KILLARA HIGH SCHOOL

PRELIMINARY COURSE FINAL EXAMINATION 2001

EXTENSION 1 MATHEMATICS

Time allowed : 2 Hours

*All questions may be attempted
*Show all necessary working
*Board approved calculators may be used
*START EACH QUESTION ON A NEW PAGE

Question 1

a. Solve i. $\frac{3}{|x+1|} < 1$ ii. $\frac{3}{x+1} < 1$ (6)

b. Given that P is the point (-3,4) and Q is (1,1), find the coordinates of the point which divides the interval PQ
i. internally in the ratio 3:2 (2)
ii. externally in the ratio 3:2 .

c. Find the angle between the lines $x - 3y + 17 = 0$ and $2x + 5y - 1 = 0$.

d. i.. For the parabola $-12y = (x-5)^2$ find
ii. the coordinates of the focus (3)
iii. the focal length
iv. the equation of the directrix .

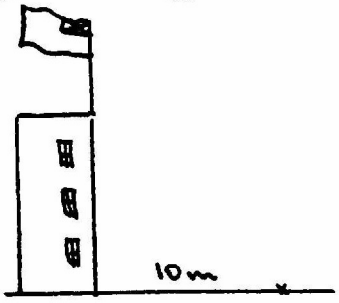
Question 2

a. Find the derivative of
i. $(2x-3)^8$
ii. $\sqrt{x^2+4}$ (8)
iii. $\frac{4x-3}{2x+5}$
iv. Show that the derivative of $(x+1)^3(x+2)^4$ is $(7x+10)(x+1)^2(x+2)^3$.

b. Given that A is the point (-2,0), B is (4,0) and P(x,y) moves such that PA is perpendicular to PB,
i. show that the locus of P is a circle. (5)
ii. Find the coordinates of the centre and the radius of the circle.

Question 3

a. An observer standing 10 metres from the base of a building, and on the same horizontal level as the base, observes the angle of elevation of the bottom and top of a flagpole, on top of the building, to be 42° and 54° respectively. Find the height of the flagpole. (3)



b. Prove that $\cot\theta - \cot 2\theta = \operatorname{cosec} 2\theta$. (4)

c. Express $4\cos\theta + 3\sin\theta$ in the form $A\sin(\theta - \partial)$ where A>0. (3)

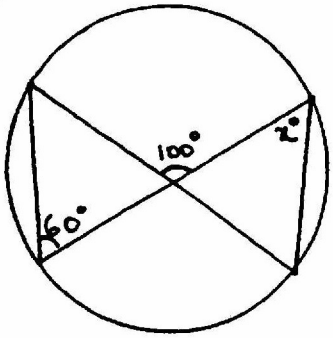
d. Using the 't' formulae (where $t = \tan\frac{x}{2}$), solve the equation (4)

$\sin x + \cos x = 1$ for $0° \leq x \leq 360°$.

Question 4

a. Find the value of the pronumerals in each of the following diagrams. (O is the center of the circle.)

i.



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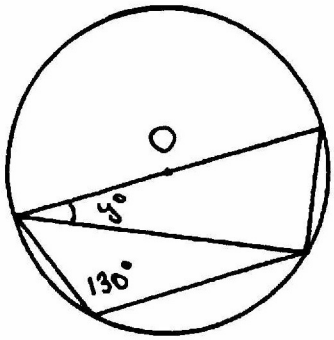
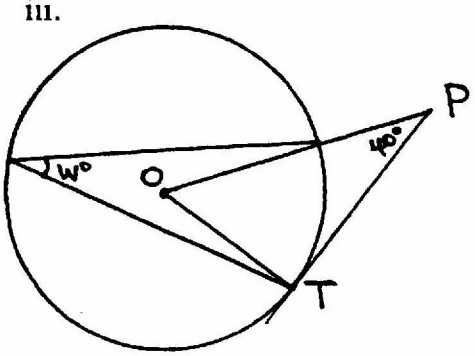
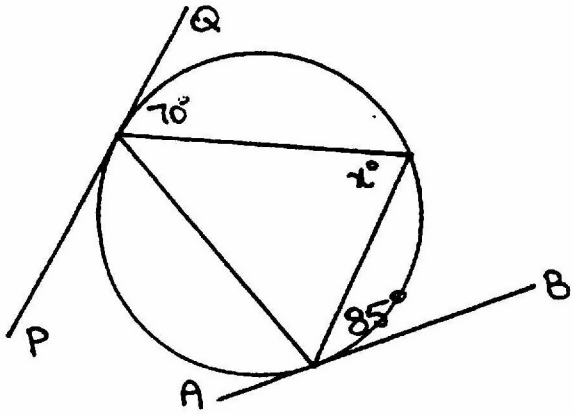


iii.



PT is a tangent.

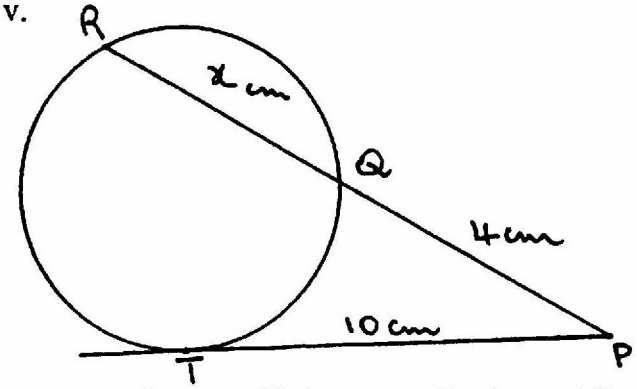
iv.



AB, PQ are tangents.

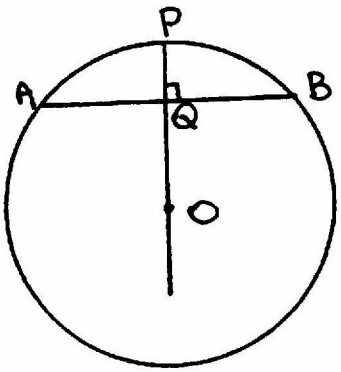
(5)

v.



b. In the diagram, OP is perpendicular to AB. If AB = 14 cm and PQ = 1 cm, find the radius of the circle.

PT is a tangent.



(2)

c. ABC is an isosceles triangle with AB=AC. P and Q are points on AB and AC respectively such that PQ is parallel to BC. Prove that the quadrilateral PBCQ is cyclic. (3)

Question 5

a. Find the quotient and remainder when $2x^3 - 4x^2 + 7x - 1$ is divided by $x - 5$. (2)

b. When $x^3 + 3x^2 - bx - 7$ is divided by $x - 2$ the remainder is 20. Find the value of b. (2)

c. Find all the zeros of the polynomial $P(x) = x^3 - 2x^2 - 11x + 12$. (3)

d. Draw a neat sketch of the polynomial function $y = (x+2)^2(x-1)^3$. (3)

a. If $x^3 - 8x^2 + kx - 12 = 0$ has one root equal to the sum of the other two, find the value of k. (4)

Question 6

a. $P(2p, p^2)$ and $Q(2q, q^2)$ are points on the parabola $x^2 = 4y$.

i. Find the equation of the chord PQ.

ii. If PQ passes through the point (0,-1), show that $pq = 1$.

iii. Write down the equation of the chord of contact of tangents to the parabola from the point (4,-12). (6)

iv. Prove that the chord of contact of tangents drawn from any point on the line y = -1 is a focal chord.

b. The tangent to the parabola at a point P on it, is perpendicular to the tangent at a point Q. If the tangents intersect at A, find the locus of A. (4)

END OF EXAMINATION

Extension 1 Mathematics - Solutions

Question 1

a. i. $\frac{3}{|x+1|} < 1$

$3 < |x+1|$ ✓

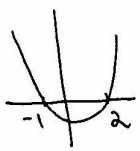
$x+1 > 3$ or $x+1 < -3$ ✓

$x > 2$ or $x < -4$ ✓

ii $\frac{3}{x+1} \times (x+1)^2 < (x+1)^2$ ✓

$3(x+1) < (x+1)^2$

$0 < (x+1)(x-2)$ ✓



$x < -1$ or $x > 2$ ✓

b. i. P(-3,4) Q(1,1)

3:2

$\Rightarrow \left(\frac{-6+3}{5}, \frac{8+3}{5}\right)$

ie. $\left(-\frac{3}{5}, \frac{11}{5}\right)$ ✓

ii. P(-3,4) Q(1,1)

3:-2

$\Rightarrow \left(\frac{6+3}{3-2}, \frac{-8+3}{3-2}\right)$

ie. $(9,-5)$ ✓

c. $x - 3y + 17 = 0 \Rightarrow m_1 = \frac{1}{3}$ ✓

$2x + 5y - 1 = 0 \Rightarrow m_2 = -\frac{2}{5}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \frac{\frac{1}{3} + \frac{2}{5}}{1 + \frac{1}{3} \times -\frac{2}{5}}$ ✓

$= \frac{11}{13}$

$\therefore \theta = 40^\circ (1\text{d.p.})$ ✓

d. i. $(5,-1)$ ✓

ii. $a = 3$ ✓

iii. $y = 2$ ✓

Question 2

a. i. $8(2x-3)^7 \times 2$ ✓ ✓

$= 16(2x-3)^7$

ii. $\frac{1}{2}(x^2+4)^{-\frac{1}{2}} \times 2x$ ✓ ✓

$= x(x^2+4)^{-\frac{1}{2}}$ ✓

iii. $\frac{(2x+5)(4) - (4x-3)(2)}{(2x+5)^2}$ ✓

$= \frac{26}{(2x+5)^2}$ ✓

iv. $(x+1)^3 \times 4(x+2)^3 + (x+2)^4 \times 3(x+1)^2$ ✓

$= (x+1)^2(x+2)^3[4(x+1) + 3(x+2)]$ ✓

$= (x+1)^2(x+2)^3(7x+10)$

b. i. $m_{PA} = \frac{y}{x+2}$, $m_{PB} = \frac{y}{x-4}$ ✓

$m_{PA} \times m_{PB} = -1$

$\frac{y^2}{(x+2)(x-4)} = -1$ ✓

$y^2 = -(x^2 - 2x - 8)$

$y^2 = -x^2 + 2x + 8$

$x^2 + y^2 - 2x - 8 = 0$ ✓

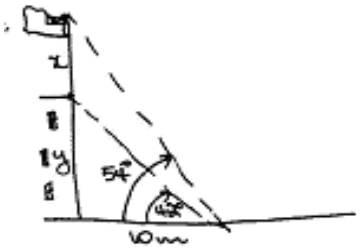
This is general equation of a circle.

ii. $(x^2 - 2x + 1) + y = 8 + 1$

$(x-1)^2 + y^2 = 9$

Centre is $(1,0)$, $R = 3$ ✓ ✓.

Question 3



$\tan 42^\circ = \frac{y}{10} \Rightarrow y = 10\tan 42^\circ$

$\tan 54^\circ = \frac{(x+y)}{10}$

$\Rightarrow x+y = 10\tan 54^\circ$

$x + 10\tan 42^\circ = 10\tan 54^\circ$ ✓

$x = 4.8\text{m}$ ✓ $(4.7597\ldots)$

b. LHS $= \frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin 2\theta}$ ✓

$= \frac{\cos\theta}{\sin\theta} - \frac{2\cos^2\theta - 1}{2\sin\theta\cos\theta}$ ✓

$= \frac{2\cos^2\theta - (2\cos^2\theta - 1)}{2\sin\theta\cos\theta}$

$= \frac{1}{2\sin\theta\cos\theta}$ ✓

$= \text{cosec}\, 2\theta$ ✓

$=$ RHS

c. $4\cos\theta + 3\sin\theta$

$\equiv A\sin\theta\cos\alpha - A\cos\theta\sin\alpha$ ✓

$A\sin\alpha = -4 \quad --①$

$A\cos\alpha = 3 \quad --②$ ✓

$①^2 + ②^2 \Rightarrow A = 5$ ✓

$① \div ② \Rightarrow \tan\alpha = -\frac{4}{3}$

$\alpha = 307^\circ\ (306^\circ 52')$ ✓

d. $\sin x + \cos x = 1$

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$ ✓

$2t + 1 - t^2 = 1 + t^2$ ✓

$2t^2 - 2t = 0$

$2t(t-1) = 0$

$t = 0 \quad \text{or} \quad t = 1$ ✓

$\tan\frac{x}{2} = 0$

$\frac{x}{2} = 0^\circ, 180^\circ, 360^\circ$

$x = 0^\circ, 360^\circ$ ✓

$\tan\frac{x}{2} = 1$

$\frac{x}{2} = 45^\circ, 225^\circ$

$x = 90^\circ,$ ✓

ie. $x = 0^\circ, 90^\circ, 360^\circ$

Question 4

a. i. $x = 40$ ✓

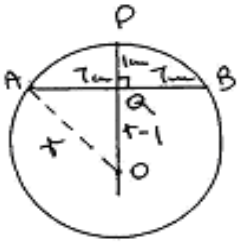
ii. $y = 40$ ✓

iii. $w = 25$ ✓

iv. $x = 25$ ✓

v. $x = 5$ ✓

b.



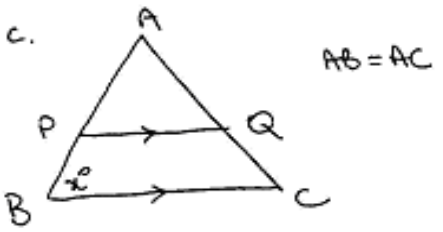
$r^2 = (r-1)^2 + 7^2$ ✓

$r^2 = r^2 - 2r + 1 + 49$

$2r = 50$

$r = 25$ ✓

c.



$AB = AC$

$\angle ACB = \angle ABC = x^\circ$ $(AB = AC)$ ✓

$\angle AQP = \angle ACB = x^\circ$ (corresp. $\angle$s & $PQ \parallel BC$) ✓

ie. $\angle AQP = \angle PBC$

$\therefore PBCQ$ is cyclic

(Ext. $\angle$ of quad. = int. opp. $\angle$ ✓)

Question 5 /15

a.
$$x-5 \;\overline{)\; 2x^3-4x^2+7x-1}$$
$2x^3-10x^2$
$6x^2+7x$
$6x^2-30x$
$37x-1$
$37x-185$
184 ✓

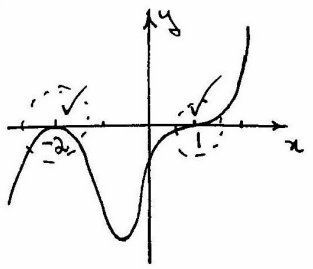
b. $P(x) = x^3+3x^2-bx-7$

$P(2) = 8+12-2b-7$
$= 13-2b$ ✓

$\therefore 13-2b = 20$
$b = -\frac{7}{2}$ ✓

c. $P(x) = x^3-2x^2-11x+12$
$P(1)=0$, $P(-3)=0$, $P(4)=0$
Zeros are $1, -3, 4$ ✓ ✓ ✓

d.



e. Let roots be $\alpha, \beta, \alpha+\beta$ ✓

$\alpha+\beta+(\alpha+\beta) = 8 \Rightarrow \alpha+\beta = 4$ --- ①

$\alpha\beta+\alpha(\alpha+\beta)+\beta(\alpha+\beta) = k$

$\Rightarrow \alpha\beta+(\alpha+\beta)^2 = k$ --- ② ✓

$\alpha\beta(\alpha+\beta) = 12$ --- ③

Subst ① into ③

$\Rightarrow \alpha\beta = 3$ --- ④ ✓

Subst. ① & ④ into ②

$\Rightarrow 3+4^2 = K$

$K = 19$ ✓

Let roots be α, β, γ & $\alpha = \beta+\gamma$

$\alpha+\beta+\gamma = 8$

$2\alpha = 8$

$\alpha = 4$ is one root

etc.

Question 6

a. i. $m_{PQ} = \frac{p^2-q^2}{2p-2q}$

$= \frac{p+q}{2}$ ✓

Equation of PQ is

$y-p^2 = \frac{p+q}{2}(x-2p)$ ✓

$y-p^2 = \frac{1}{2}(p+q)x - p^2 - pq$

ie. $y = \frac{1}{2}(p+q)x - pq$

ii. If PQ passes thro' $(0,-1)$

$-1 = \frac{1}{2}(p+q)(0) - pq$ ✓

ie. $pq = 1$

iii. $xx_0 = 2a(y+y_0)$

$4x = 2(y-12)$ ✓

(or $2x-y+12=0$, $y=2x+12$)

iv. Let $(x_1,-1)$ be any pt. on $y=-1$.

Chord of contact is

$x_1x = 2(y-1)$ ✓

Focus is $(0,1)$.

Subst. coords of focus

$\Rightarrow 0x = 2(1-1)$ ✓

$\therefore$ Focal chord.

b. $y = \frac{x^2}{4a} \Rightarrow y' = \frac{x}{2a}$

At $P(2ap, ap^2)$ $y' = p$ ✓

Tangent at P is

$y-ap^2 = p(x-2ap)$

ie. $y = px-ap^2$ — ① ✓

Similarly, tangent at Q is

$y = qx-aq^2$ — ②

For pt. of intersection

① $\times q \Rightarrow qy = pqx-apq$ --- ③

② $\times p \Rightarrow py = pqx-apq^2$ --- ④

④ − ③ $\Rightarrow (p-q)y = -apq(p-q)$

ie. $y = +apq$ ✓

[x is a $(p+q)$]

Since tangents at P+Q are perpendicular

$pq = -1$

$\therefore y = +ax-1$

ie. $y = -a$ ✓

is the equation of the locus.