INTERNATIONAL GRAMMAR SCHOOL
UNITY THROUGH DIVERSITY

2020 **PRELIMINARY EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 1.5 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In questions 11 – 13, show relevant mathematical reasoning and / or calculations

Total Marks: 60

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 45 marks (pages 6-17)

- Attempt Questions 11–13
- Allow about 1 hour and 15 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 A PIN is to be chosen using four of the digits 0 through 9. Digits cannot be repeated.

How many possible PINs are there?

A. 256

B. 210

C. 10 000

D. 5 040

2 The function $f(x)$ is defined as $f(x)=x^3+2$

Which expression is equal to $f^{-1}(x)$?

A. $\frac{1}{x^3}-2$

B. $\frac{1}{x^3-2}$

C. $\sqrt[3]{x}-2$

D. $\sqrt[3]{x-2}$

3 What is the domain of $2\cos^{-1}(3x)$?

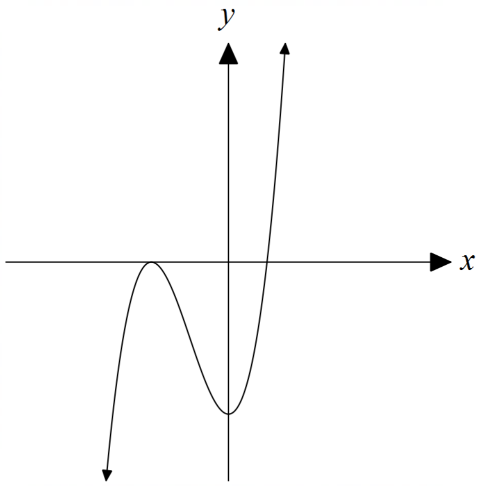
A. $[-3,3]$

B. $[0,\pi]$

C. $\left[-\frac{1}{3},\frac{1}{3}\right]$

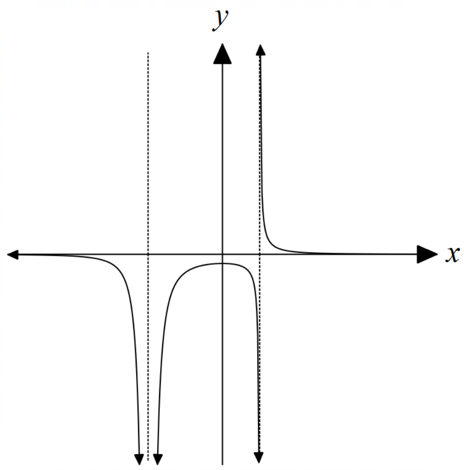
D. $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

4 The graph of the function $f(x)$ is shown below.

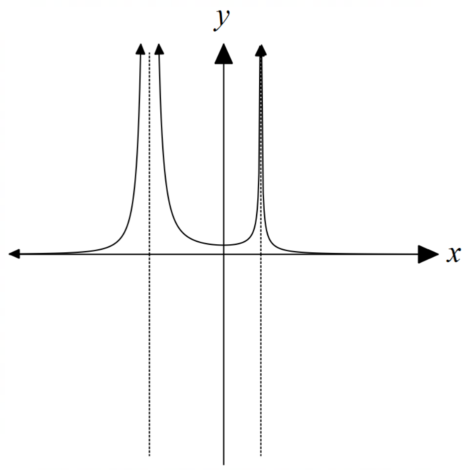


Which of the following graphs best represents $\dfrac{1}{f(x)}$?

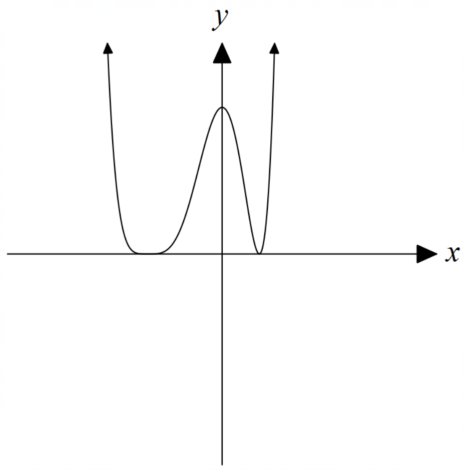
A.



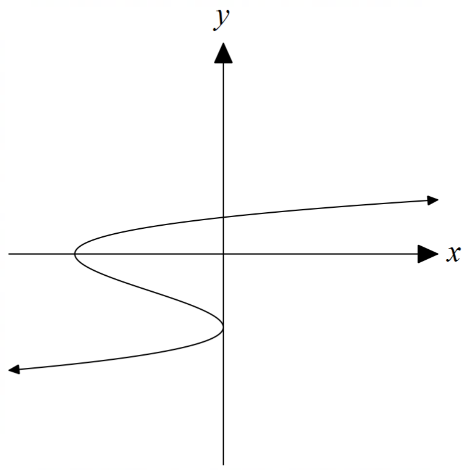
B.



C.

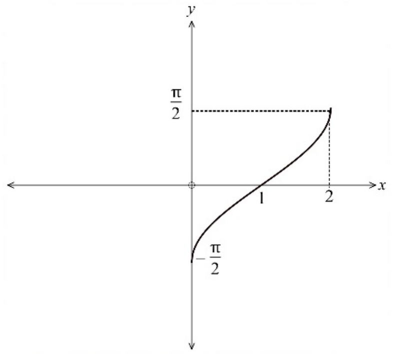


D.

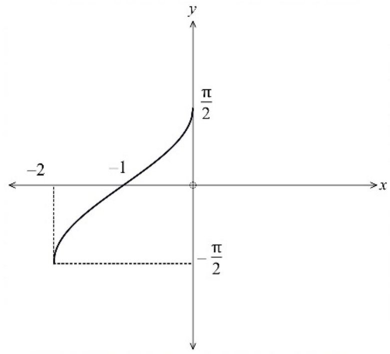


5 Which graph shows the curve $y = \sin^{-1}(x+1)$?

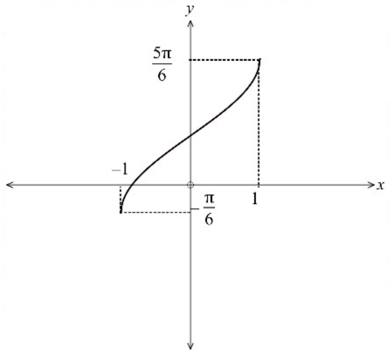
A.



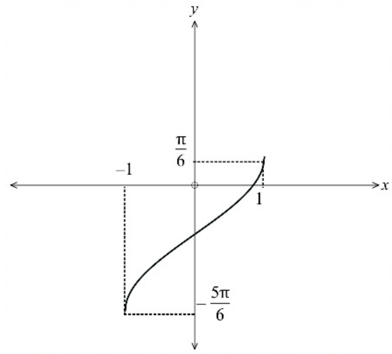
B.



C.



D.



6 Which of the following is a factor of $6x^3 - 5x^2 - 34x - 15$?

A. $x+1$

B. $x-3$

C. $x-2$

D. $x+5$

7 Given $N = 50 + 200e^{-kt}$, where k is a positive constant, what value does N approach as t approaches infinity?

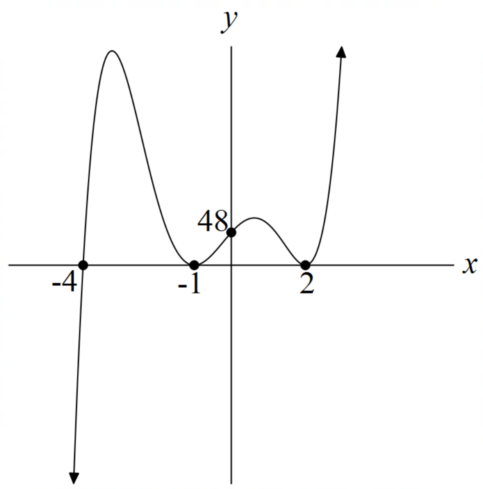
A. 50

B. 200

C. 250

D. 150

8 The diagram shows the graph of $y=a(x+b)^2(x+c)^2(x+d)$.



What are possible values of a, b, c and d?

A. $a=3,\ b=4,\ c=1,\ d=2$

B. $a=6,\ b=4,\ c=1,\ d=2$

C. $a=3,\ b=-2,\ c=1,\ d=4$

D. $a=6,\ b=-2,\ c=1,\ d=4$

9 What is the coefficient of x^7 in the expansion of $\left(x+\dfrac{4}{x}\right)^{13}$?

A. 18 304

B. 45 760

C. 286

D. 64

10 The gradient of the tangent to the curve $y = 2\cos\left(\frac{x}{2}\right)$ at the point where $x = \frac{\pi}{3}$ is $-\frac{1}{2}$.

What is the gradient on the curve where $x = \frac{11\pi}{3}$?

Hint: sketch the function.

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. 2

Section II

60 marks
Attempt Questions 11–13
Allow about 1 hour and 15 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.
In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet.

(a) Solve $|3x-6| \geq 15$ **2**

(b) Write the expansion of $(x-2y)^4$. **2**

(c) What is the exact value of $\tan^{-1}\left(2\sin\left(-\frac{2\pi}{3}\right)\right)$? **2**

(d) If $\sin\theta = \frac{5}{6}$ for $\frac{\pi}{2} \leq \theta \leq \pi$, what is the exact value of $\sin 2\theta$? **2**

(e) Four Year 12 students, three Year 11 students, and two Year 10 students are seated around a circular table randomly.

(i) How many seating arrangements are possible? **1**

(ii) What is the probability that the three groups are seated together? **2**

(f) The polynomial $P(x) = x^3 - 10x^2 - 39x + 180$ has roots α, β, γ.

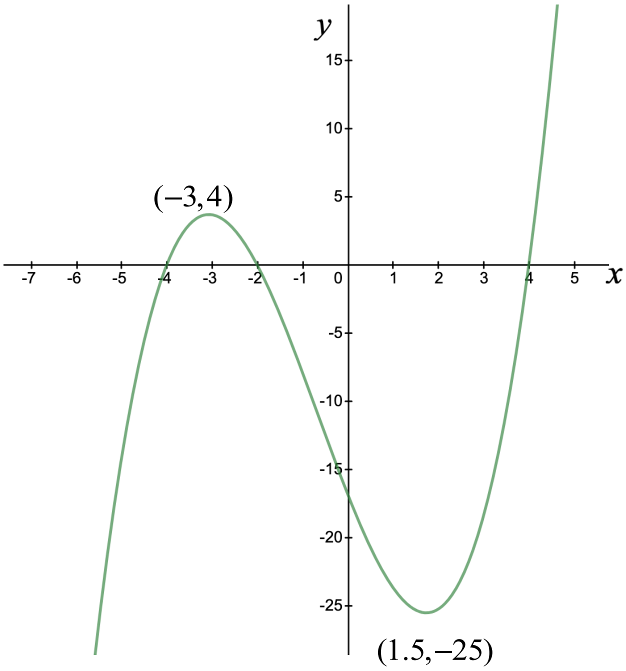
(i) Find $\alpha^2 + \beta^2 + \gamma^2$. **2**

(ii) Find $(\alpha\beta)^{-2} + (\alpha\gamma)^{-2} + (\beta\gamma)^{-2}$. **3**

Question 12 (14 marks) Use the Question 12 Writing Booklet.

(a) Show that $\sin\theta\sin\left(\frac{\pi}{2}-\theta\right)=\frac{1}{2}\cos\left(2\theta-\frac{\pi}{2}\right)$ **2**

(b) The diagram shows the graph of a cubic function $y=f(x)$.



Draw separate one-third of a page sketches of the following functions, clearly showing any transformations of the turning points and *x*- and *y*-intercepts, as applicable.

(i) $y=f(\,|x|\,)$ **2**

(ii) $y^2=f(x)$ **2**

Question 12 continues on page 9

Question 12 (continued)

(c) A particle is moving along the x-axis. Its position, x metres, at time t seconds is given by $x(t) = -\frac{t^3}{3} + \frac{11t^2}{2} - 28t + 2$ for $t \geq 0$.

(i) When is the particle at rest? **2**

(ii) For what values of t is the particle moving to the right? **1**

(d) A curve in the xy-plane is defined parametrically by $x = 4t^2$ and $y = 8t$, where t is the parameter.

(i) Draw the curve for $-3 \leq t \leq 3$, showing the beginning and ending coordinates and any intercepts. **2**

(ii) For what value or values of t will the parabola intersect the line $y = 6 - 2x$? **3**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Five points are scattered over a $2m$ by $2m$ square. Show that there is at least one pair of points which are a distance at most $\sqrt{2}\,m$ apart. **2**

(b) Given $t = \tan 15°$,

(i) show that $\dfrac{2t}{1-t^2} = \dfrac{1}{\sqrt{3}}$. **1**

(ii) Hence, find the exact value of $\tan 15°$. **2**

(c) A bottle of soft drink has been taken out of a refrigerator and is initially $2°C$. The soft drink is placed on a counter in a room, where the ambient temperature is $28°C$.

After being in the room for 10 minutes, the temperature of the soft drink is $6°C$.

The change in temperature of the soft drink can be modelled by $\dfrac{dT}{dt} = k(T-28)$, where T is the temperature of the soft drink, t is the time in minutes after the soft drink is taken out of the refrigerator, and k is a constant.

(i) Show that $T = 28 + Ae^{kt}$, where A is a constant, satisfies $\dfrac{dT}{dt} = k(T-28)$. **1**

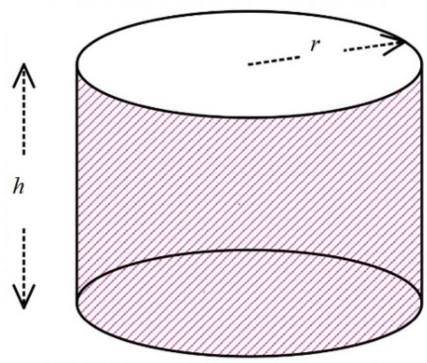
(ii) Show that $k = -0.0167$ (3sf) **2**

(iii) After 30 minutes, at what rate is the temperature of the soft drink changing? **2**

Question 13 continues on page 11

Question 13 (continued)

(d) The solid shown below is a cylinder which has its height equal to its diameter.



A virtual 3-D model is created which maintains the ratio of height and diameter.
In an animation, the model is being scaled up so that its volume is increasing at a steady rate of 120 cm^3 per second.

Note: The volume of a cylinder is given by $V = \pi r^2 h$

(i) Find the rate at which the radius is increasing when it reaches 2 cm. **4**

(ii) What is the radius when $\dfrac{dr}{dt} = \dfrac{80}{\pi}$ cm s^{-1}? **1**

End of paper

2020 Year 11 Assessment Task 3 Student Name: . . . .

Mathematics Extension

Section I Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

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Solutions

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1 A PIN is to be chosen using four of the digits 0 through 9. Digits cannot be repeated.

How many possible PINs are there?

A. 210

B. 256

C. 10 000

D. 5 040

$10 \times 9 \times 8 \times 7 = 5040$
Or
${}^{10}P_4 = 5040$

2 The function $f(x)$ is defined as $f(x) = x^3 + 2$

Which expression is equal to $f^{-1}(x)$?

A. $\frac{1}{x^3} - 2$

B. $\frac{1}{x^3 - 2}$

C. $\sqrt[3]{x} - 2$

D. $\sqrt[3]{x - 2}$

$y^3 + 2 = x$
$y^3 = x - 2$
$y = \sqrt[3]{x - 2}$

3 What is the domain of $2\cos^{-1}(3x)$?

A. $[-3, 3]$

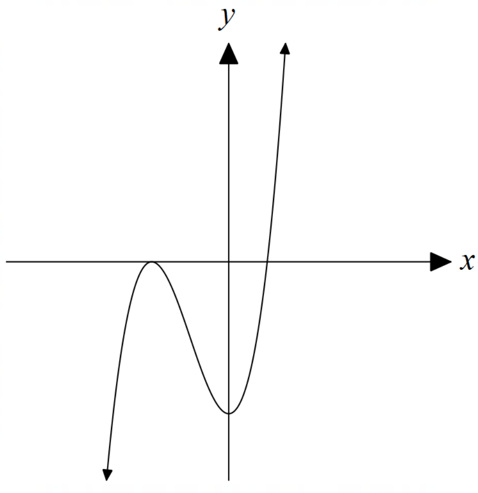
B. $[0, \pi]$

C. $\left[-\frac{1}{3}, \frac{1}{3}\right]$

D. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$-1 \le 3x \le 1$
$\frac{-1}{3} \le x \le \frac{1}{3}$

4 The graph of the function $f(x)$ is shown below.



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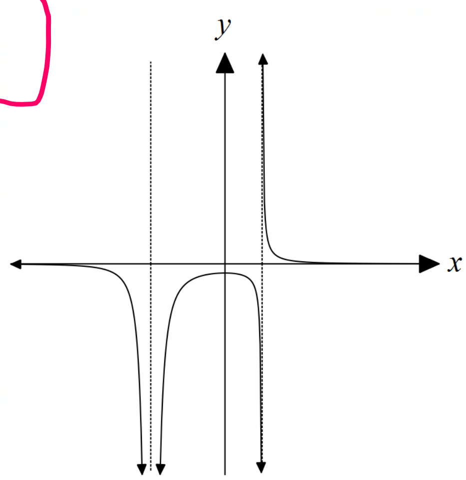
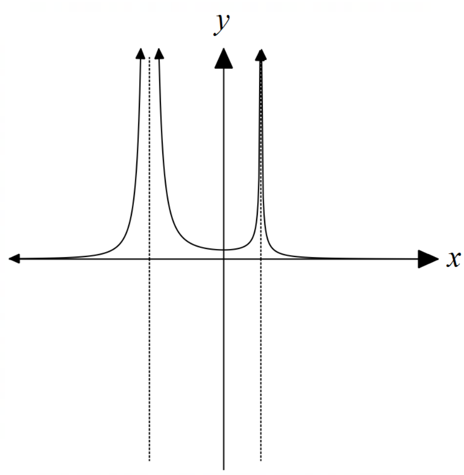
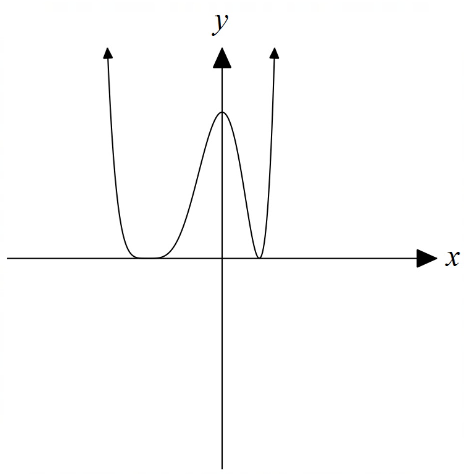


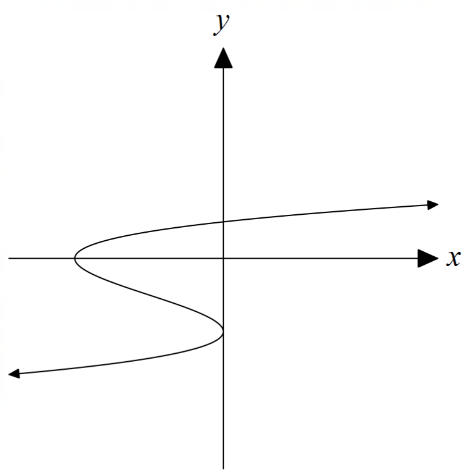
B.



C.

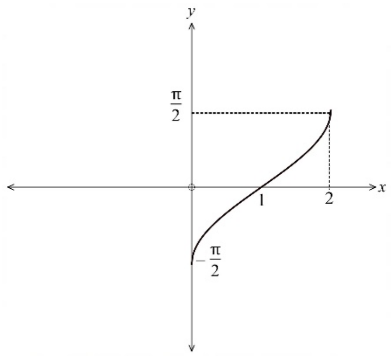


D.

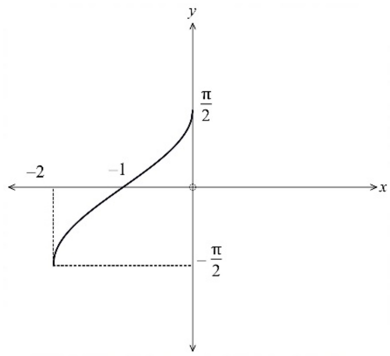


5 Which graph shows the curve $y=\sin^{-1}(x+1)$?

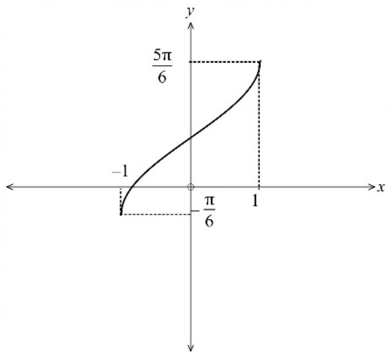
A.



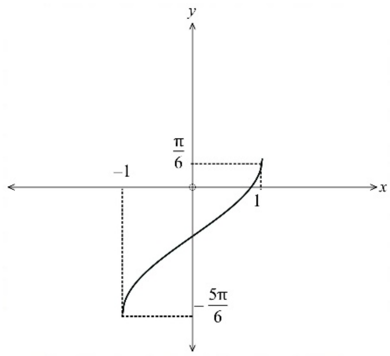
B.



C.



D.



Transformation is 1 to the left

6 Which of the following is a factor of $6x^3-5x^2-34x-15$?

A. $x+1$

B. $x-3$

C. $x-2$

D. $x+5$

$$6\times 3^3-5\times 3^2-34\times 3-15=0$$

7 Given $N=50+200e^{-kt}$, where k is a positive constant, what value does N approach as t approaches infinity?

A. 50

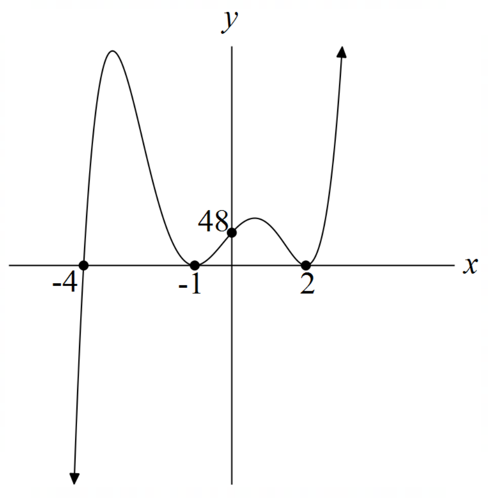
B. 200

C. 250

D. 150

$$\lim_{t\to\infty}\left(e^{-kt}\right)=0$$
$$\lim_{t\to\infty}\left(50+200e^{-kt}\right)=50+200\times 0$$
$$=50$$

8 The diagram shows the graph of $y = a(x+b)^2(x+c)^2(x+d)$.



What are possible values of a, b, c and d?

A. $a=3$, $b=4$, $c=1$, $d=2$

B. $a=6$, $b=4$, $c=1$, $d=2$

C. $a=3$, $b=-2$, $c=1$, $d=4$

D. $a=6$, $b=-2$, $c=1$, $d=4$

Single root at -4, so d=4
Double root at -1 and 2, so b and c are 1 and -2

$$48 = a(b)^2(c)^2(d)$$
$$48 = a\times(-2)^2(1)^2(4)$$
$$48 = a\times 16$$
$$a = 3$$

9 What is the coefficient of x^7 in the expansion of $\left(x+\frac{4}{x}\right)^{13}$?

A. 18 304

B. 45 760

C. 286

D. 64

$$\binom{13}{r}x^{13-r}\left(\frac{4}{x}\right)^r$$
$$13-r-r=7$$
$$6=2r$$
$$r=3$$
$$\binom{13}{3}x^{10}\left(\frac{4}{x}\right)^3 = 18304x^7$$

10 The gradient of the tangent to the curve $y = 2\cos\left(\frac{x}{2}\right)$ at the point where $x = \frac{\pi}{3}$ is $-\frac{1}{2}$.

What is the gradient on the curve where $x = \frac{11\pi}{3}$?

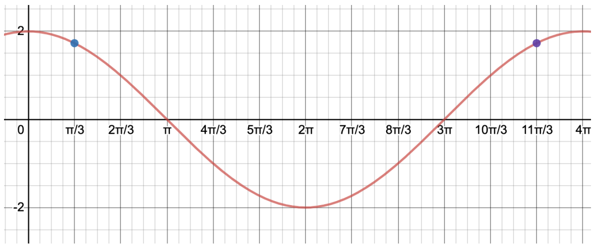
Hint: sketch the function.

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. 2



By the symmetry of the curve, it must be ½

Question	Answer
1	D
2	D
3	C
4	A
5	B
6	B
7	A
8	C
9	A
10	B

Section II

60 marks
Attempt Questions 11–13
Allow about 1 hour and 15 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.
In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet.

Question 11a	Marks
• Provides the correct solution	2
• Writes two inequalities based on the given inequality	1

(a) Solve $|3x-6| \geq 15$ **2**

$$3x-6 \geq 15 \qquad 3x-6 \leq -15$$
$$3x \geq 21 \quad \text{or} \quad 3x \leq -9$$
$$x \geq 7 \qquad x \leq -3$$

Question 11b	Marks
• Provides the correct solution	2
• Uses the binomial theorem, or equivalent merit	1

(b) Write the expansion of $(x-2y)^4$. **2**

$$^4C_0x^4 + {}^4C_1x^3(-2y) + {}^4C_2x^2(-2y)^2 + {}^4C_3x(-2y)^3 + {}^4C_4(-2y)^4$$
$$= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$$

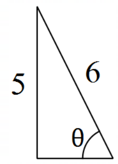
Question 11c	Marks
• Provides the correct solution	2
• Finds an angle that is a multiple of $\frac{\pi}{3}$.	1

(c) What is the exact value of $\tan^{-1}\left(2\sin\left(-\frac{2\pi}{3}\right)\right)$? **2**

$$\tan^{-1}\left(2\sin\left(-\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(2\times\frac{-\sqrt{3}}{2}\right)$$
$$= \tan^{-1}\left(-\sqrt{3}\right)$$
$$= -\frac{\pi}{3}$$

Question 11d	Marks
• Provides the correct solution	2
• Uses Pythagoras' Theorem to find the third side of the right triangle, or equivalent merit	1

(d) If $\sin\theta = \frac{5}{6}$ for $\frac{\pi}{2} \le \theta \le \pi$, what is the exact value of $\sin 2\theta$? **2**



$$\sqrt{6^2 - 5^2} = \sqrt{11}$$

$$\sin\theta = \frac{5}{6}, \ \cos\theta = -\frac{\sqrt{11}}{6},$$

$$\begin{aligned} \sin 2\theta &= 2\sin\theta\cos\theta \\ &= 2 \times \frac{5}{6} \times -\frac{\sqrt{11}}{6} \\ &= -\frac{5\sqrt{11}}{18} \end{aligned}$$

(e) Four Year 12 students, three Year 11 students, and two Year 10 students are seated around a circular table randomly.

Question 11e (i)	Marks
• Provides the correct solution	1

(i) How many seating arrangements are possible? **1**

$$8! = 40320$$

Question 11e (ii)	Marks
• Provides the correct solution	2
• Finds the number of ways each group can be arranged, or equivalent merit	1

(ii) What is the probability that the three groups are seated together? **2**

$$\begin{aligned} \frac{4! \times 3! \times 2! \times 2!}{40320} &= \frac{576}{40320} \\ &= \frac{1}{70} \end{aligned}$$

Question 11f (i)	Marks
• Provides the correct solution	2
• Uses $(\alpha+\beta+\gamma)^2$ or equivalent merit	1

(f) The polynomial $P(x)=x^3-10x^2-39x+180$ has roots α, β, γ.

(i) Find $\alpha^2+\beta^2+\gamma^2$. **2**

$$(\alpha+\beta+\gamma)^2=\alpha^2+\alpha\beta+\alpha\gamma+\alpha\beta+\beta^2+\gamma\beta+\alpha\gamma+\beta\gamma+\gamma^2$$

$$(\alpha+\beta+\gamma)^2=(\alpha^2+\beta^2+\gamma^2)+2(\alpha\beta+\alpha\gamma+\gamma\beta)$$

$$\left(\frac{-b}{a}\right)^2=(\alpha^2+\beta^2+\gamma^2)+2\left(\frac{c}{a}\right)$$

$$(10)^2=(\alpha^2+\beta^2+\gamma^2)+2\times-39$$

$$\alpha^2+\beta^2+\gamma^2=178$$

Question 11f (ii)	Marks
• Provides the correct solution	3
• Makes significant progress towards the solution	2
• Expresses each term as a fraction and attempts to use a common denominator	1

(ii) Find $(\alpha\beta)^{-2}+(\alpha\gamma)^{-2}+(\beta\gamma)^{-2}$. **3**

$$\begin{aligned}\frac{1}{(\alpha\beta)^2}+\frac{1}{(\alpha\gamma)^2}+\frac{1}{(\beta\gamma)^2}&=\frac{\gamma^2}{(\alpha\beta\gamma)^2}+\frac{\beta^2}{(\alpha\beta\gamma)^2}+\frac{\alpha^2}{(\alpha\beta\gamma)^2}\\&=\frac{\alpha^2+\beta^2+\gamma^2}{(\alpha\beta\gamma)^2}\\&=178\div\left(\frac{-d}{a}\right)^2\\&=178\times\frac{1}{32400}\\&=\frac{89}{16200}\end{aligned}$$

Question 12 (14 marks) Use the Question 12 Writing Booklet.

Question 12a	Marks
• Provides the correct proof	2
• Applies an appropriate trigonometric identity, or equivalent merit	1

(a) Show that $\sin\theta\sin\left(\frac{\pi}{2}-\theta\right)=\frac{1}{2}\cos\left(2\theta-\frac{\pi}{2}\right)$ **2**

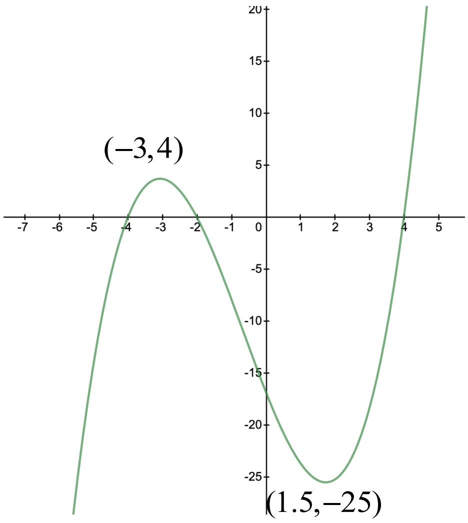
Method 1

$$\begin{aligned} LHS &= \sin\theta\sin\left(\frac{\pi}{2}-\theta\right) \\ &= \frac{1}{2}\times 2\sin\theta\cos\theta \\ &= \frac{1}{2}\sin 2\theta \\ &= \frac{1}{2}\cos\left(\frac{\pi}{2}-2\theta\right) \\ &= \frac{1}{2}\cos\left(2\theta-\frac{\pi}{2}\right) \\ &= RHS \end{aligned}$$

Method 2

$$\begin{aligned} LHS &= \sin\theta\sin\left(\frac{\pi}{2}-\theta\right) \\ &= \frac{1}{2}\left(\cos\left(\theta-\left(\frac{\pi}{2}-\theta\right)\right)-\cos\left(\theta+\left(\frac{\pi}{2}-\theta\right)\right)\right) \\ &= \frac{1}{2}\left(\cos\left(2\theta-\frac{\pi}{2}\right)-\cos\left(\frac{\pi}{2}\right)\right) \\ &= \frac{1}{2}\left(\cos\left(2\theta-\frac{\pi}{2}\right)-0\right) \\ &= \frac{1}{2}\cos\left(2\theta-\frac{\pi}{2}\right) \\ &= RHS \end{aligned}$$

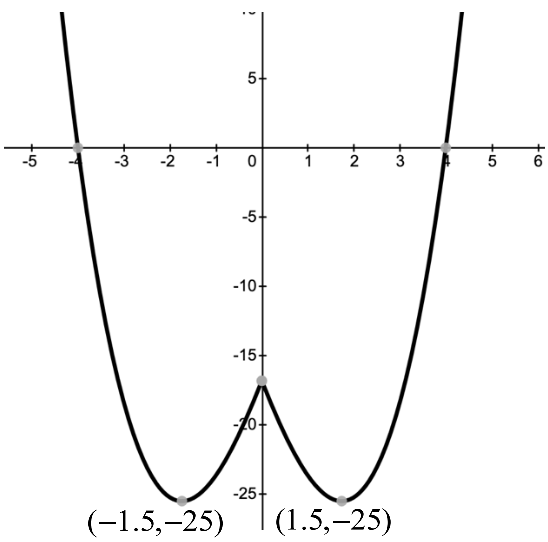
(b) The cubic function $y = f(x)$ is graphed below.



Question 12b (i)	Marks
• Provides the correct graph	2
• Graphs an absolute value graph based on the given graph	1

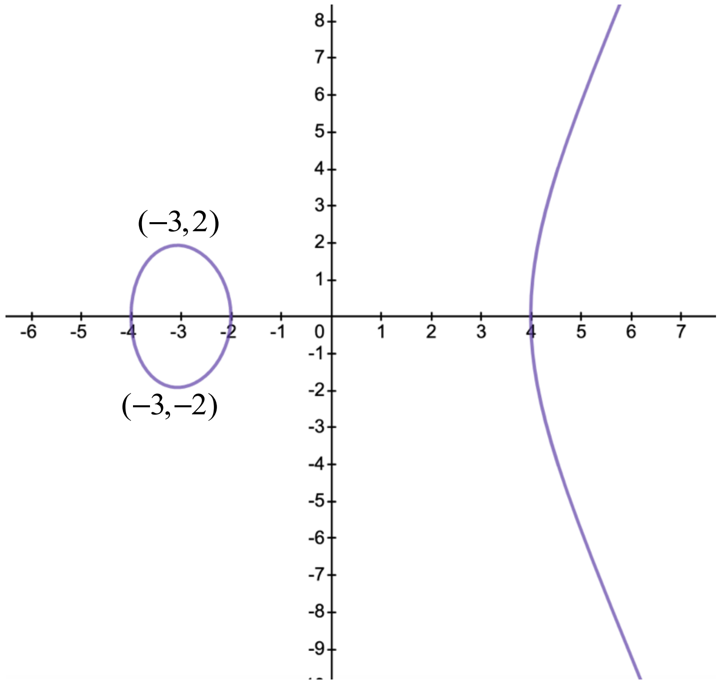
Draw separate one-third of a page sketches of the following functions, clearly showing any transformations of the turning points and *x*- and *y*-intercepts, as applicable.

(i) $y = f(\,|x|\,)$ **2**



Question 12b (ii)	Marks
• Provides the correct graph	2
• Some sections of the graph correct	1

(ii) $y^2 = f(x)$ **2**



(c) A particle is moving along the x-axis. Its position, x metres, at time t seconds is given by $x(t) = -\frac{t^3}{3} + \frac{11t^2}{2} - 28t + 2$ for $t \geq 0$.

Question 12c (i)	Marks
• Provides the correct solution	2
• Attempts to solve the first derivative equal to 0	1

(i) When is the particle at rest? **2**

$$v(t) = -t^2 + 11t - 28$$

$$t^2 - 11t + 28 = 0$$

$$(t-4)(t-7) = 0$$

$$t = 4\text{s}, t = 7\text{s}$$

Question 12c (ii)	Marks
• Provides the correct solution	1

(ii) For what values of t is the particle moving to the right? **1**

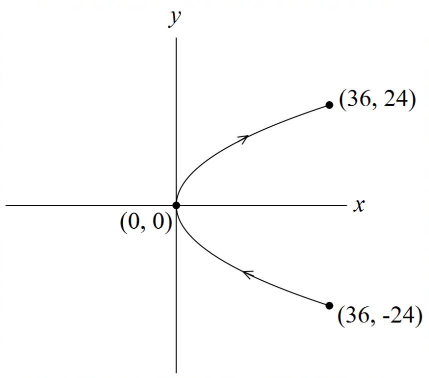
$$v(t) \geq 0$$

$$4\text{s} < t < 7\text{s}$$

(d) A curve in the xy-plane is defined parametrically by $x = 4t^2$ and $y = 8t$, where t is the parameter.

Question 12d (i)	Marks
• Provides the correct graph, with end points and turning point labelled	2
• Provides the correct shape of the graph or x and y reversed	1

(i) Draw the curve for $-3 \le t \le 3$, showing the beginning and ending coordinates and any intercepts. **2**



Question 12d (ii)	Marks
• Provides the correct solutions	3
• Provides the correct solutions in terms of x and y rather than t	2
• Develops a quadratic equation to find t	2
• Substitutes the parametric equations into the line, or equivalent merit	1

(ii) For what value or values of t will the parabola intersect the line $y = 6 - 2x$? **3**

$$t = \frac{y}{8}$$

$$x = 4 \times \left(\frac{y}{8}\right)^2$$

$$y^2 = 16x$$

$$t = \frac{y}{8}$$

$$t = \frac{6 - 2\left(4t^2\right)}{8}$$

$$8t = 6 - 8t^2$$

$$8t^2 + 8t - 6 = 0$$

$$2\left(4t^2 + 4t - 3\right) = 0$$

$$2\left(4t^2 + 6t - 2t - 3\right) = 0$$

$$2\left(2t(2t+3) - (2t+3)\right) = 0$$

$$2(2t+3)(2t-1) = 0$$

$$t = \frac{-3}{2}, t = \frac{1}{2}$$

Question 13 (15 marks) Use the Question 13 Writing Booklet.

Question 13a	Marks
• Provides the correct proof	2
• Correctly applies the Pigeonhole Principle, or equivalent merit	1

(a) Five points are scattered over a $2m$ by $2m$ square. Show that there is at least one pair of points which are a distance at most $\sqrt{2}\,m$ apart. **2**

Divide the square into four $1m \times 1m$ squares.

By the pigeon hole principle, at least one of the squares must contain 2 points. The furthest distance between the 2 points is the diagonal of the smaller square, which is $\sqrt{2}\,m$

(b) Given $t = \tan 15°$,

Question 13b (i)	Marks
• Provides a correct proof	1

(i) show that $\dfrac{2t}{1-t^2} = \dfrac{1}{\sqrt{3}}$. **1**

$$\tan 30 = \frac{2\tan 15}{1-\tan^2 15}$$

$$\frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$$

Question 13b (ii)	Marks
• Provides the correct solution	2
• Develops and attempts to solve a quadratic equation, or equivalent merit	1

(ii) Hence, find the exact value of $\tan 15°$. **2**

$$\frac{2t}{1-t^2} = \frac{1}{\sqrt{3}}$$

$$2\sqrt{3}t = 1 - t^2$$

$$t^2 + 2\sqrt{3}t - 1 = 0$$

$$t = \frac{-2\sqrt{3} \pm \sqrt{\left(2\sqrt{3}\right)^2 + 4}}{2}$$

$$= -\sqrt{3} \pm 2$$

(c) A bottle of soft drink has been taken out of a refrigerator and is initially $2°C$. The soft drink is placed on a counter in a room, where the ambient temperature is $28°C$.

After being in the room for 10 minutes, the temperature of the soft drink is $6°C$.

The change in temperature of the soft drink can be modelled by $\frac{dT}{dt} = k(T-28)$, where T is the temperature of the soft drink, t is the time in minutes after the soft drink is taken out of the refrigerator, and k is a constant.

Question 13c (i)	Marks
• Provides correct proof	1

(i) Show that $T = 28 + Ae^{kt}$, where A is a constant, satisfies $\frac{dT}{dt} = k(T-28)$. **1**

$$T = 28 + Ae^{kt}$$
$$\frac{dT}{dt} = kAe^{kt}$$
$$\frac{dT}{dt} = k\left(T-28\right)$$

Question 13c (ii)	Marks
• Provides the correct solution	2
• Finds A, or equivalent merit.	1

(ii) Show that $k = -0.0167 \text{ (3sf)}$ **2**

$$t = 0, T = 2$$
$$T = 28 + Ae^{kt}$$
$$2 = 28 + A$$
$$A = -26$$

$$t = 10, T = 6$$
$$6 = 28 - 26e^{k\times10}$$
$$-22 = -26e^{10k}$$
$$e^{10k} = \frac{11}{13}$$
$$10k = \ln\frac{11}{13}$$
$$k = \frac{1}{10}\ln\frac{11}{13}$$
$$= -0.0167 \text{ (3sf)}$$

Question 13c (iii)	Marks
• Provides the correct solution	2
• Finds the temperature at $t = 30$, or equivalent merit	1

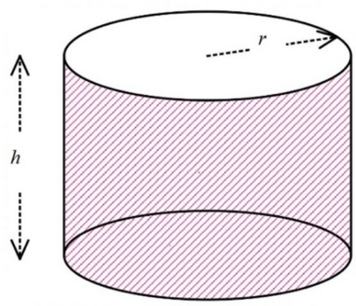
(iii) After 30 minutes, at what rate is the temperature of the soft drink changing? **2**

$$T = 28 - 26e^{-0.0167\times 30}$$
$$= 12.25$$
$$\frac{dT}{dt} = -0.0167(12.25 - 28)$$
$$= 0.263°C / \min$$

or

$$\frac{dT}{dt} = kAe^{kt}$$
$$= -0.0167 \times (-26)e^{-0.0167\times 30}$$
$$= 0.263°C / \min$$

(d) The solid shown below is a cylinder which has its height equal to its diameter.



A virtual 3-D model is created which maintains the ratio of height and diameter.
In an animation, the model is being scaled up so that its volume is increasing at a steady rate of 120 cm^3 per second.

Note: The volume of a cylinder is given by $V = \pi r^2 h$

Question 13d (i)	Marks
• Provides the correct solution with units	4
• Correctly uses $\frac{dV}{dt} = 120$	3
• Differentiates with respect to t or r correctly, or equivalent merit	2
• Substitutes $2r = h$ into volume formula	1

(i) Find the rate at which the radius is increasing when it reaches 2 cm. **4**

$$V = 2\pi r^2(2r)$$
$$= 2\pi r^3$$
$$\frac{dV}{dt} = 6\pi r^2 \frac{dr}{dt}$$
$$120 = 6\pi(2)^2 \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{5}{\pi}\,\text{cm s}^{-1}$$

or

$$\frac{dV}{dr} = 6\pi r^2$$
$$\frac{dV}{dr} \times \frac{dr}{dt} = \frac{dV}{dt}$$
$$6\pi(2)^2 \times \frac{dr}{dt} = 120$$
$$\frac{dr}{dt} = \frac{5}{\pi}\,\text{cm s}^{-1}$$

Question 13d (ii)	Marks
• Provides the correct solution	1

(ii) What is the radius when $\dfrac{dr}{dt} = \dfrac{80}{\pi}$ cm s^{-1}? **1**

$$V = 2\pi r^3$$
$$\frac{dV}{dt} = 6\pi r^2 \frac{dr}{dt}$$
$$120 = 6\pi r^2 \frac{80}{\pi}$$
$$120 = 6r^2 \times 80$$

or

$$V = 2\pi r^3$$
$$\frac{dV}{dr} = 6\pi r^2$$
$$\frac{dV}{dr} = \frac{dV}{dr} \times \frac{dr}{dt}$$
$$120 = 6\pi r^2 \times \frac{80}{\pi}$$
$$120 = 6r^2 \times 80$$

$$r^2 = \frac{1}{4}$$
$$r = \frac{1}{2} cm$$

(can't be negative)

End of paper

Mapping Grid

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
1	1	ME-A1 Working with Combinatorics	ME11-5	E2
2	1	ME-F1 Further Work with Functions	ME11-2	E2
3	1	ME-T1 Inverse Trigonometric Functions	ME11-3	E2
4	1	ME-F1 Further Work with Functions	ME11-2	E3
5	1	ME-T1 Inverse Trigonometric Functions	ME11-3	E2
6	1	ME-F2 Polynomials	ME11-2	E3
7	1	ME-C1 Rates of Change	ME11-4	E3
8	1	ME-F2 Polynomials	ME11-2	E3
9	1	ME-A1 Working with Combinatorics	ME11-5	E3
10	1	MA-C1 Introduction to Differentiation MA-T1 Trigonometry and Measure of Angles	ME11-1	E4
11a	2	ME-F1 Further Work with Functions	ME11-2	E2
11b	2	ME-A1 Working with Combinatorics	ME11-5	E2
11c	2	ME-T1 Inverse Trigonometric Functions	ME11-3	E2-E3
11d	2	ME-T2 Further Trigonometric Identities	ME11-3	E2-E3
11e (i)	1	ME-A1 Working with Combinatorics	ME11-5	E2
11e (ii)	2	ME-A1 Working with Combinatorics MA-S1 Probability and Discrete Probability Distributions	ME11-5	E2-E3
11f (i)	2	ME-F2 Polynomials	ME11-2	E3
11f (ii)	3	ME-F2 Polynomials	ME11-2	E3
12a	2	ME-T2 Further Trigonometric Identities	ME11-3	E2-E3
12b (i)	2	ME-F1 Further Work with Functions	ME11-1 ME11-2	E2-E3
12b (ii)	2	ME-F1 Further Work with Functions	ME11-1 ME11-2	E2-E3
12c (i)	2	ME-C1 Rates of Change	ME11-4	E2
12c (ii)	1	ME-C1 Rates of Change ME-F1 Further Work with Functions	ME11-4 ME11-7	E2-E3
12d (i)	2	ME-F1 Further Work with Functions	ME11-2	E2-E3
12d (ii)	3	ME-F1 Further Work with Functions	ME11-2	E3
13a	2	ME-A1 Working with Combinatorics	ME11-1 ME11-5 ME11-7	E3-E4
13b (i)	1	ME-T2 Further Trigonometric Identities	ME11-3	E2
13b (ii)	2	ME-T2 Further Trigonometric Identities	ME11-1 ME11-3	E2-E3
13c (i)	1	ME-C1 Rates of Change	ME11-4	E2
13c (ii)	2	ME-C1 Rates of Change	ME11-4	E3
13c (iii)	2	ME-C1 Rates of Change	ME11-4	E3
13d (i)	4	ME-C1 Rates of Change	ME11-1 ME11-4	E2-E4
13d (ii)	1	ME-C1 Rates of Change	ME11-4	E2

2020 Year 11 Assessment Task 3 **Student Name: ANSWERS**

Mathematics Extension

Section I Multiple-Choice Answer Sheet

1	A ○	B ○	C ○	D ●
2	A ○	B ○	C ○	D ●
3	A ○	B ○	C ●	D ○
4	A ●	B ○	C ○	D ○
5	A ○	B ●	C ○	D ○
6	A ○	B ●	C ○	D ○
7	A ●	B ○	C ○	D ○
8	A ○	B ○	C ●	D ○
9	A ●	B ○	C ○	D ○
10	A ○	B ●	C ○	D ○