International Grammar School

Name: ______________________

Year 11

Mathematics Extension 1

Assessment Task 3

2018

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- In Questions 11 – 13, show relevant mathematical reasoning and/or calculations

Total marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.
- Answer on the provided Multiple Choice Answer Sheet.

Section II

45 marks

- Attempt Questions 11 – 13
- Allow about 75 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which one of the following functions is *not* even?

A) $f(x) = x^2$

B) $f(x) = \dfrac{1}{1 + x^2}$

C) $f(x) = \sqrt{4 - x^2}$ when $|x| \leq 2$

D) $f(x) = x\sqrt{1 - x^2}$ when $|x| \leq 1$

2 State the natural domain of $y = \dfrac{1}{\sqrt{4 - x^2}}$:

A) Domain: $\{x : -2 < x < 2\}$

B) Domain: $\{x : -2 \leq x \leq 2\}$

C) Domain: $\{x : x < -2, x > 2\}$

D) Domain: $\{x : x < \pm 2\}$

3 In how many ways can 5 girls and 3 boys be arranged in a row if there are 7 girls and 4 boys to select from?

A) 84

B) 60 480

C) 3 386 880

D) 2 438 553 600

4 How many points on the curve of the function $y = \frac{1}{x-1} - \frac{4}{x-4}$ have a gradient of zero?

A) 0

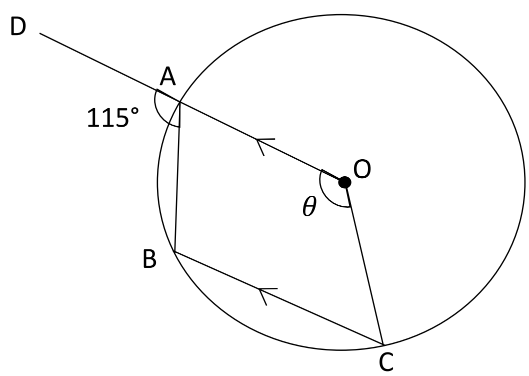
B) 1

C) 2

D) 4

5 The diagram shows a circle with centre O.

A, B and C are points on the circle, and OA is produced to D.
DO is parallel to BC, and ∠DAB = 115°.



Find the size of angle θ.
A) 100°

B) 115°

C) 130°

D) 230°

6 If $t = \tan\frac{1}{2}\theta$ which of the following expressions is equivalent to $\sec\theta$?

A) $\frac{1+t^2}{1-t^2}$

B) $\frac{2t}{1+t^2}$

C) $\frac{1-t^2}{1+t^2}$

D) $\frac{2t}{1-t^2}$

7 If θ is an acute angle, where $\sin\theta = \frac{1}{\sqrt{5}}$, find the exact value of $\sin 2\theta$.

A) $\frac{4}{5}$

B) $\frac{2}{\sqrt{5}}$

C) $\frac{1}{2\sqrt{5}}$

D) $\frac{2}{5}$

8 Which of the following is an expression for $\cos(A-B)-\cos(A+B)$

A) $-2\cos A\cos B$

B) $-2\sin A\sin B$

C) $2\cos A\cos B$

D) $2\sin A\sin B$

9 Which of the following is *not* a function?

A) $y=\sqrt{x}$

B) $x=\sqrt{y}$

C) $y=2$

D) $x=2$

10 $2^x\times5^x$ is equal to:

A) 7^x

B) 7^{2x}

C) 10^x

D) 10^{2x}

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Section II
45 marks
Attempt Questions 11–13
Allow about 75 minutes for this section

Answer each question in a **separate writing booklet**. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11: 15 marks

a) Find the exact value $\cos 105^\circ$ **2**

b) Differentiate $y = \dfrac{\sqrt{x+1}}{x}$, leaving your answer in simplest fraction form – ie. As a single fraction. **3**

c) Solve $\dfrac{2x+5}{x+3} \leq 1$. **2**

d) i) Show that $\dfrac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2\theta$ when $\cos 2\theta \neq -1$. **3**

ii. Hence show that the exact value of $\tan 15°$ is $2 - \sqrt{3}$. **2**

e) i. In how many ways can 6 friends sit around a circular table? **1**

ii. In how many ways can the 6 friends sit around a circular table if 2 of the friends insist on sitting next to each other? **1**

iii. In how many ways can the 6 friends sit around a circular table if 2 of the friends have had a fight and refuse to sit next to each other. **1**

End of question 11

Question 12: 15 marks – Start a new booklet

a) Consider the function $y = \dfrac{x}{2-x}$

i. State the equation of the horizontal asymptote. **1**

ii. Sketch the graph of the function including all intercepts and asymptotes **2**

b) How many distinct arrangements can be made by using the letters of the word POSSIBILITY***:***

i. With no restrictions? **1**

ii. If the letter ***s*** does not occupy the first or last place? **2**

iii. If the letters ***s*** are together? **1**

c) Vanessa needs to choose five representatives from the nine exchange students at her school. **2**

The exchange students consist of 4 Americans, 2 Japanese and 3 Germans.

In how many ways can she select the representatives if she wishes to include at least one student of each nationality?

d) The point $P(x, y)$ divides the line joining the points $A(5, -2)$ and $B(-1, -5)$ externally in the ratio $1:2$. Find the coordinates of P. **2**

e) i. Sketch the graph of $y = |2x + 1|$. **1**

ii. Hence, or otherwise, solve $|2x + 1| \geq |x - 2|$. **3**

End of question 12

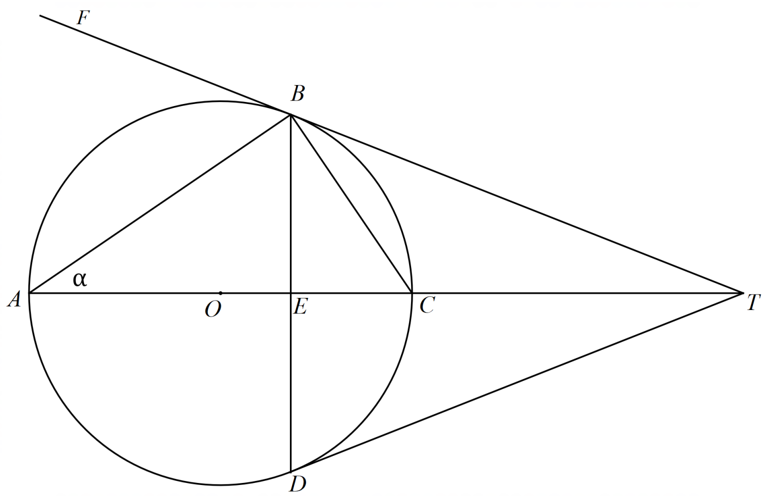
Question 13: 15 marks – Start a new booklet

a) The point *O* is the centre of the circle *ABCD*. *FT* is a tangent at *B*. *DT* is a tangent at *D*.

The points *A*, *O*, *E*, *C* and *T* are collinear.

$BD \perp AT$ and $\angle BAE = \alpha$.

Copy the diagram into your booklet.



i. Explain why $\angle CBT = \alpha$. **1**

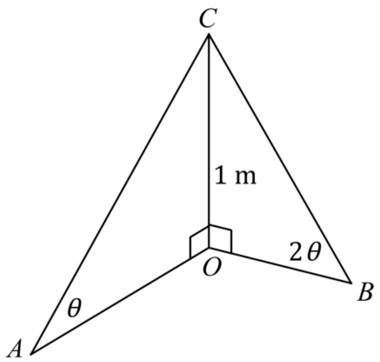
ii. Prove that BC bisects $\angle EBT$. **2**

iii. Prove that AB bisects $\angle EBF$. **1**

b) The curves $y = x^2$ and $y = x^2 - 3x + 6$ meet at the point $(2, 4)$. What is the acute angle between the tangents to the curves at this point, to the nearest minute? **3**

Question 13 continues on the next page

c) The diagram below shows a vertical pole *OC* of height 1 metre. It stands on horizontal ground. The angles of elevation of the top (*C*) of the pole, from points *A* and *B*, on the ground are θ and 2θ respectively.



Not to scale

i. Show that $OA = \cot\theta, OB = \cot 2\theta$ and $BC = \operatorname{cosec} 2\theta$ **1**

ii. Hence or otherwise, show that $OA - OB = BC$ **2**

d) i. Express $\sin(2x) - \cos(2x)$ in the form $R\sin(\theta - \alpha)$ **2**

ii. Hence or otherwise, solve the equation $\sin(2x) - \cos(2x) = 1$ on the domain $0 \le x \le 360^\circ$. **3**

End of examination

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Use the multiple-choice answer sheet for Questions 1–10.

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B) $f(x) = \dfrac{1}{1+x^2}$

C) $f(x) = \sqrt{4-x^2}$ when $|x| \leq 2$

D) $f(x) = x\sqrt{1-x^2}$ when $|x| \leq 1$

$f(x) = x\sqrt{1-x^2}$ when $|x| \leq 1$

$f(-x) = -x\sqrt{1-(-x^2)}$
$f(-x) = -x\sqrt{1-x^2} = -\left(x\sqrt{1-x^2}\right)$
$= -f(x)$
$\therefore$ it is an odd function

D

2 State the natural domain and range of $y = \dfrac{1}{\sqrt{4-x^2}}$:

A) Domain: $\{x : -2 < x < 2\}$

B) Domain: $\{x : -2 \leq x \leq 2\}$

C) Domain: $\{x : x < -2, x > 2\}$

D) Domain: $\{x : x < \pm 2\}$

A

3 In how many ways can 5 girls and 3 boys be arranged in a row if there are 7 girls and 4 boys to select from?

A) 84

B) 60 480

C) 3 386 880

D) 2 438 553 600

5 girls can be selected from 7 girls in ${}^7C_5$ ways. **C**

3 boys can be selected from 4 boys in ${}^4C_3$ ways.

Each selected group can then be arranged in 8! ways.

Total arrangements = ${}^7C_5 . {}^4C_3 . 8!$

$= 3\ 386\ 880$

4 How many points on the curve $y = \dfrac{1}{x-1} - \dfrac{4}{x-4}$ have a gradient of zero?

A) 0

B) 1

C) 2

D) 4

C

$$y = \frac{1}{x-1} - \frac{4}{x-4}$$ C

$$y = (x-1)^{-1} - 4(x-4)^{-1}$$

$$y' = \frac{-1}{(x-1)^2} + \frac{4}{(x-4)^2}$$

$$y' = \frac{-(x-4)^2 + 4(x-1)^2}{(x-1)^2(x-4)^2}$$

$$y' = \frac{-x^2 + 8x - 16 + 4x^2 - 8}{(x-1)^2(x-4)^2}$$

$$y' = \frac{3x^2 - 12}{(x-1)^2(x-4)^2}$$

$$3x^2 - 12 = 0 \text{ for stationary point}$$

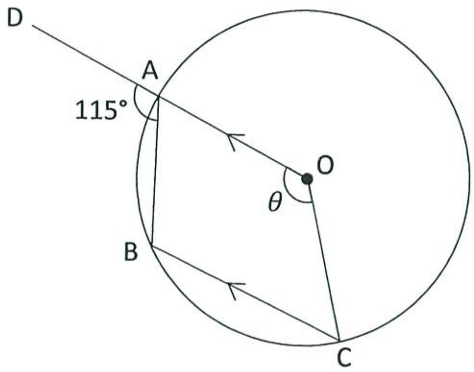
$$3(x^2 - 4) = 0$$

$$x = \pm 2 \ (\therefore 2 \text{ stat points})$$

5 The diagram shows a circle with centre O.

A, B and C are points on the circle, and OA is produced to D.

DO is parallel to BC, and $\angle DAB = 115°$.



Find the size of angle θ.

A) 100°

B) 115°

C) 130°

D) 230°

$\angle ABC = 115°$ (alternate angles on parallel lines)

$reflex\angle AOC = 230°$ (reflex angle at the centre is 2 times the angle subtending the same major arc)

$\theta = 360° - 230° = 130°$

C

6 If $t = \tan\frac{1}{2}\theta$ which of the following expressions is equivalent to $\sec\theta$?

A) $\frac{1+t^2}{1-t^2}$

B) $\frac{2t}{1+t^2}$

C) $\frac{1-t^2}{1+t^2}$

D) $\frac{2t}{1-t^2}$

A

7 If θ is an acute angle, where $\sin\theta = \frac{1}{\sqrt{5}}$, find the exact value of $\sin 2\theta$.

A) $\frac{4}{5}$

B) $\frac{2}{\sqrt{5}}$

C) $\frac{1}{2\sqrt{5}}$

D) $\frac{2}{5}$

$\sin\theta = \frac{1}{\sqrt{5}} \rightarrow$

Pythagorean theorem: $b^2 = \sqrt{5}^2 - 1^2 = 4 \rightarrow b = 2$

Double angle identity:
$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{4}{5}$

A

8 Which of the following is an expression for $\cos(A-B)-\cos(A+B)$

$\cos(A-B) - \cos(A+B)$
$= \cos A\cos B + \sin A\sin B - (\cos A\cos B - \sin A\sin B)$
$= \cancel{\cos A\cos B} + \sin A\sin B - \cancel{\cos A\cos B} + \sin A\sin B$
$= 2\sin A\sin B$

A) $-2\cos A\cos B$

B) $-2\sin A\sin B$

C) $2\cos A\cos B$

D) $2\sin A\sin B$

D

9 Which of the following is *not* a function?

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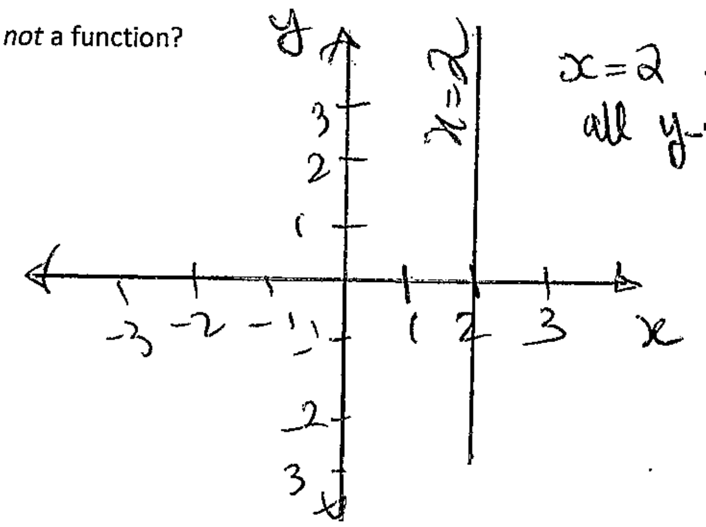
B) $x=\sqrt{y}$

C) $y=2$

D) $x=2$



$x = 2$ for all y-values

D

10 $2^x \times 5^x$ is equal to:

$2^x \times 5^x = (2\times5)^x$
$= 10^x$

A) 7^x

B) 7^{2x}

C) 10^x

D) 10^{2x}

C

Section II

45 marks
Attempt Questions 11–13
Allow about 75 minutes for this section

Answer each question in a **separate writing booklet**. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

a) Find the exact value $\cos 105°$ 2

$$\cos 105° = \cos(60° + 45°)$$
$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$
$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$
$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

or $\frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{2}-\sqrt{6}}{4}$

b) Differentiate $y = \frac{\sqrt{x+1}}{x}$, leaving your answer in simplest fraction form – ie. As a single fraction. 3

$u = (x+1)^{\frac{1}{2}}$, $v = x$
$u' = \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $v' = 1$

$$u' = \frac{1}{2\sqrt{x+1}}$$
$$y' = \frac{vu' - uv'}{v^2}$$
$$y' = \frac{x \cdot \frac{1}{2\sqrt{x+1}} - \sqrt{x+1}}{x^2}$$
$$y = \frac{\frac{x}{2\sqrt{x+1}} - \sqrt{x+1}}{x^2}$$
$$y' = \frac{\frac{x - \sqrt{x+1} \cdot 2\sqrt{x+1}}{2\sqrt{x+1}}}{x^2}$$
$$y' = \frac{x - 2(x+1)}{2\sqrt{x+1}} \div x^2$$
$$y' = \frac{x - 2x - 2}{2\sqrt{x+1}} \times \frac{1}{x^2}$$
$$y' = \frac{-x-2}{2x^2\sqrt{x+1}}$$

or

$$y' = -\frac{x+2}{2x^2\sqrt{x+1}}$$

c) Solve $\dfrac{2x+5}{x+3} \leq 1.$ 2

$$\frac{2x+5}{x+3} \leq 1$$

$$(x+3)^2\,\frac{2x+5}{x+3} \leq 1(x+3)^2$$

$$(x+3)(2x+5) \leq (x+3)^2$$

$$2x^2+11x+15 \leq x^2+6x+9$$

$$x^2+5x+6 \leq 0$$

$(x+3)(x+2) \leq 0$ test a point

$-3 < x \leq -2$

or $x \neq -3$

$2x+5 = x+3$

$x = -2$

Test:

$x=-4$: $\dfrac{2\times -4+5}{-4+3} \leq 1$, $\dfrac{-3}{-1} \leq 1$ False

$x=0$: $\dfrac{0+5}{0+3} \leq 1$ False

$x=-2.5$: $\dfrac{2\times(-2.5)+5}{-2.5+3} \leq 1$, $0 \leq 1$ True

$\therefore -3 \leq x \leq -2$

d) i) Show that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$ when $\cos 2\theta \neq -1$. 3

$$LHS = \frac{1-(\cos^2\theta - \sin^2\theta)}{1+(\cos^2\theta-\sin^2\theta)}$$

$= \dfrac{1-\cos^2\theta+\sin^2\theta}{1+\cos^2\theta-\sin^2\theta}$ divide through by co

$$= \frac{\sec^2\theta - 1 + \tan^2\theta}{\sec^2\theta + 1 - \tan^2\theta}$$

$$= \frac{(\tan^2\theta+1)-1+\tan^2\theta}{(\tan^2\theta+1)+1-\tan^2\theta}$$

$$= \frac{2\tan^2\theta}{2}$$

$= \tan^2\theta$

$=$ RHS

3 marks for proof reaching the required result.

2 marks for a proof with one or two minor errors or which doesn't not quite reach the required result

1 mark for a proof with some correct working towards result

ii. Hence show that the exact value of $\tan 15°$ is $2-\sqrt{3}$. 2

$$\tan^2(15°)=\frac{1-\cos 2(15°)}{1+\cos 2(15°)}$$

$$\tan^2(15°)=\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}$$

$$\tan^2(15°)=\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}$$

$$\tan^2(15°)=\frac{2-\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\tan^2(15°)=\frac{(2-\sqrt{3})^2}{2^2-(\sqrt{3})^2}$$

$$\tan^2(15°)=\frac{(2-\sqrt{3})^2}{4-3}$$

$$\tan^2(15°)=(2-\sqrt{3})^2$$

Since $\tan 15°$ is positive ($1st$ quadrant)

$$\tan(15°)=2-\sqrt{3}$$

2 marks for the correct solution

1 mark for use of the derived formula with an error resulting in an incorrect result

e) i. In how many ways can 6 friends sit around a circular table? 1

$$(n-1)!=(6-1)!=5!=120$$

ii. In how many ways can the 6 friends sit around a circular table if 2 of the friends insist on sitting next to each other? 1

There are 2! ways for 2 friends to sit next to each other.
There are 4! ways for the rest of the friends to sit next to each other.

$$2!\cdot 4!=48$$

iii. In how many ways can the 6 friends sit around a circular table if 2 of the friends have had a fight and refuse to sit next to each other? 1

There are 120 ways for the 6 friends to sit. 48 involve 2 sitting next to each other.

$120-48=72$

Question 12:

a) Consider the function $y = \dfrac{x}{2-x}$

i. State the equation of the horizontal asymptote 1

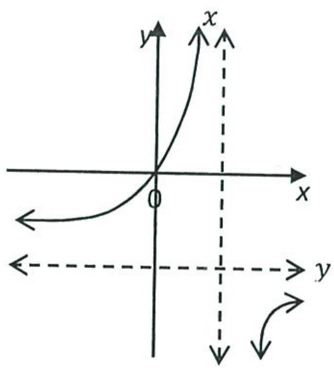
$$y = \frac{x}{2-x} = \frac{x}{-(x-2)} = \frac{x-2+2}{-(x-2)} = -\frac{(x-2)+2}{(x-2)} = -\left(\frac{x-2}{x-2} + \frac{2}{x-2}\right)$$

$= -\frac{2}{x-2} - 1$, which is a hyperbola, reflected vertically, shifted right 2 and down 1 unit.

The horizontal asymptote is $y = -1$

ii. Sketch the graph of the function including all intercepts and asymptotes 2

When $x = 0, y = 0$



1 for intercepts (0,0)
1 for vertical asymptote and correct curve

b) How many distinct arrangements can be made by using the letters of the word ***possibility***:

i. With no restrictions? 1

Arrangements $= \dfrac{11!}{2!3!} = 3\ 326\ 400$

ii. If the letter **s** does *not* occupy the first or last place? 2

Number of ways first and last place can be filled = $9 \times 8 = 72$

Total arrangements = $\dfrac{72 \times 9!}{2! \times 3!} = 2\ 177\ 280$

2 marks for correct answer

1 mark for a solution which includes some correct and relevant working toward an answer.

iii. If the letters *s* are together? 1

Treat the 2 s's as one letter, so there are 10 letters with one letter repeated 3 times.

$$\text{Arrangements} = \frac{10!}{3!} = 604800$$

c) Vanessa needs to choose five representatives from the nine exchange students at her school. 2

The exchange students consist of 4 Americans, 2 Japanese and 3 Germans.

In how many ways can she select the representatives if she wishes to include at least one student of each nationality?

$$\begin{aligned}\text{Selections} &= {}^4C_1 \times {}^2C_2 \times {}^3C_2 + {}^4C_1 \times {}^2C_1 \times {}^3C_3 + {}^4C_2 \times {}^2C_1 \times {}^3C_2 \\ &\quad + {}^4C_2 \times {}^2C_2 \times {}^3C_1 + {}^4C_3 \times {}^2C_1 \times {}^3C_1 \\ &= 12 + 8 + 36 + 18 + 24 \\ &= 98\end{aligned}$$

2 marks for correct solution

1 mark for working which includes some terms missing, or incorrect but has basic logic correct or if the calculation is done incorrectly

d) The point $P(x, y)$ divides the line joining the points $A(5, -2)$ and $B(-1, -5)$ externally in the ratio $1:2$. Find the coordinates of P. 2

(x_1, y_1) (x_2, y_2)

$(5, -2)$ $(-1, -5)$

$m : n$

$-1 : 2$

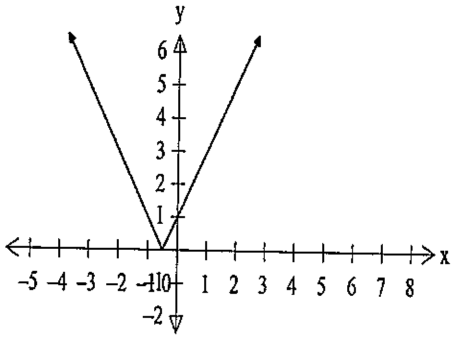
$$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$P\left(\frac{(-1)(-1)+2(5)}{-1+2}, \frac{(-1)(-5)+2(-2)}{-1+2}\right)$$

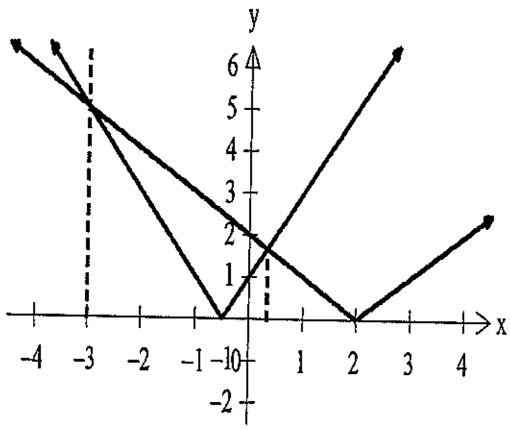
$$P\left(\frac{1+10}{1}, \frac{5-4}{1}\right)$$

$$P(11, 1)$$

e) i. Sketch the graph of $y = |2x + 1|$. 1



ii. Hence, or otherwise, solve $|2x + 1| \geq |x - 2|$. 3



$$2x + 1 = x - 2 \quad \text{or} \quad 2x + 1 = -x + 2$$

$$x = -3 \qquad\qquad x = \frac{1}{3}$$

$\therefore$ from the graph and equation $x \geq \frac{1}{3}$ and $x \leq -3$

3 marks for correct solution by any method.

2 marks for a worked solution with a minor error or which not quite complete

1 mark for an attempt with some correct working towards result

Test: $x = -4$ $\quad x = 0$ $\quad x = 1$

$|2\times -4 + 1| \geq |-4-2|$, $\quad |2\times 0 + 1| \geq |0-2|$ $\quad |2\times 1 + 1| \geq |1-2|$

$|-8+1| \geq |-6|$ $\quad 1 \geq 2$ $\quad 3 \geq 1$

$7 \geq 6$ $\quad$ False $\quad$ True

True

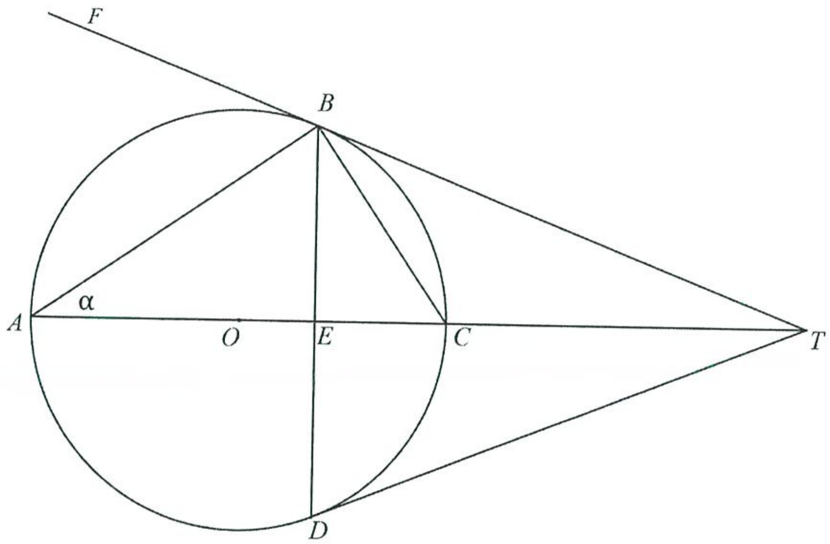
$\therefore x \leq -3$ or $x \geq \frac{1}{3}$

Question 13:

a) The point O is the centre of the circle $ABCD$. FT is a tangent at B. DT is a tangent at D.

The points A, O, E, C and T are collinear.

$BD \perp AT$ and $\angle BAE = \alpha$.



i. Explain why $\angle CBT = \alpha$. 1

$\angle CBT = \angle BAC = \alpha$ (ang in alternate seg on chord BC)

ii. Prove that BC bisects $\angle EBT$. 2

$BD \perp AT$ ($\Delta EBT \equiv \Delta EDT$)

$\therefore$ $\angle BET = 90°$

$\angle ABC = 90°$ ($\angle$ in semi circle)

$\angle ACB = 180° - (90° + \alpha)$ ($\angle$ sum of ΔABC)

$= 90° - \alpha$

$\therefore$ $\angle EBC = 180° - (90° + 90° - \alpha)$ ($\angle$ sum of ΔBCE)

$= \alpha$

since $\angle CBT = \angle EBC = \alpha$

BC bisects $\angle EBT$

2 marks for correct solution.

1 mark for a fully worked solution with a minor error in logic or calculation or for a solution which is incomplete but includes working of merit.

iii Prove that AB bisects $\angle EBF$. 1

$\angle ABF = \angle ECB = 90° - \alpha$ ($\angle$ in alt seg on chord AB)
$\angle ABE = 90° - \alpha$ (proved above)
$\therefore$ AB bisects $\angle EBF$

b) The curves $y = x^2$ and $y = x^2 - 3x + 6$ meet at the point $(2,4)$. What is the acute angle between the tangents to curves at this point to the nearest minute? 3

1 Mark for gradients

$y = x^2$
$y' = 2x$
$\therefore m_1 = 2 \times 2$
$m_1 = 4$

$y = x^2 - 3x$
$y' = 2x - 3$
$\therefore m_2 = 2 \times 2 - 3$
$m_2 = 1$

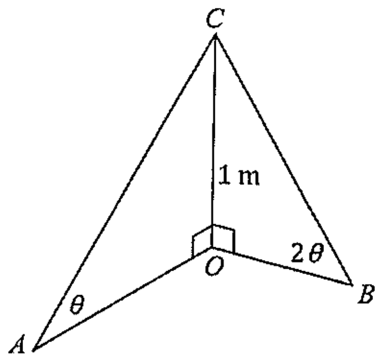
$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\theta = \dfrac{4-1}{1+4}$

$\tan\theta = \dfrac{3}{5}$

$\therefore \theta = 30.9637$ 1
$\theta = 30° 57' 49''$
$\theta = 30° 58'$ 1

c) The diagram below shows a vertical flagpole OC of height 1 metre. It stands on horizontal ground. The angles of elevation of the top (C) of the flagpole, from points A and B, on the ground are θ and 2θ respectively.



Not to scale

i. Show that $OA = \cot\theta, OB = \cot 2\theta$ and $BC = \operatorname{cosec} 2\theta$ 1

$\tan\theta = \dfrac{1}{OA}$ $\therefore OA = \cot\theta$

$\tan 2\theta = \dfrac{1}{OB}$ $\therefore OB = \cot 2\theta$

$\sin 2\theta = \dfrac{1}{BC}$ $BC = \operatorname{cosec} 2\theta$

ii. Hence or otherwise, show that $OA - OB = BC$ **2**

$$
\begin{aligned}
OA - OB &= \cot\theta - \cot 2\theta \\
&= \frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin 2\theta} \\
&= \frac{\cos\theta}{\sin\theta} - \frac{(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} \\
&= \frac{2\cos^2\theta - (\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta + \sin^2\theta}{\sin 2\theta} \\
&= \operatorname{cosec} 2\theta \\
&= BC
\end{aligned}
$$

2 marks: Correct answer.

1 mark: Simplifies the expression by using at least one double angle formula.

d) i. Express $\sin(2x) - \cos(2x)$ in the form $R\sin(\theta - \alpha)$ **3**

$\sin(2x) - \cos(2x) = R\sin(2x - \alpha)$ (we can let the auxiliary angle's sign be determined by the resulting triangle)

Dividing both sides by R:

$\frac{1}{R}\sin(2x) - \frac{1}{R}\cos(2x) = \sin(2x - \alpha)$

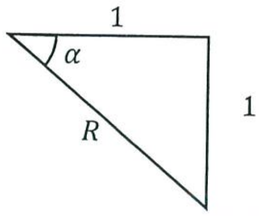
Using angle sum identity on RHS:

$\frac{1}{R}\sin(2x) - \frac{1}{R}\cos(2x) = \sin(2x)\cos\alpha + \cos(2x)\sin\alpha$

Equating the coefficients of $\sin(2x)$ and $\cos(2x)$:

$\sin\alpha = \frac{1}{R}$

$\cos\alpha = \frac{1}{R}$



$R = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$\tan\alpha = \frac{1}{1} \rightarrow \alpha = 45°$

$\sin(2x) - \cos(2x) = \sqrt{2}\sin(2x - 45°)$

ii. Hence or otherwise, solve the equation $\sin(2x) - \cos(2x) = 1$ on the domain $0 \le x \le 360°$.

$\sin(2x) - \cos(2x) = 1 \rightarrow \sqrt{2}\sin(2x - 45°) = 1$ (by part (i))

Adjusting the domain:

$0 \le x \le 360° \rightarrow 0 \le 2x \le 720° \rightarrow -45° \le 2x - 45° \le 675°$

$\sin(2x - 45°) = \frac{1}{\sqrt{2}}$, and $\sin\theta = \frac{1}{\sqrt{2}}$ at $\theta = 45, 135°, \ldots$ (sine is positive in Quadrants I and II)

$2x - 45° = 45°, 135°, 405°, 495°$

$2x = 90°, 180°, 450°, 540°$

$x = 45°, 90°, 225°, 270°$