QUESTION 1 (15 MARKS)

Marks

(a) Given one zero of the polynomial $P(x) = x^3 - 2x^2 - ax + 6$ occurs at $x = 3$

(i) Find the value of a **1**

(ii) Hence, find the value of the other two zeros of the polynomial. **2**

(b) A box contains 7 identical coloured discs of which 2 are green, 2 are yellow and 3 are red.

(i) Find the total number of arrangements of all the discs. **1**

(ii) If the discs are put in a line, how many arrangements do <u>not</u> have a red disc at each end of the line? **2**

(c) Differentiate with respect to x the following:

(i) $e^x \sin^2 x$ **2**

(ii) $log_{10}(3x + 2)$ **2**

(d) Evaluate: **2**

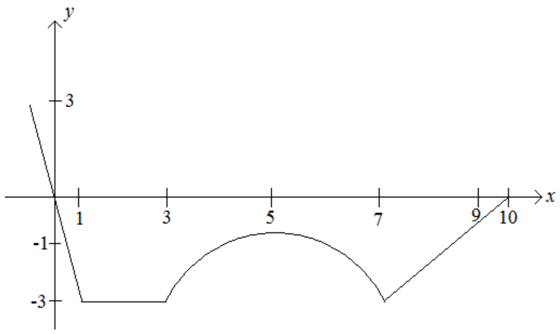
$$\lim_{x \to 0} \left(\frac{\sin 3x}{\tan \frac{x}{3}} \right)$$

(e) Simplify $\dfrac{\tan 75° - \tan 15°}{\tan 75° + \tan 15°}$ leaving your answer in surd form. **3**

QUESTION 2 (15 MARKS) Start a new page

Marks

(a) Given the graph of $y = f(x)$ which consists of three straight lines and a section of a concave down parabola for $3 \leq x \leq 7$.



Draw separate sketches of:

(i) $y = f'(x)$ **3**

(ii) $y = f''(x)$ **2**

(b) A ten-member Committee consist of 5 students, 3 teachers and 2 parents. The Committee meets around a circular table.

(i) How many different arrangements of the 10 members around the table are possible if the students sit together as a group and so do the teachers, but no teacher sits next to a student? **2**

(ii) One student and one parent are related. Given that all arrangements in part (i) are equally likely, find the number of ways these two members can sit next to each other. **3**

(c) Using the fact that $\cos(180 - x)° = -\cos x°$, find the general solution of $\sin 2x° + \cos x° = 0$. **3**

(d) Given *P(x)* is a polynomial and when divided by $x^2 - 4$ the remainder is $5x - 2$. Find the remainder when *P(x)* is divided by $+2$. **2**

QUESTION 3 (15 MARKS) Start a new page

Marks

(a) The angle of elevation of a hill *OA*, at the point *P* due south of the hill, is 37°. From *Q*, due west of *P*, the angle of elevation is 26° and the distance *PQ* is 5 km.

(i) Draw a neat diagram showing all the above information. **1**

(ii) Find the height of the hill to the nearest 10 metres. **3**

(b) Find the value of *n* so that:

$${}^{n}C_3 - 2\ .\ {}^{n+1}C_2 = {}^{n}C_1$$ **3**

(c) Using the '*t*' method, where $t = \tan\frac{x}{2}$ and $0° \leq x \leq 360°$, find the values of x when $y = -2$ in the equation $y = 2\cos x° - 3\sin x°$. Give your answer correct to the nearest minute. **3**

(d) *L* and *M* are midpoints of arcs *AB* and *AC* respectively of a circle. These arcs do not overlap. *LM* cuts the lines *AB* and *AC* in *E* and *F*.

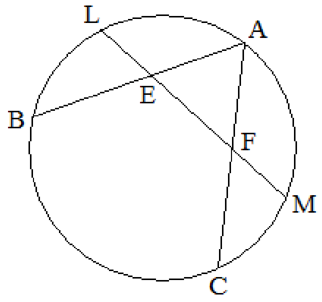


Diagram not to scale

Draw the diagram on your answer page

(i) Show that $\angle ABM + \angle LMB = \angle ACL + \angle CLM$ **2**

(ii) Hence, or otherwise show that $AE = AF$. **3**

QUESTION 4 (15 MARKS) Start a new page

Marks

(a) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$

(i) If the angle that chord PQ makes with the x-axis is $45°$, show that $p + q = 2$ **2**

(ii) Find the equation of the locus of M, the midpoint of chord PQ as it moves about the parabola. **2**

(iii) Clearly state any restrictions on the locus of M. **2**

(b) If A is an angle such that $\tan A + \sec A = 2$, evaluate $\cos A$. **3**

(c) A sheet metal fabricator wants to make a container in the shape of a right triangular prism that has a fixed volume V cubic units. The cross-section of the prism is an equilateral triangle of side length a, and the length of the prism is b.

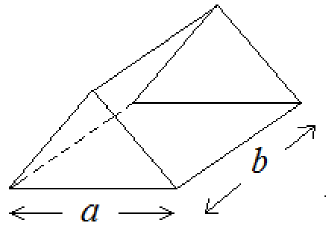


Diagram not to scale

(i) Show that the height of the triangular face is $\frac{\sqrt{3}}{2}\,a$ units. **1**

(ii) Show that the total surface area of the prism is a minimum when **5**

$a = (4V)^{\frac{1}{3}}$ and $b = \dfrac{(4V)^{\frac{1}{3}}}{\sqrt{3}}$.

END OF PAPER

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
a) (i) $P(x) = x^3 - 2x^2 - ax + 6$ $P(3) = 0$ $0 = 27 - 18 - 3a + 6$ $3a = 15$ $a = 5$	1	$\frac{1}{2}$ method. $\frac{1}{2}$ correct answer
(ii) $(x-3)\,\overline{)\,x^3 - 2x^2 - 5x + 6}$ quotient $x^2 + x - 2$ $x^3 - 3x^2$ $x^2 - 5x$ $x^3 - 3x$ $-2x + 6$ $-2x + 6$ 0 $P(x) = (x-3)(x+2)(x-1)$ $\therefore$ other zeros $x = -2$ or 1	2	1 for division $\frac{1}{2} + \frac{1}{2}$ for each zero.
b) (i) No of arrangements $= \dfrac{7!}{3!\,2!\,2!} = 210$		
(ii) Arrangements with red on ends 5 places in between 1R, 2G, 2Y No of ways $= \dfrac{5!}{2!\,2!} = 30$ No of ways no red on both ends $= 210 - 30$ $= 180$ ways.		
c) (i) $y = e^x \sin^2 x$ $\dfrac{dy}{dx} = 2e^x \sin x \cos x + e^x \sin^2 x$	2	1 + 1 for each part of derivative no marks deducted for incorrect simplifying
(ii) $y = \log_{10}(3x+2) = \dfrac{\log_e(3x+2)}{\log_e(10)}$ $\dfrac{dy}{dx} = \dfrac{3}{(3x+2)\log_e 10}$	2	$\frac{1}{2}$ change of base but not necessary to show. 1 $\dfrac{3}{3x+2}$ 1 $\dfrac{1}{\log_e 10}$
d) $\lim\limits_{x \to 0}\left(\dfrac{\sin 3x}{\tan \frac{x}{3}}\right) = \lim\limits_{x \to 0}\left[\dfrac{\sin 3x}{3x} \times \dfrac{3x}{x/3} \times \dfrac{\frac{x}{3}}{\tan\frac{x}{3}}\right]$ $= 1 \times 3 \times 3 \times 1$ $= 9$	2	1 expansion in limit 1 showing $1 \times 3 \times 3 \times 1$ Answer only $\frac{1}{2}$
e) $\dfrac{\tan 75^\circ - \tan 15^\circ}{\tan 75 + \tan 15} = \dfrac{\frac{\sin 75}{\cos 75} - \frac{\sin 15}{\cos 15}}{\frac{\sin 75}{\cos 75} + \frac{\sin 15}{\cos 15}}$ $= \dfrac{\sin 75 \cos 15 - \cos 75 \sin 15}{\sin 75 \cos 15 + \cos 75 \sin 15} = \dfrac{\sin(75-15)}{\sin(75+15)} = \dfrac{\sin 60}{\sin 90} = \dfrac{\sqrt{3}}{2}$.	3	1 for each step. Answer only = 1

other methods possible

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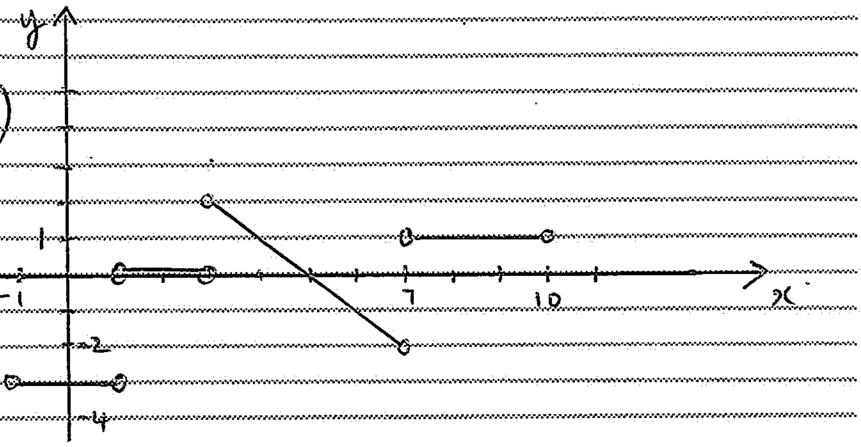
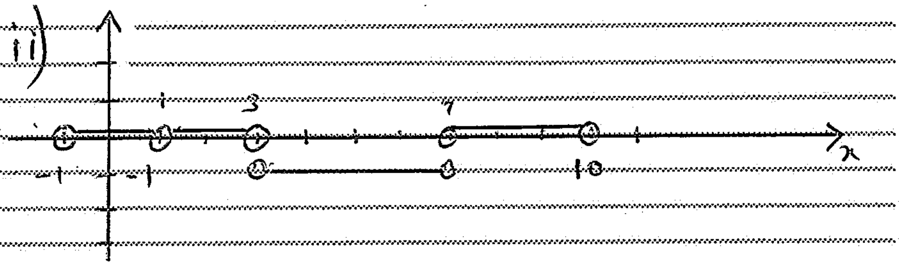
MATHEMATICS Extension 1 : Question 2

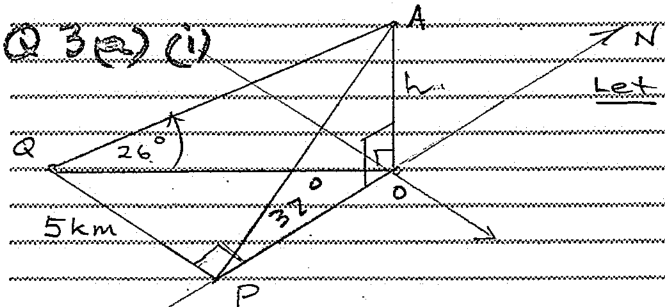
Suggested Solutions	Marks	Marker's Comments
a) i)  ii) 	1½ shape 1 value ½ ends 1 shape ½ value ½ ends	Many did not appreciate "parabola" means degree 2. (I didn't worry about far end points as it was slightly ambiguous). There were silly number of blank spaces where $f'=0$.
b) i) Place one parent at table Student Block can be placed in 2 ways. Students can be mixed in $5!$ Positions of other parent & teacher fixed Teachers can be mixed in $3!$ $\therefore 2\times 5!\times 3! = \underline{\underline{1440 \text{ ways}}}$ ii) Place related parent in 1 place. Student block can be placed in 2 ways Related student placed at end Other student placed in $4!$ ways Positions of other parent & teacher fixed Teachers can be fixed in $3!$ ways $\therefore 2\times 4!\times 3! = \underline{\underline{288 \text{ ways}}}$ Alternatively Result (i) is varied only by taking the related student and placing them next to their parent. The other students can mix in $4!$ (instead of $5!$) Thus divide answer by 5. $1440 \div 5 = \underline{\underline{288}}$	2 3	Correct answer = 2 Wrong answer = 0 unless some reasoning given. (This is more than a couple of factorials). If the second answer is first divide by 5, then some cfe marks may be awarded.

MATHEMATICS Extension 1 : Question 2 (cont)

Suggested Solutions	Marks	Marker's Comments
c) $\sin 2x = -\cos x^\circ$ $= \cos(180^\circ - x^\circ)$ $= \sin(90^\circ-(180-x))$ $= \sin(x^\circ - 90^\circ)$ $2x = 180^\circ n + (-1)^n(x-90^\circ)$ $x(2-(-1)^n) = 180^\circ n \quad (-1)^n 90^\circ$ $x = \dfrac{180^\circ n - (-1)^n 90^\circ}{(2-(-1)^n)} \quad n \in \mathbb{Z}$	3	The "hint" made it more difficult. ½ marks were docked for i) not using hint ii) including radians iii) not putting $n \in \mathbb{Z}$
Altenatively (for 2½ marks only) $\sin 2x + \cos x = 0$ $2\sin x \cos x + \cos x = 0$ $\cos x(2\sin x + 1) = 0$ $\cos x = 0$ or $\sin x = -\frac{1}{2}$ $x = 360^\circ n \pm 90^\circ$ or $x = 180^\circ m + (-1)^m(-30^\circ)$ $n \in \mathbb{Z}$ $\quad$ $m \in \mathbb{Z}$	(2½)	
d) $P(x) = (x^2-4)Q(x) + 5x - 2$ Remainder when divide by $(x+2)$ is $P(-2)$ by Remainder Theorem $P(-2) = 0 + 5(-2) - 2$ $= -12$	2	Many people wrote a lot here and got no marks. There were very few part marks given.

Y11 M. EXT1 PRELIMINARY TEST, 2011

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
Q 3(a) (i)		$\frac{1}{2}$ For $\angle OPQ = 90^\circ$
		$\frac{1}{2}$ For Diagram with $\angle OQA = 26^\circ$, $\angle OPA = 37^\circ$
Let $OA = h$ km		$\boxed{1}$
(ii) In $\triangle OPA$: $\tan 37^\circ = \frac{OA}{OP}$		
$\therefore OP = \frac{OA}{\tan 37^\circ} \equiv h\cot 37^\circ = h\tan 53^\circ$		$\frac{1}{2}$
In $\triangle OAQ$: $OQ = \frac{h}{\tan 26^\circ} \equiv h\cot 26^\circ = h\tan 64^\circ$		
$OP^2 + PQ^2 = OQ^2$ (Pythag. Thm)	$\frac{1}{2}$	$\boxed{3}$
$\therefore h^2\tan^2 53^\circ + 5^2 = h^2\tan^2 64^\circ$		
So $h^2[\tan^2 64^\circ - \tan^2 53^\circ] = 5^2$		
$h^2 = \frac{5^2}{[\tan^2 64^\circ - \tan^2 53^\circ]} = \frac{5^2}{[\cot^2 26^\circ - \cot^2 37^\circ]}$		$= \frac{5^2}{\frac{1}{\tan^2 26^\circ} - \frac{1}{\tan^2 37^\circ}} = \frac{5^2\tan^2 26^\circ \tan^2 37^\circ}{\tan^2 37^\circ - \tan^2 26^\circ}$
$h^2 = 10.2345\,8536\ldots$		
$\therefore h = 3.1991\,5385\ldots$ in km	$\frac{1}{2}$	IF $\angle POQ = 90^\circ$
$3199.15385\ldots$ in m		2 – $3\frac{1}{2}$ Mks
$\therefore$ height $= 3200$ m // 3.20 km	$\frac{1}{2}$	"height = 2050 m"
(b) ${}^{n}C_3 - 2\cdot{}^{n+1}C_2 = {}^{n}C_1$		
$n \geq 3$ $\quad n+1 \geq 2$ $\quad n \geq 1$ $\Rightarrow n \geq 3$		$\frac{1}{2}$ For at least 2 correct
$\frac{n!}{3!(n-3)!} - 2\times\frac{(n+1)!}{2!(n-1)!} = \frac{n!}{(n-1)!}$		(NOT UNDERSTAND HOW TO WORK WITH FACTORIALS !!!!)
SIMPLIFY TO:		
$\frac{n(n-1)(n-2)}{6} - (n+1)n = n$	1	
$n(n-1)(n-2) - 6n(n+1) = 6n$		
$n[(n-1)(n-2) - 6(n+1) - 6] = 0$		$\boxed{3}$
$n[n^2 - 3n + 2 - 6n - 6 - 6] = 0$		
$n[n^2 - 9n - 10] = 0$		
$n(n+1)(n-10) = 0$	1	
$n = 0, -1$ or 10		
but $n \geq 3$		
$\therefore n = 10$ only	$\frac{1}{2}$	

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Suggested Solutions	Marks	Marker's Comments

Q3 (c) $y = 2\cos x - 3\sin x \quad 0^\circ \le x \le 360^\circ$

As $y = -2$

$\therefore 2\cos x - 3\sin x = -2$

TEST $x = 180^\circ$

LHS $= 2\cos 180^\circ - 3\sin 180^\circ$

$= 2(-1) - 3\times 0$

$= -2$

$=$ RHS

$\therefore x = 180^\circ$ — 1 — 1 For testing 180° is a solution

Using t-results

$$\frac{2(1-t^2)}{1+t^2} - 3\times\frac{2t}{1+t^2} = -2$$

$\therefore 2 - \cancel{2t^2} - 6t = -2(1+t^2) = -2 - \cancel{2t^2}$

$2 - 6t = -2$

$\therefore 6t = 4$

$t = \frac{4}{6} = \frac{2}{3}$ ✓ — 1

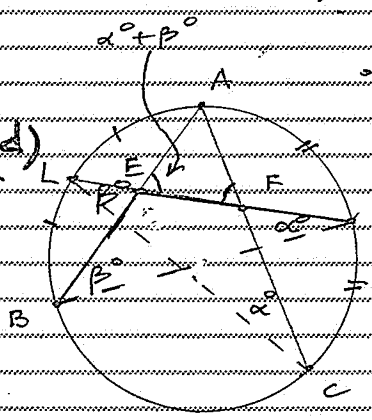
ie $\tan\frac{x}{2} = \frac{2}{3}$ ✓ $0^\circ \le \frac{x}{2} \le 180^\circ$

$\therefore \angle\frac{x}{2} = 33^\circ 41' 24.24..$

$\therefore x = 67^\circ 23'$ or 180° ✓ — 1 For $67^\circ 23'$, $67^\circ 22'$

$\boxed{3}$

(d)



(i) arc BL $= \overset{\frown}{BL} = \overset{\frown}{LA}$
and $\overset{\frown}{AM} = \overset{\frown}{MC}$

$\therefore \angle BML = \angle LCA = \alpha^\circ$ (Equal angles at the circumference standing on equal arcs: BL and BL, AM + MC) — 1

Similarly $\angle ABM = \angle MBC = \beta^\circ$ — $\frac{1}{2}$

$\therefore \angle ABM + \angle LMB = \beta + \alpha = \angle CLM + \angle ACL$ — $\frac{1}{2}$ (Angle addition)

$\boxed{2}$

(ii) THE BEST:

$\angle AEF = \alpha^\circ + \beta^\circ$ (Exterior angle of $\triangle EBM$ equals sum of interior opposite angles) — 1

Similarly $\angle AFE = \alpha^\circ + \beta^\circ$ — $\frac{1}{2}$

$\therefore \angle AEF = \angle AFE = \alpha^\circ + \beta^\circ$ — $\frac{1}{2}$

$\therefore AE = AF$ (equal sides are opposite equal Angles in $\triangle AEF$) — 1

$\boxed{3}$

OTHER: using similar $\triangle$s $\triangle BEM \,|||\, \triangle LFC$ with straight angles at E and F // Angle sum of $\triangle$s

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

(a)(i) $M_{PQ} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$

$= \dfrac{a(p-q)(p+q)}{2a(p-q)}$

$= \dfrac{p+q}{2}$ — (1)

now $\tan 45^\circ = 1 = m$ (as $m = \tan\theta$)

$\therefore 1 = \dfrac{p+q}{2}$

$\therefore p+q = 2$ — (1)

a lot of students did not write that $\tan 45^\circ = 1$

(ii) $M\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{2ap+2aq}{2}, \dfrac{ap^2+aq^2}{2}\right)$

$= \left(a(p+q), \frac{a}{2}(p^2+q^2)\right)$ — (1)

now $p+q = 2$ from (i)

$\therefore x = 2a$

$\therefore$ the locus is the straight line $x = 2a$ — (1)

If they subbed $x = 2a$ into y and got a parabola $\rightarrow$ 0 marks

(iii) M lies inside the parabola — (½)

$\therefore 4ay > x^2$

$4ay > (2a)^2$ — (1)

$4ay > 4a^2$

$y > a$ is the restriction — (½)

(b) $\tan A + \sec A = 2$

Method 1

$\dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A} = 2$ $(\cos A \neq 0)$ — (½)

$\sin A + 1 = 2\cos A$

$(\sin A + 1)^2 = (2\cos A)^2$

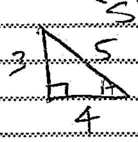
$\sin^2 A + 2\sin A + 1 = 4\cos^2 A$

$\sin^2 A + 2\sin A + 1 = 4 - 4\sin^2 A$ — (½)

$5\sin^2 A + 2\sin A - 3 = 0$

$(5\sin A - 3)(\sin A + 1) = 0$ — (½)

$\sin A = 3/5$ or $\sin A = -1$ — (½)



$\therefore \cos A = 4/5$ (½) or $\cos A = 0$ but $\cos A \neq 0$ as $\sec A$ is undefined

$\therefore \cos A = 4/5$ only. (½)

2/4

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

(b) Method 2

using Auxillary angle method

$\therefore \sin A - 2\cos A = -1$

$\sin A - 2\cos A \equiv \sqrt{5}\sin(A + 296°34')$ (1)

$\sqrt{5}\sin(A + 296°34') = -1$

$A = 36°52'$ or $270°$ (1)

$\therefore \cos A = \frac{4}{5}$ or $\cos A = 0$ (1/2)

$\cos A = \frac{4}{5}$ only as $\frac{1}{0}$ is undefined.

Method 3

using the t-results

$\sin A - 2\cos A = -1$

$\frac{2t}{1+t^2} - 2\left(\frac{1-t^2}{1+t^2}\right) = -1$

$2t - 2 + 2t^2 = -1 - t^2$

$2t - 2 + 2t^2 + 1 + t^2 = 0$

$3t^2 + 2t - 1 = 0$

$(3t-1)(t+1) = 0$ (1)

$t = \frac{1}{3}$ or $t = -1$ (1)

$\cos A = \frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} = \frac{8/9}{10/9} = \frac{8}{10} = \frac{4}{5}$ (1/2)

(1/2) or $\cos A = \frac{1-1}{1+1} = 0$ but $\cos A \neq 0$ as $\sec A$ is undefined

(c) (i) In $\triangle ACD$

$\angle ACD = 60°$ (all angles in an equilateral triangle are 60°)

$AD \perp CD$ (altitude in a triangle)

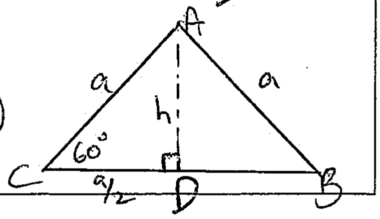
$CD = BD$ (altitude in an equilateral triangle bisects the base) (1/2)

$AD^2 + CD^2 = AC^2$ (by Pythagoras)

$\therefore h^2 = -\left(\frac{a}{2}\right)^2 + a^2$

$h^2 = \frac{3}{4}a^2$ (1/2)

$h = \frac{\sqrt{3}}{2}a$ units as $h > 0$



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MATHEMATICS Extension 1 : Question 4		
Suggested Solutions	**Marks**	**Marker's Comments**

(ii) $V = \frac{1}{2}ahH$

$= \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a \times b$

$\therefore V = \frac{\sqrt{3}a^2b}{4}$ — (1/2)

$S.A. = 2 \times \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a + 3 \times a \times b$

$= \frac{\sqrt{3}a^2}{2} + 3ab$ — (1/2)

now $b = \frac{4V}{\sqrt{3}a^2}$

$\therefore SA = \frac{\sqrt{3}a^2}{2} + 3a\frac{4V}{\sqrt{3}a^2}$ — (1/2)

$\therefore A = \frac{\sqrt{3}a^2}{2} + \frac{4\sqrt{3}V}{a}$ as $a>0$ — (1/2)

$\frac{dS}{dA} = \sqrt{3}a + \frac{-12V}{\sqrt{3}a^2}$ — (1/2)

Stationary points when $\frac{dS}{dA} = 0$ — (1/2)

$\therefore \sqrt{3}a - \frac{12V}{\sqrt{3}a^2} = 0$

$\sqrt{3}a = \frac{12V}{\sqrt{3}a^2}$

$3a^3 = 12V$

$a^3 = 4V$

$a = (4V)^{1/3}$ — (1/2)

when $a = (4V)^{1/3}$ $b = \frac{4V}{\sqrt{3}(4V)^{2/3}}$

$= \frac{4V}{\sqrt{3}\,4^{2/3}V^{2/3}}$

$= \frac{(4V)^{1/3}}{\sqrt{3}}$ — (1/2)

$\frac{d^2S}{dA^2} = \sqrt{3} + \frac{24V}{\sqrt{3}a^3}$

when $a = (4V)^{1/3}$ $\frac{d^2S}{dA^2} = \sqrt{3} + \frac{24V}{\sqrt{3}(4V)}$

$= \sqrt{3} + 2\sqrt{3}$

$= 3\sqrt{3}$

>0 $\therefore$ concave up

$\therefore$ relative minimum at $a = (4V)^{1/3}$ — (1)

$\therefore$ Minimum Surface area occurs when $a = (4V)^{1/3}$ and $b = \frac{(4V)^{1/3}}{\sqrt{3}}$

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MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

Or, Since the function, $\frac{ds}{da}$ are continuous for $a>0$, test the nature of the stationary point using the first derivative.

a	$v^{1/3}$	$(4v)^{1/3}$	$(4v)^{1/2}$
$\frac{ds}{dA}$	$\sqrt{3}v^{1/3} - 4\sqrt{3}v^{1/3} = -3\sqrt{3}v^{1/3}$	0	$2\sqrt{3}v - \sqrt{3}$
slope	\	—	/

$\therefore$ relative minimum at $a = (4v)^{1/3}$

If they had no numbers they got zero marks.