than once? **1**

(ii) What is the probability that a numberplate produced at random will be PQ707E? **1**

(f) Solve $\sin 2x = \sin x$, given $0 \leq x \leq 360$. **3**

End of Question 11

Question 12 (15 marks)

(a) The graphs of the curve $f(x) = x^3$ and the line $y = -x + 2$ intersect at $(1,1)$.

(i) Find the equation of the tangent to the curve $f(x) = x^3$ at $(1,1)$. **3**

(ii) Find the value of the acute angle between the given line and the tangent to the curve at the point of intersection. **2**

Give your answer correct to the nearest degree.

(b) An extension Maths class has 12 students. 5 students are chosen at random to represent the class in an equation solving competition. **2**

What is the probability that the chosen students are the first 5 on the roll alphabetically?

(c) Find the points on the curve $f(x) = x^3 - 3x^2 + 4x$ where the tangent is parallel to the line $y = 4x - 5$. **3**

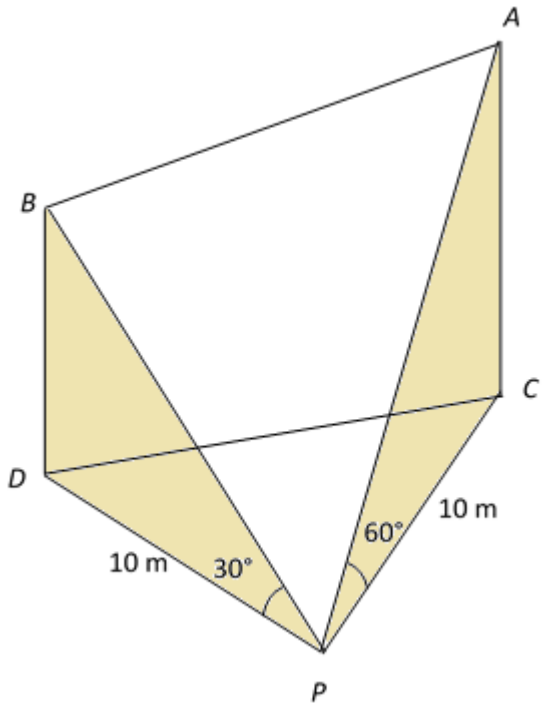
(d) (i) Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $r\sin(\theta + \alpha)$ where α is acute. **2**

(ii) Hence or otherwise, solve $\cos x + \sqrt{3}\sin x = 1$, given $0° \le x \le 360°$. **3**

End of Question 12

Question 13 (15 marks)

(a) Two drones, drone A and drone B, are hovering above a flat field. From a point P on the field, the angle of elevation of drone A is 60° and the angle of elevation of drone B is 30° on the field such that PC and PD are each 10 metres. Drone A is directly above the point C on the field and drone B is directly above the point D.



(i) The bearing of drone A from P is 040° and the bearing of drone B from P is 310°. Show that $\angle CPD = 90°$ and hence $CD = 10\sqrt{2}$ m. **2**

(ii) Show that $3BD = AC$ **2**

(iii) Using (i) and (ii), or otherwise, find the angle of elevation of drone A from drone B. Give your answer correct to the nearest degree **2**

Question 13 continues on page 11

Question 13 Continued

(b) By first factorising, sketch the graph of $y = \dfrac{x^2 - x - 6}{x - 3}$. **2**

Use about half a page for your graph.

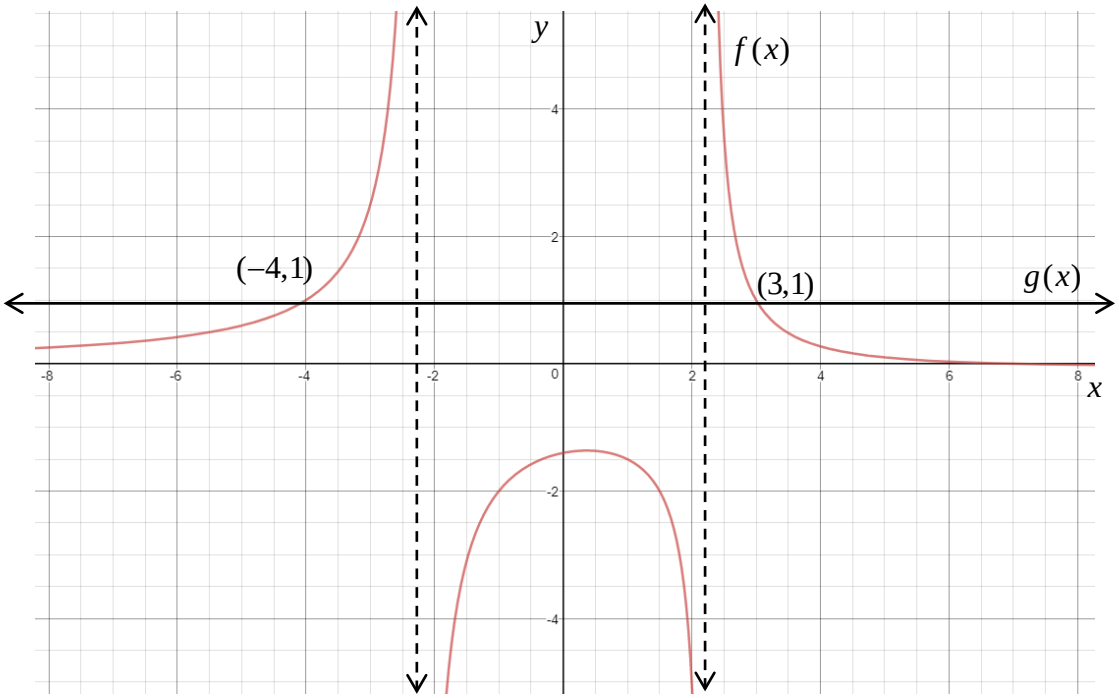
(c) Consider the function $y = \dfrac{x^2 + 2}{x}$

(i) Show that $\dfrac{x^2 + 2}{x} = x + \dfrac{2}{x}$. **1**

(ii) State any vertical, horizontal or oblique asymptotes of the function. **2**

(iii) Without using calculus, graph the function, showing all asymptotes. **2**
Use about half a page.

(d) The functions $f(x) = \dfrac{7 - x}{x^2 - 5}$ and $y = 1$ are graphed below. The vertical asymptotes of $f(x)$ are at $x = \pm\sqrt{5}$.



Using the graph, give all solutions to the inequation $\dfrac{7 - x}{x^2 - 5} \leq 1$ **2**

End of Paper

2017 Year 11 Assessment Task 3 **Student Name: . . . . . .**

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

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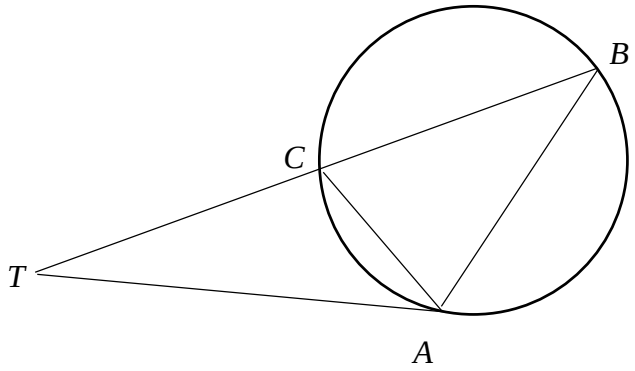

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Question 11 (15 marks)

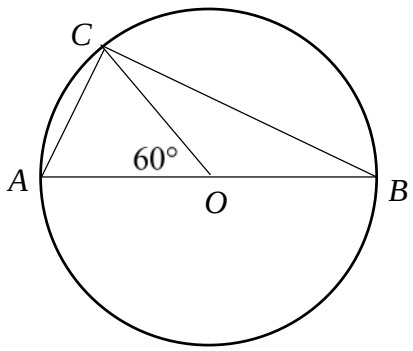
(a) Solve $\dfrac{4}{x-3} \geq -2$ **3**

$$\frac{4}{x-3} \geq -2$$
$$x \neq 3$$
$$4 = -2(x-3)$$
$$4 = -2x+6$$
$$2x = 2$$
$$x = 1$$
$$x \leq 1,\ x > 3$$

or

$$\frac{4}{x-3} \geq -2$$
$$4(x-3) \geq -2(x-3)^2$$
$$4(x-3)+2(x-3)^2 \geq 0$$
$$2(x-3)[2+(x-3)] \geq 0$$
$$2(x-3)(x-1) \geq 0$$
$$x \leq 1,\ x > 3$$

(b) In the diagram below, AB is a diameter, passing through the centre O and $\angle AOC = 60°$



(i) Find the size of $\angle ABC$, give a reason for your answer **2**

$\angle ABC = 30°$ (Angle at the centre is twice the angle at the circumference subtended by the same chord/arc)

(ii) Find the size of $\angle ACB$, give a reason for your answer **2**

$\angle ACB = 90°$ (Angle in a semicircle)

(c) In how many ways can the letters of the word EXTENSION be arranged in a line? **2**

$$\frac{9!}{2!2!} = 90720$$

(d) A class consists of 13 students. At the end of a lesson, 1 student is to be chosen to clean the board and another student to put up the chairs. **1**

In how many different ways can the two student be chosen?

$$13 \times 12 = 156 \quad \text{or} \quad {}^{13}P_2 = 156$$

(e) A country's numberplates consist of 2 upper case letters, 3 digits and then another upper case letter such as PQ707E.

(i) How many different numberplates are possible given letters and digits may be used more than once? **1**

$$26 \times 26 \times 10 \times 10 \times 10 \times 26 = 17576000$$

(ii) What is the probability that a numberplate produced at random will be PQ707E? **1**

$$\frac{1}{17576000}$$

(f) Solve $\sin 2x = \sin x$, given $0 \le x \le 360$. **3**

$$\begin{aligned}
\sin 2x &= \sin x \\
2\sin x \cos x &= \sin x \\
2\sin x \cos x - \sin x &= 0 \\
\sin x[2\cos x - 1] &= 0 \\
\sin x &= 0 \\
x &= 0°, 180°, 360° \\
\cos x &= \frac{1}{2} \\
x &= 60°, 300°
\end{aligned}$$

End of Question 11

Question 12 (15 marks)

(a) The graphs of the curve $f(x)=x^3$ and the line $y=-x+2$ intersect at $(1,1)$.

(i) Find the equation of the tangent to the curve $f(x)=x^3$ at $(1,1)$. **3**

$$f'(x)=3x^2$$
$$f'(1)=3(1)^2$$
$$=3$$

$$y-y_1=m(x-x_1)$$
$$y-1=3(x-1)$$
$$y=3x-2$$
$$3x-y-2=0$$

(ii) Find the value of the acute angle between the given line and the tangent to the curve at the point of intersection. **2**

Give your answer correct to the nearest degree.

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$\tan\theta=\left|\frac{3-(-1)}{1+3(-1)}\right|$$
$$\tan\theta=2$$
$$\theta=63°$$

(b) An extension Maths class has 12 students. 5 students are chosen at random to represent the class in an equation solving competition. **2**

What is the probability that the chosen students are the first 5 on the roll alphabetically?

$$\frac{1}{{}^{12}C_5}=\frac{1}{792}$$

(c) Find the points on the curve $f(x)=x^3-3x^2+4x$ where the tangent is parallel to the line $y=4x-5$. **3**

$$f'(x)=3x^2-6x+4$$
$$3x^2-6x+4=4$$
$$3x^2-6x=0$$
$$3x(x-2)=0$$
$$x=0,2$$

$$f(0)=(0)^3-3(0)^2+4(0)$$
$$=0$$
$$(0,0)$$
$$f(2)=(2)^3-3(2)^2+4(2)$$
$$=4$$
$$(2,4)$$

(d) (i) Express $\cos\theta+\sqrt{3}\sin\theta$ in the form $r\sin(\theta+\alpha)$ where α is acute. **2**

$$
\begin{aligned}
r\sin(\theta+\alpha) &= r\sin\theta\cos\alpha + r\sin\alpha\cos\theta \\
\cos\theta+\sqrt{3}\sin\theta &= \sqrt{3}\sin\theta+\cos\theta \\
r\cos\alpha &= \sqrt{3} \\
r\sin\alpha &= 1 \\
r &= \sqrt{\sqrt{3}^2+1^2} \\
&= 2 \\
\tan\alpha &= \frac{1}{\sqrt{3}} \\
\alpha &= 30^\circ \\
\cos\theta+\sqrt{3}\sin\theta &= 2\sin(\theta+30)^\circ
\end{aligned}
$$

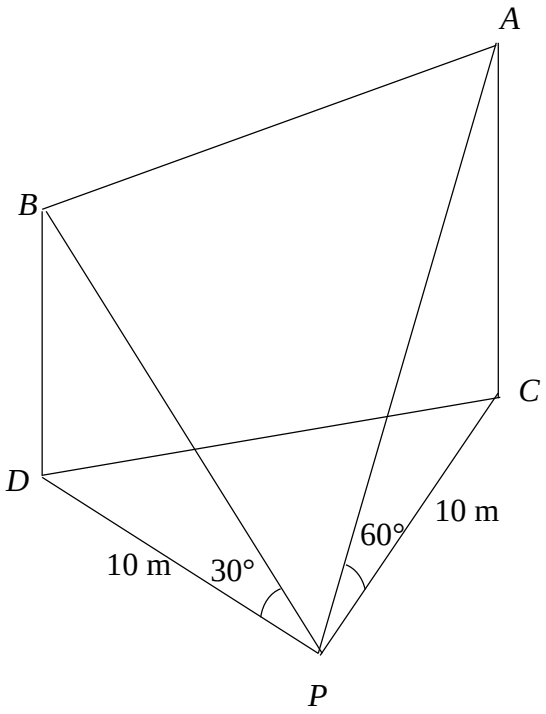
(ii) Hence or otherwise, solve $\cos x+\sqrt{3}\sin x=1$, given $0^\circ\le x\le 360^\circ$. **3**

$$
\begin{aligned}
2\sin(\theta+30)^\circ &= 1 \\
\sin(\theta+30)^\circ &= \frac{1}{2} \\
(\theta+30)^\circ &= 30^\circ, 150^\circ \\
\theta &= 0^\circ, 120^\circ
\end{aligned}
$$

End of Question 12

Question 13 (15 marks)

(a) Two drones, drone A and drone B, are hovering above a flat field. From a point P on the field, the angle of elevation of drone A is 60° and the angle of elevation of drone B is 30° on the field such that PC and PD are each 10 metres. Drone A is directly above the point C on the field and drone B is directly above the point D.



(i) The bearing of drone A from P is 040° and the bearing of drone B from P is 310°. Show that $\angle CPD = 90°$ and hence $CD = 10\sqrt{2}$ m. **2**

(Let N be the point due North of P)

$$\angle CPD = \angle DPN + \angle NPC$$
$$= (360 - 310)° + 40°$$
$$= 90°$$

$$CD^2 = PD^2 + PC^2$$
$$= 10^2 + 10^2$$
$$= 200$$
$$CD = \sqrt{200}$$
$$= 10\sqrt{2}\ m$$

(ii) Show that $3BD = AC$ **2**

$$\tan 30 = \frac{BD}{PD} \qquad \tan 60 = \frac{AC}{PC} \qquad \frac{AC}{3} = \frac{10\sqrt{3}}{3}$$

$$\frac{1}{\sqrt{3}} = \frac{BD}{10} \qquad \sqrt{3} = \frac{AC}{10} \qquad = \frac{10}{\sqrt{3}}$$

$$BD = \frac{10}{\sqrt{3}} \qquad AC = 10\sqrt{3} \qquad = BD$$

(iii) Using (i) and (ii), or otherwise, find the angle of elevation of drone A from drone B. Give your answer correct to the nearest degree **2**

BD is y metres, AC is 3y metres, so AC is 2y units taller than BD

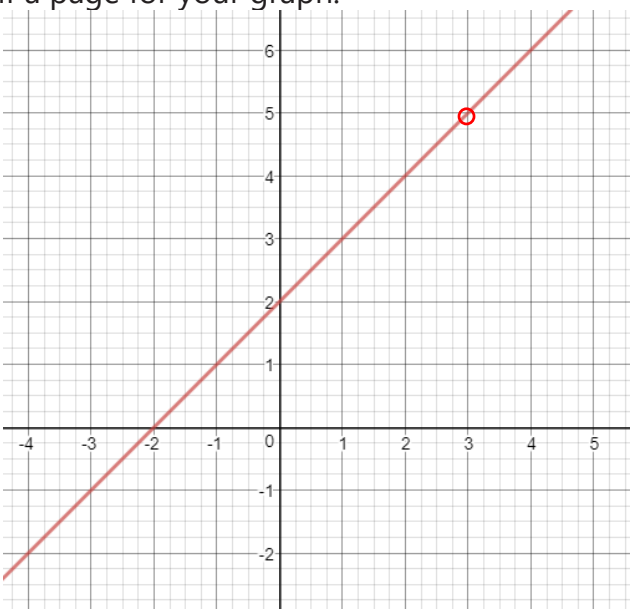
$$\tan\theta = \frac{\frac{2}{3} \times \sqrt{3}x}{\sqrt{2}x}$$

$$\tan\theta = \frac{2\sqrt{3}}{3\sqrt{2}}$$

$$\theta = 39°$$

(b) By first factorising, sketch the graph of $y = \dfrac{x^2 - x - 6}{x - 3}$. **2**

Use about half a page for your graph.



p

(c) Consider the function $y = \dfrac{x^2+2}{x}$

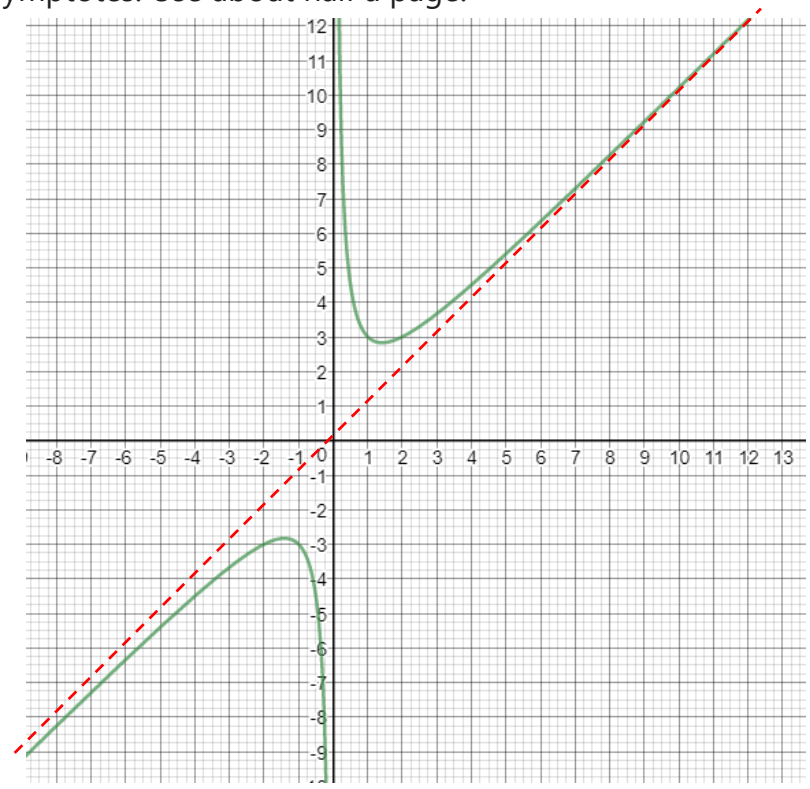
(i) Show that $\dfrac{x^2+2}{x} = x + \dfrac{2}{x}$. **1**

$$\frac{x^2+2}{x} = \frac{x^2}{x} + \frac{2}{x}$$
$$= x + \frac{2}{x}$$

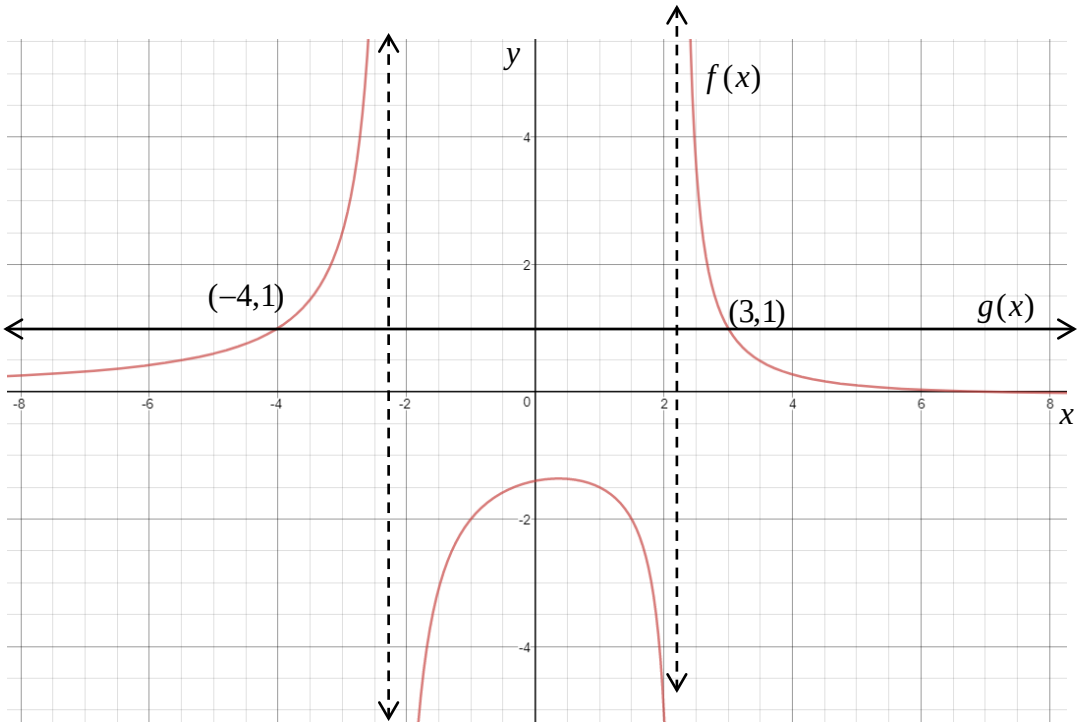
(ii) State any vertical, horizontal or oblique asymptotes of the function. **2**

Vertical at $x = 0$, Oblique at $y = x$

(iii) Without using calculus, graph the function, showing all asymptotes. Use about half a page. **2**



(d) The functions $f(x)=\dfrac{7-x}{x^2-5}$ and $g(x)=1$ are graphed below. The vertical asymptotes of $f(x)$ are at $x=\pm\sqrt{5}$.



Using the graph, give all solutions to the inequation $\dfrac{7-x}{x^2-5}\le 1$. **2**

$x\le -4,\quad -\sqrt{5}<x<\sqrt{5},\quad x\ge 3$

End of Paper

2017 Year 11 Assessment Task 3 **Student Name: . . . . . .**

Mathematics Extension 1

Section I Multiple-Choice Answer Sheet

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