



THE
KING'S
SCHOOL

2019

Year 11
Assessment Task 3 (AT3)
September 2019

Mathematics Extension 1

General Instructions:

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Write your **NAME** where indicated

Total marks: 70

Section I 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II 60 marks

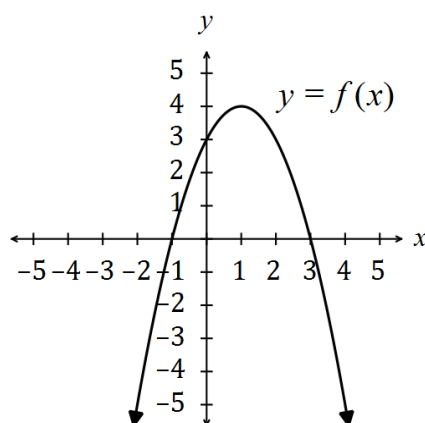
- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

Examiners' Use Only			Setter/Checker: External/BDB		
Question	Calculus	Functions	Trigonometric Functions	Total	
Section I					
1-10	8 / 1	1, 3-5, 7, 9 / 6	2, 6, 10 / 3		/10
Section II					
11		a-c /12	d / 3		/15
12	b / 7	e / 4	a, c, d / 4		/15
13	c, d / 7	a / 4	b / 4		/15
14	e / 5	c, d / 6	a, b / 4		/15
Total	/20	/32	/18		/70

Section I
10 marks

Attempt Questions 1-10
Allow about 15 minutes for this section

- 1 Consider the function $y = f(x)$ below:



What is the domain of $y = \frac{1}{f(x)}$?

- (A) $(-\infty, 0) \cup \left[\frac{1}{4}, \infty\right)$
(B) $(-\infty, 0) \cup \left(\frac{1}{4}, \infty\right)$
(C) $[-\infty, -1) \cup (-1, 3) \cup (3, \infty]$
(D) $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

- 2 Simplify $4\sin 2\theta \cos 2\theta$

- (A) $\sin \theta$
(B) $2\sin \theta$
(C) $\sin 4\theta$
(D) $2\sin 4\theta$

3 Solve $|3 - 2x| \leq 1$

(A) $1 \leq x \leq 2$

(B) $x \leq 1$ or $x \geq 2$

(C) $-2 \leq x \leq -1$

(D) $x \leq -2$ or $x \geq -1$

4 Find the inverse relation of $y = x^2 - 4x + 17$

(A) No real solution

(B) $y = 2x - 4$

(C) $y = 4 \pm \sqrt{x - 1}$

(D) $y = 2 \pm \sqrt{x - 13}$

5 If α, β, γ and δ are the roots of $x^4 - x^3 + 4x^2 - 2x + 4 = 0$, evaluate $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 1

(D) -1

6 Solve: $\sin^{-1} x + 2\sin x \cos x + \cos^{-1} x = \frac{1 + \pi}{2}, 0 \leq x \leq \frac{\pi}{4}$

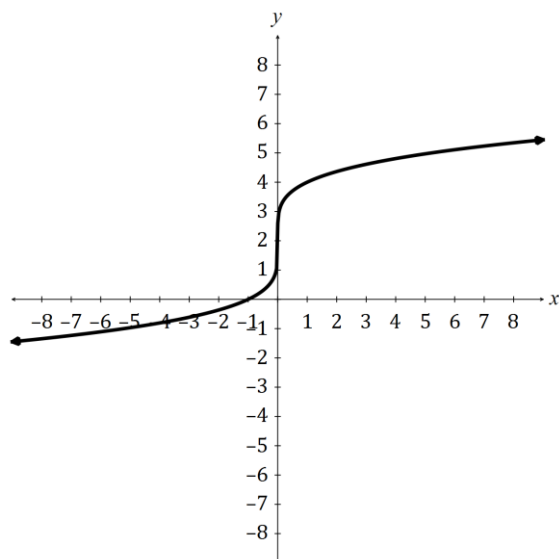
(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

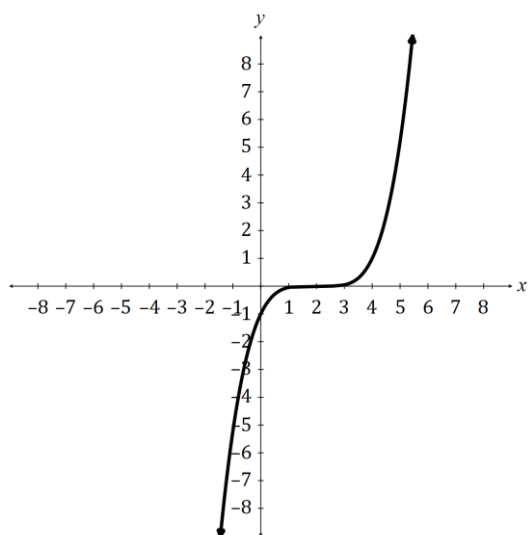
(D) No real solution

7 Consider the graph $y = f(x)$ below.

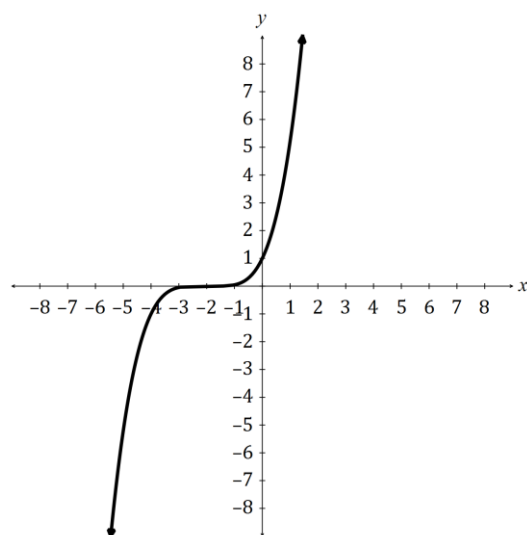


Which graph represents $y = f^{-1}(x)$?

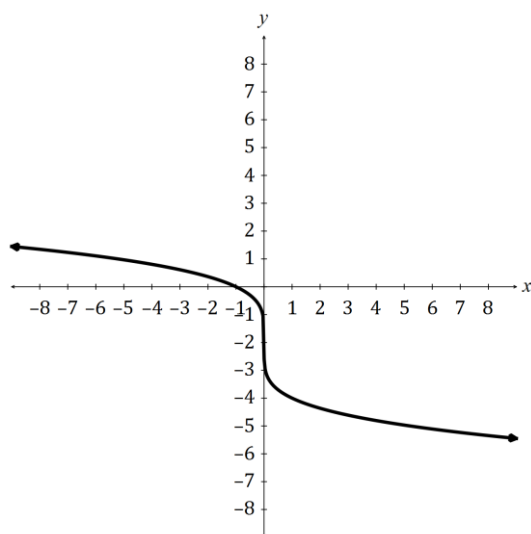
(A)



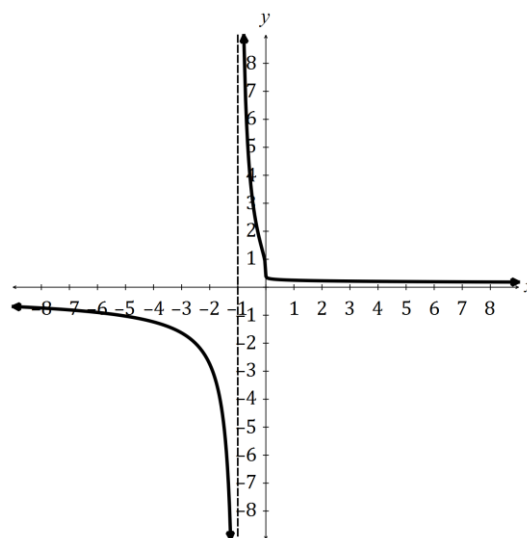
(B)



(C)



(D)



- 8 The velocity of a particle which is travelling along the x -axis is given by

$$v = 3t^2 + 4t + 5, \text{ where } t \text{ is the time in seconds.}$$

Which of the following could be the function which gives the position of the particle with respect to time?

(A) $x = t^3 + 2t^2 + 5t + 10$

(B) $x = 3t^3 + 4t^2 + 5t + 10$

(C) $x = 6t + 4$

(D) $x = 3t + 4$

- 9 If $P(x) = 0$ has a root of multiplicity 3 at $x = k$, then $P'(k) = 0$ has

(A) a root of multiplicity ≤ 3

(B) a root of multiplicity 3

(C) a root of multiplicity 2

(D) a rational root of multiplicity ≤ 2

- 10 If $\sin\theta = \sqrt{\frac{x}{3}}$ ($x \geq 0$), find the exact value of $\cos 2\theta$

(A) $\frac{3 - 2x}{3}$

(B) $\frac{2\sqrt{3x - x^2}}{3}$

(C) $\frac{-2\sqrt{3x - x^2}}{3}$

(D) $\sqrt{\frac{3 - 2x}{3}}$

End of Section I

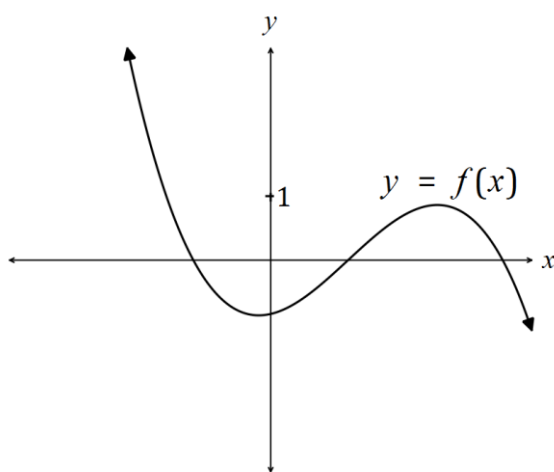
Section II
60 marks

Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section

Question 11 (15 marks)

Marks

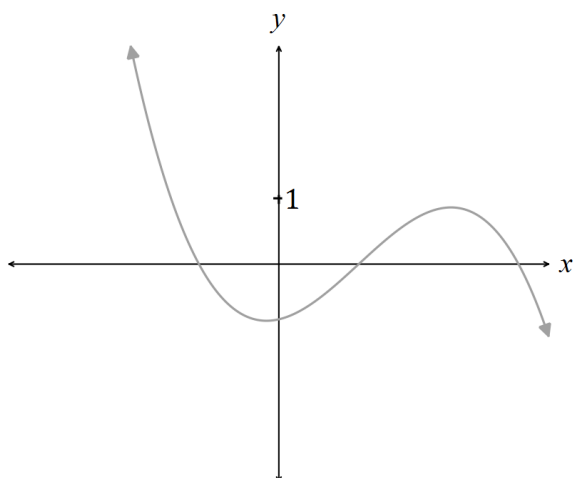
(a) Consider the function $y = f(x)$ below:



On each number plane provided, sketch the function indicated:

(i) $y = |f(x)|$

1

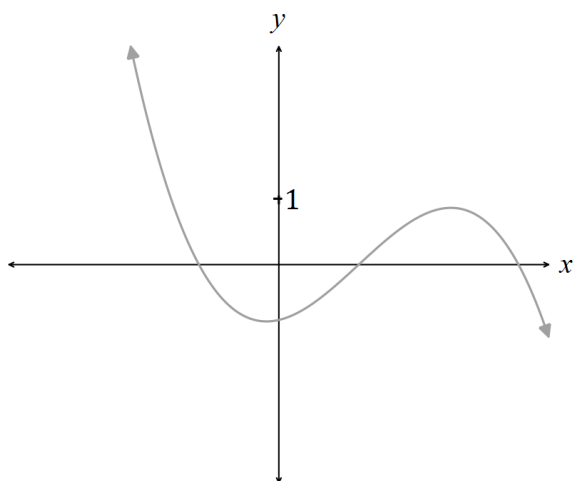


Question 11 (continued)

Marks

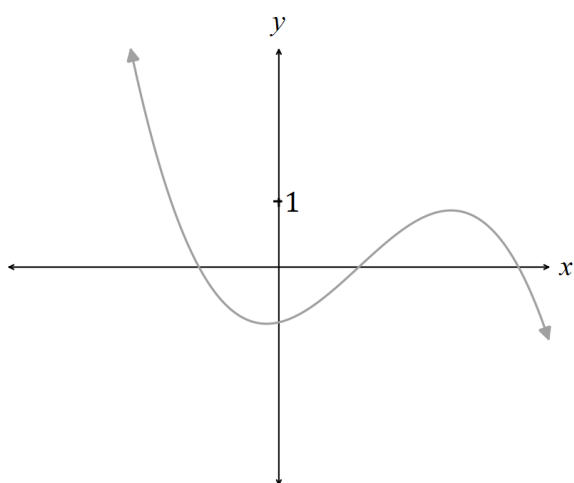
(a) (ii) $y = f(|x|)$

1



(iii) $y^2 = f(x)$

1



Question 11 continues on the next page

Question 11 (continued)

Marks

(b) Solve:

(i) $x^2 < 6 - x$

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(ii) $\frac{x + 3}{1 - x} \geq 1$

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Question 11 continues on the next page

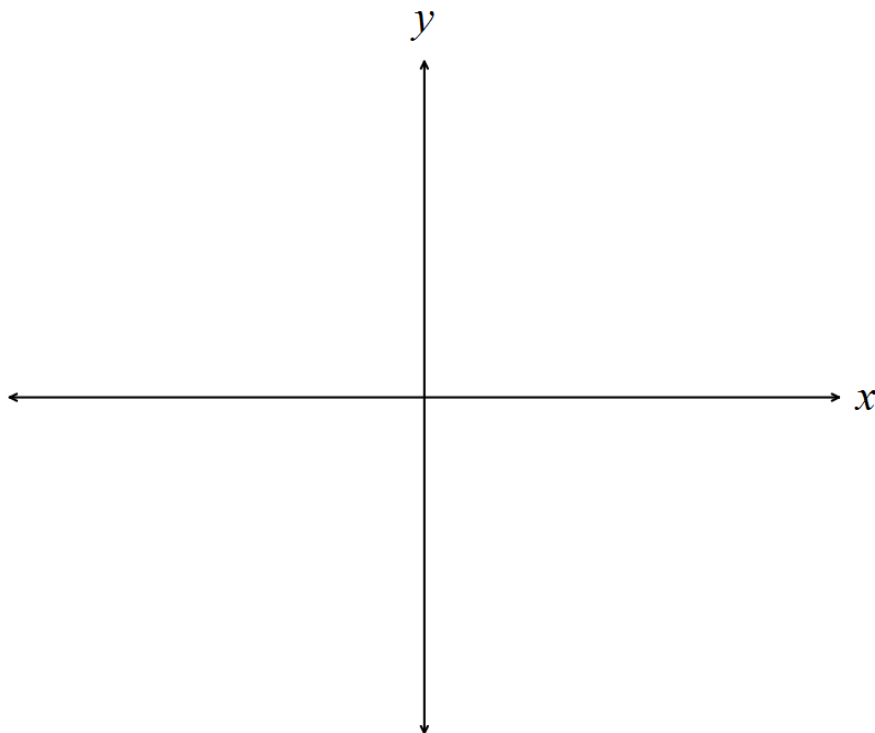
Question 11 (continued)

Marks

(c) For the polynomial function $P(x) = x^2 (2 - x)^3 (3 + x)$

(i) Sketch $y = P(x)$ on the number plane provided.

2



(ii) Hence, solve $P(x) \leq 0$

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Question 11 continues on the next page

Question 11 (continued)

Marks

(d) Find the exact value of $\sin 15^\circ$

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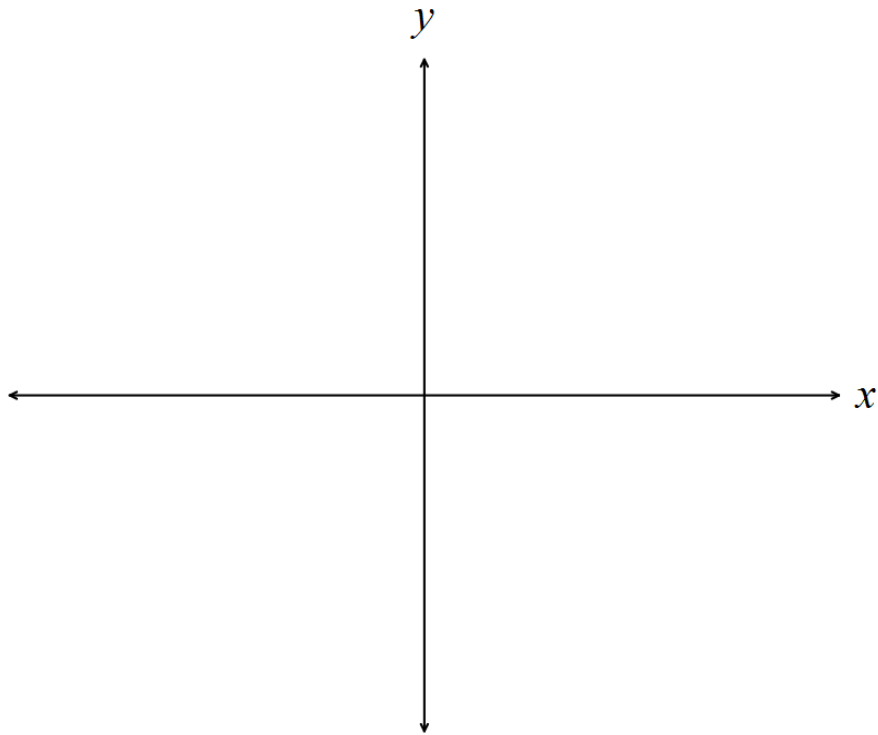
End of Question 11

Question 12 (15 marks)

Marks

(a) Sketch the graph $y = \cos^{-1} x$

2



Question 12 continues on the next page

Question 12 (continued)

Marks

- (b) The displacement of a particle moving along the x -axis is given by $x = (t - 1)^3$, where t is time in seconds and x is displacement in cm.

- (i) Find the equations for velocity ($\dot{x}$) and acceleration ($\ddot{x}$). **2**

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- (ii) Find the acceleration of the particle after 10 seconds. **1**

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- (iii) When is the particle at rest? **1**

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- (iv) What is the minimum acceleration of the particle? **1**

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- (v) Describe the motion of the particle. **2**

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Question 12 continues on the next page

Question 12 (continued)

Marks

(c) Simplify $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

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(d) For what domain does $\tan (\tan^{-1} x) = x$?

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Question 12 continues on the next page

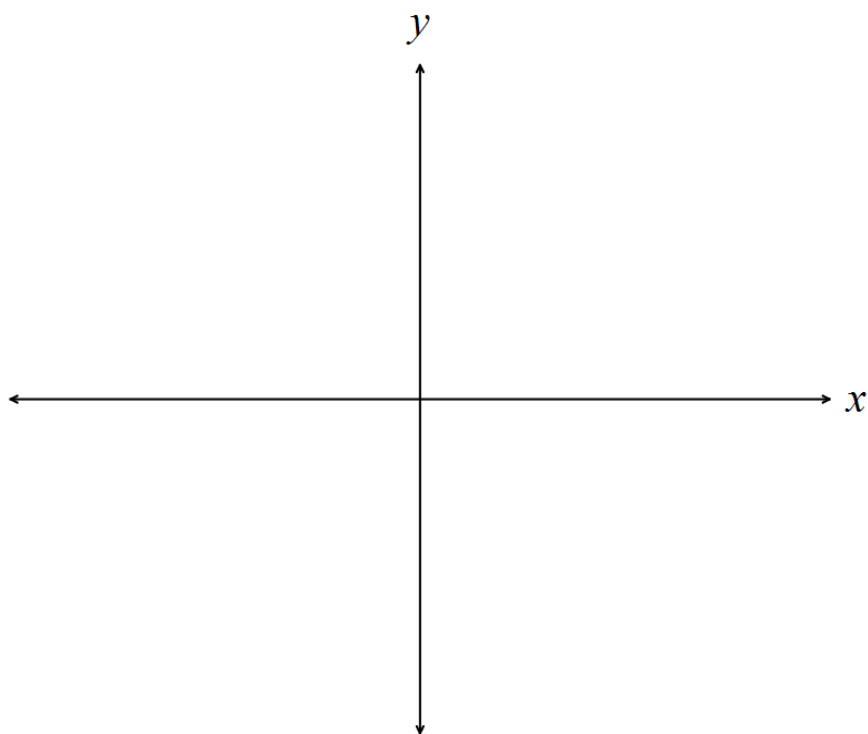
Question 12 (continued)

Marks

(e) Consider the Cartesian equation $x^2 + y^2 - 10y - 11 = 0$

(i) Sketch the graph.

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(ii) Given $x = 6 \sin \theta$, find y in terms of θ .

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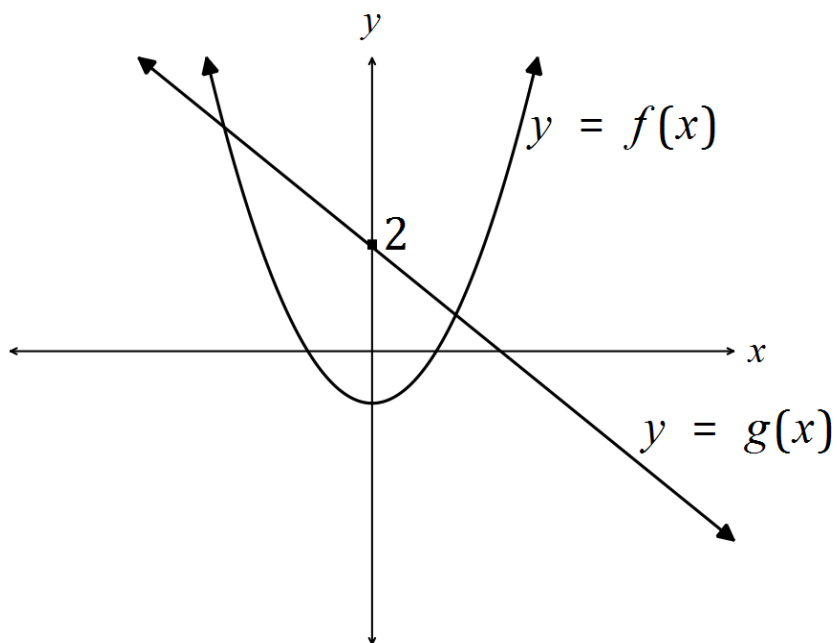
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End of Question 12

Question 13 (15 marks)

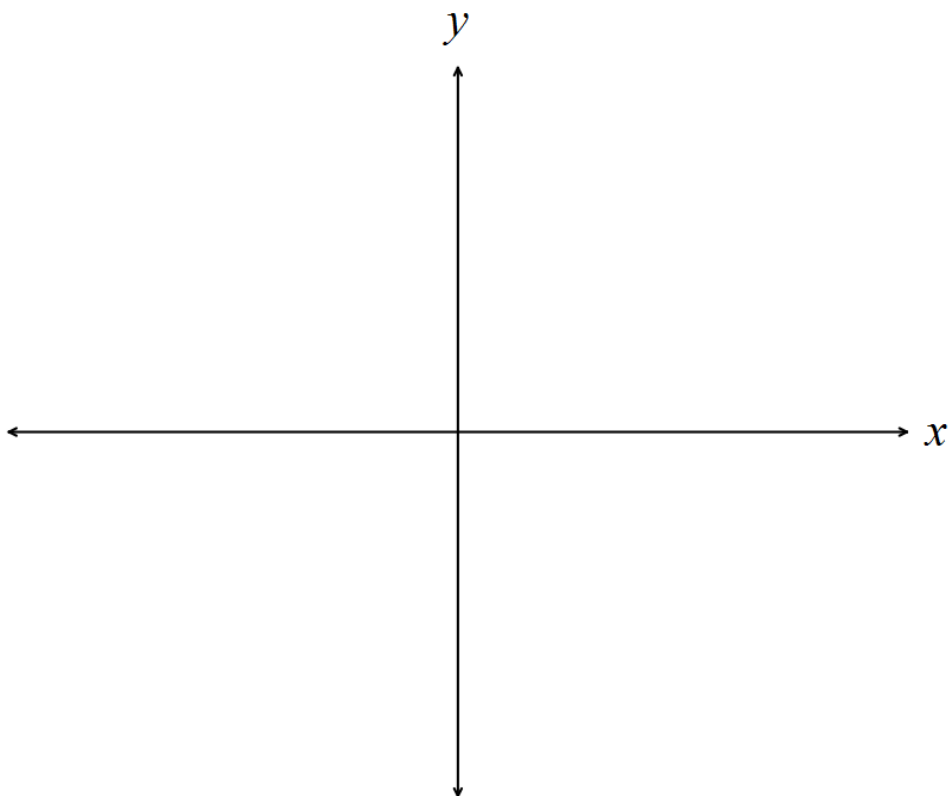
Marks

(a) The graphs of $y = f(x)$ and $y = g(x)$ are shown:



(i) Sketch $y = f(x) + g(x)$

2



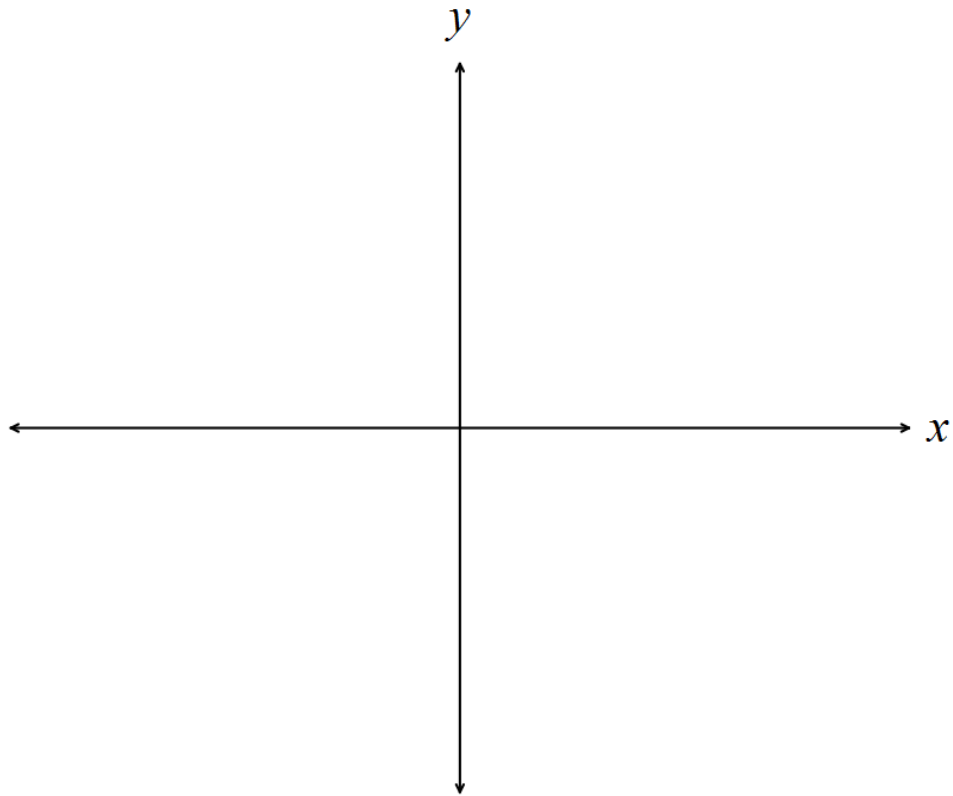
Question 13 continues on the next page

Question 13 (continued)

Marks

(a) (ii) Sketch $y = f(x) \cdot g(x)$

2



Question 13 continues on the next page

Question 13 (continued)

Marks

(b) (i) Prove $\cos(A + B) + \cos(A - B) \equiv 2\cos A \cos B$

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(ii) Hence, find the exact value of $\cos \frac{5\pi}{12} + \cos \frac{\pi}{12}$

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Question 13 continues on the next page

Question 13 (continued)

Marks

(c) The number of bacteria in a culture is given by $N = N_0 e^{0.28t}$ where t is in hours.

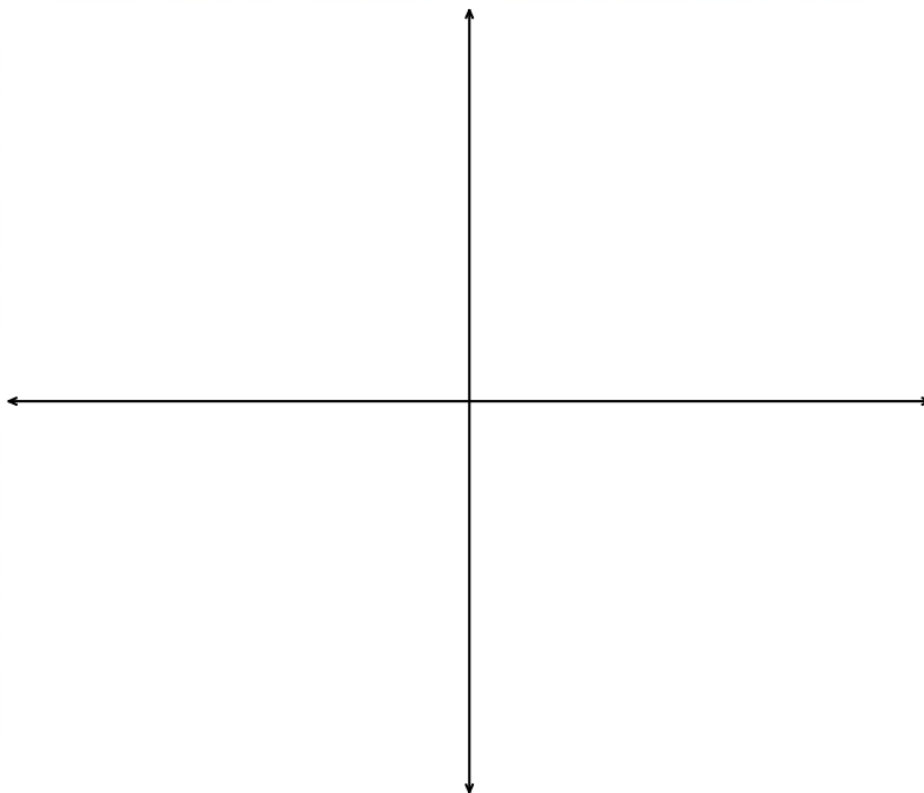
(i) If there are initially 100 bacteria, how many will there be after 10 hours? **1**

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(ii) How many hours will it take for the population to reach 200 000? Answer to the nearest hour. **2**

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(iii) Graph the relationship between N and t for $N \in [100, 200\,000]$ **1**



Question 13 continues on the next page

Question 13 (continued)

Marks

- (d) A spherical balloon has a radius r mm. It has a small hole, causing the radius to shrink at a rate of 0.2 mm s^{-1} .

Calculate the rate of change of the surface area of the balloon when the radius is 100 mm.

Hence, describe what is happening to the surface area of the balloon.

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End of Question 13

Question 14 (15 marks)

Marks

(a) Prove $\sin^{-1}(-x) = -\sin^{-1}x$

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(b) Show that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$, for $0 \leq x < 1$

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Question 14 continues on the next page

Question 14 (continued)

Marks

(c) Given $P(x) = 6x^3 - 11x^2 - 3x + 2$

(i) Show $x - 2$ is a factor of $P(x)$

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(ii) Hence, factorise $P(x)$ and find the roots of $P(x) = 0$

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Question 14 continues on the next page

Question 14 (continued)

Marks

- (d) Consider the polynomial $P(x) = (x - 2)(x - 5)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x - 2)$ the remainder is 4, and when it is divided by $(x - 5)$, the remainder is 13.

Find a and b

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Question 14 continues on the next page

Question 14 (continued)

Marks

- (e) The blood temperature of a live animal is 38.6°C .

An animal is found dead with a blood temperature of 31.1°C , in a barn of constant temperature $P^{\circ}\text{C}$. 10 minutes later, the animal's blood temperature is 30.7°C .

The cooling rate is proportional to the difference between the blood temperature and the room temperature.

- (i) By using the formula for Newton's Law of cooling, $T = P + Ae^{kt}$, find an expression for k in terms of A .

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- (ii) Given $P = 26^{\circ}$, show that $k = -0.008$ to three decimal places.

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Question 14 continues on the next page

Question 14 (continued)

Marks

- (e) (iii) Hence, if the animal was found at 8 pm, what was the time of death? Answer to the nearest minute.

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End of Examination Paper



THE
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SCHOOL

2019

Year 11
Assessment Task 3 (11AT3)
September 2019

Multiple Choice Answer Sheet
Mathematics Extension 1

Select the alternative A, B, C or D that best answers the question and indicate your choice by filling in the appropriate response oval, completely.

					Examiner's Use Only	
1	(A)	(B)	(C)	(D)	Functions	/ 1
2	(A)	(B)	(C)	(D)	Trigonometric Functions	/ 1
3	(A)	(B)	(C)	(D)		
4	(A)	(B)	(C)	(D)		
5	(A)	(B)	(C)	(D)	Functions	/ 3
6	(A)	(B)	(C)	(D)	Trigonometric Functions	/ 1
7	(A)	(B)	(C)	(D)	Functions	/ 1
8	(A)	(B)	(C)	(D)	Calculus	/ 1
9	(A)	(B)	(C)	(D)	Functions	/ 1
10	(A)	(B)	(C)	(D)	Trigonometric Functions	/ 1
					Total	/10

Examiners' Use Only				Setter/Checker: External/BDB	
Question	Calculus	Functions	Trigonometric Functions	Total	
Section I					
1-10	8 / 1	1, 3-5, 7, 9 / 6	2, 6, 10 / 3	/10	
Total	/20	/32	/18	/70	

**2019 Year 11 Examination
Mathematics Extension 1
Marking Guidelines**

Section I

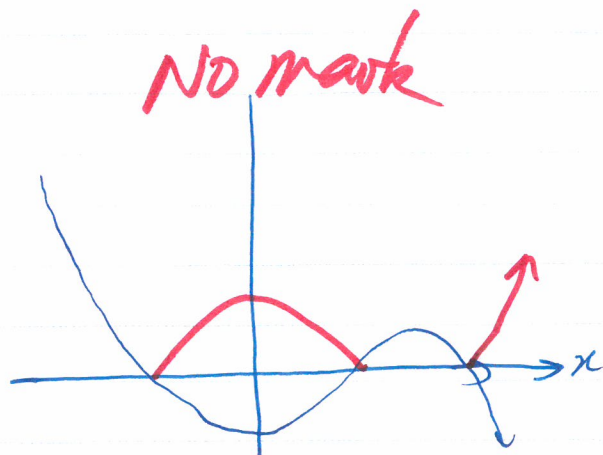
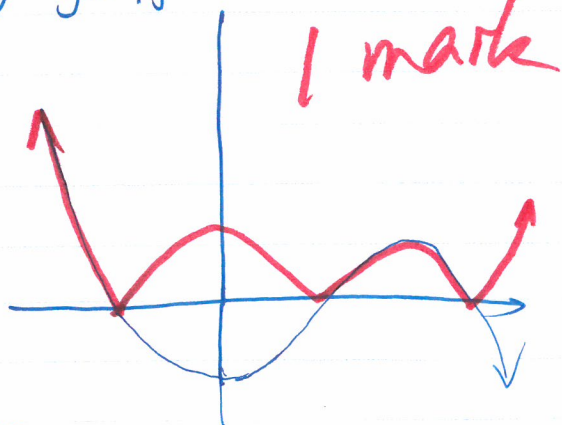
AT3

Multiple-choice Answer Key

Question	Answer
1	D
2	D
3	A
4	D
5	B
6	A
7	A
8	A
9	C
10	A

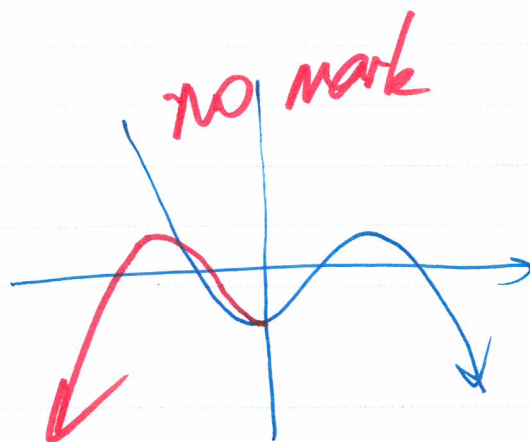
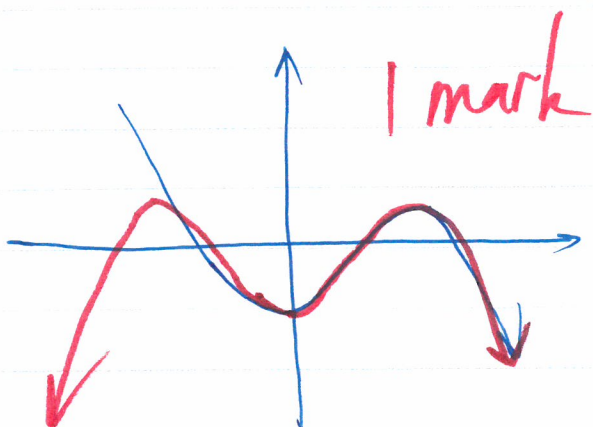
Year 11 AT3 Ex) Question 11 MARKING.

(a) i) $y = |f(x)|$

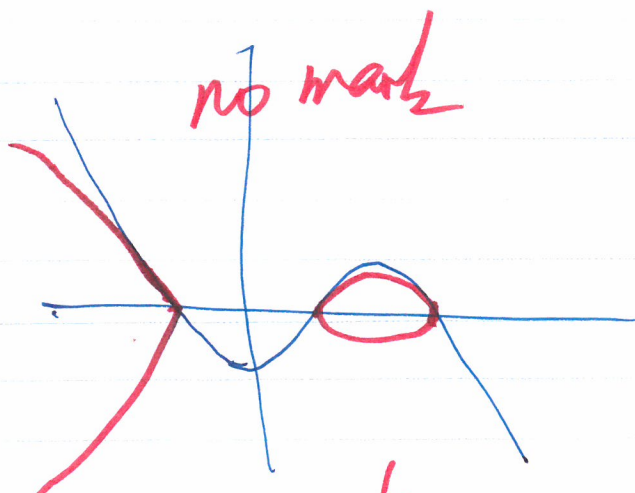
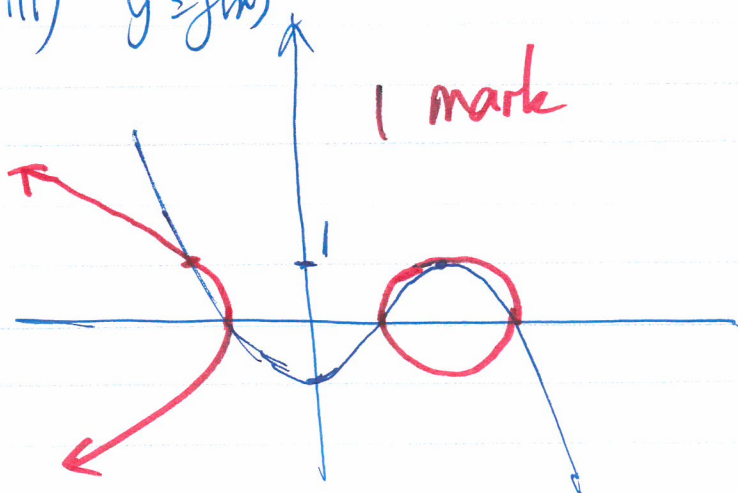


(ii)

$y = f(|x|)$



(iii) $y^2 = f(x)$

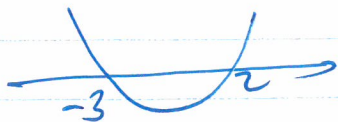


other variations
= no mark

(b) (i)

$$x^2 + x - 6 < 0$$
$$(x+3)(x-2) < 0$$

← 1 for factored quad.



$$-3 < x < 2 \quad \text{or} \quad x \in (-3, 2).$$

← 1 correct answer

2

(ii)

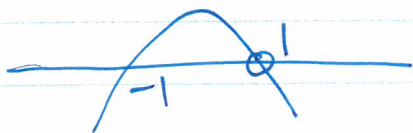
$$\frac{x+3}{1-x} \geq 1$$

$$(x+3)(1-x) \geq (1-x)^2$$

$$(x+3)(1-x) - (1-x)^2 \geq 0$$

$$2(1-x)(x+1) \geq 0.$$

1 mark for progress

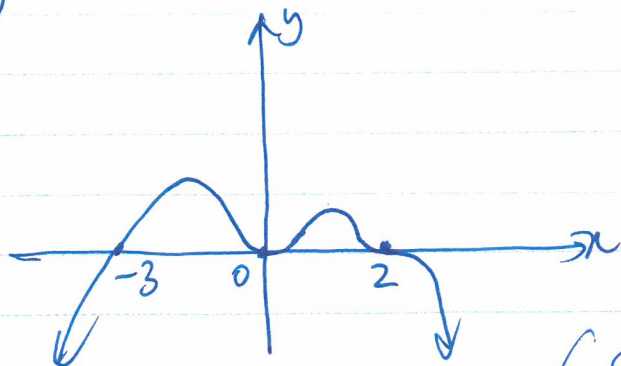


$$-1 \leq x \leq 1$$

← 2 marks correct answer
(-1 for $-1 \leq x \leq 1$)
mark 3

{ "Critical points method" correctly solve the problem,
showing working to $-1 \leq x < 1$ 3
 $-1 \leq x \leq 1$ 2 }
~~completely wrong~~ Very few used this method.

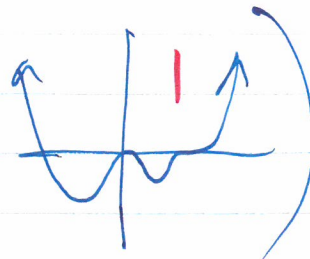
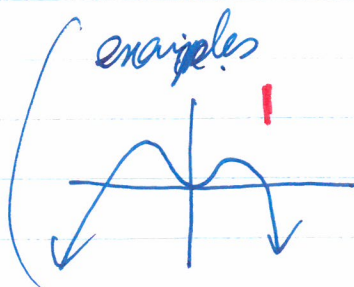
(c) (i)



Correct x intercepts
and shape at intercepts 1

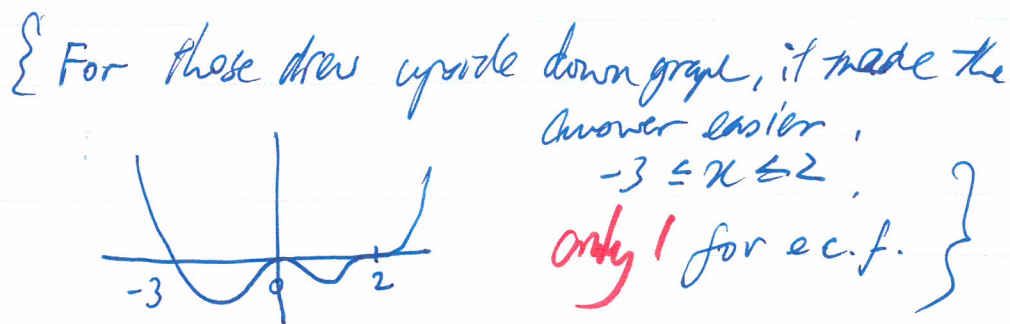
Correct ^{even} degree graph

1 1/2



(c) (ii) $x \leq -3, x = 0, x \geq 2$ /2
 -1 mark for ~~side~~ mistake

example $x \leq -3, x \geq 2$ only 1 mark



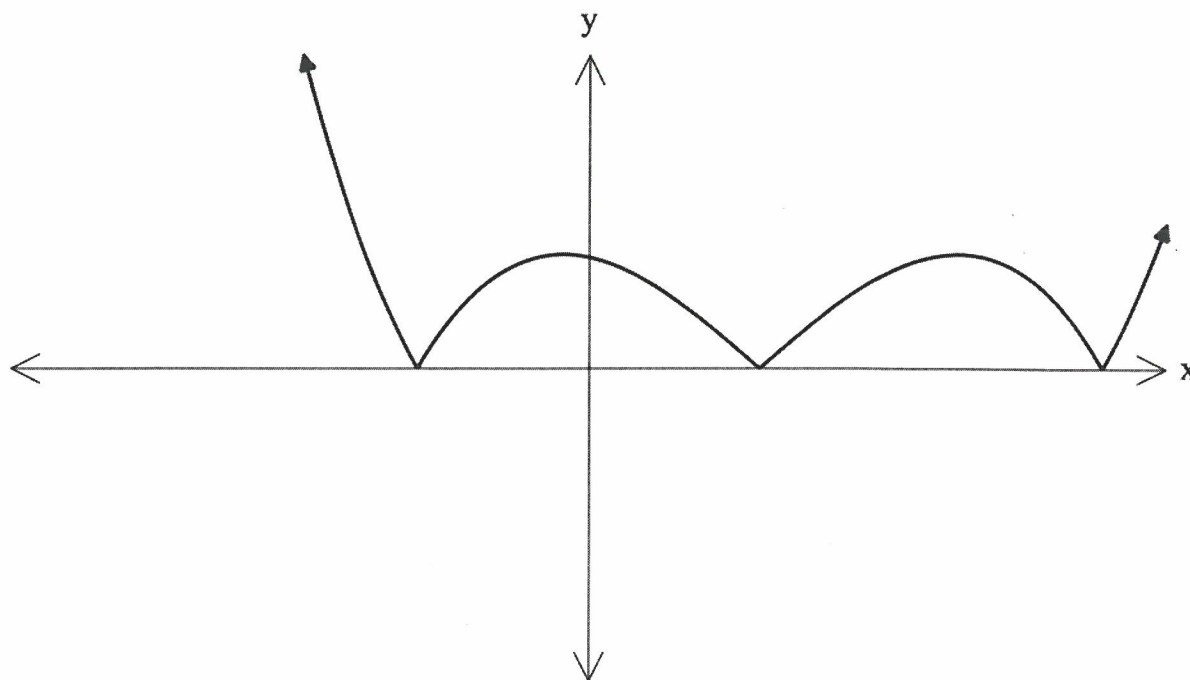
(d) $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ ← 1 mark for correct use of $\sin(A-B)$.
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
 $= \frac{\sqrt{3}-1}{2\sqrt{2}}$ ← 2 mark with working shown. /3
 (or $\frac{\sqrt{6}-\sqrt{2}}{4}$)
 -1 mark for error.

{ 0 mark for $\sin 15^\circ = \sin 45^\circ - \sin 30^\circ$ }

Question 11

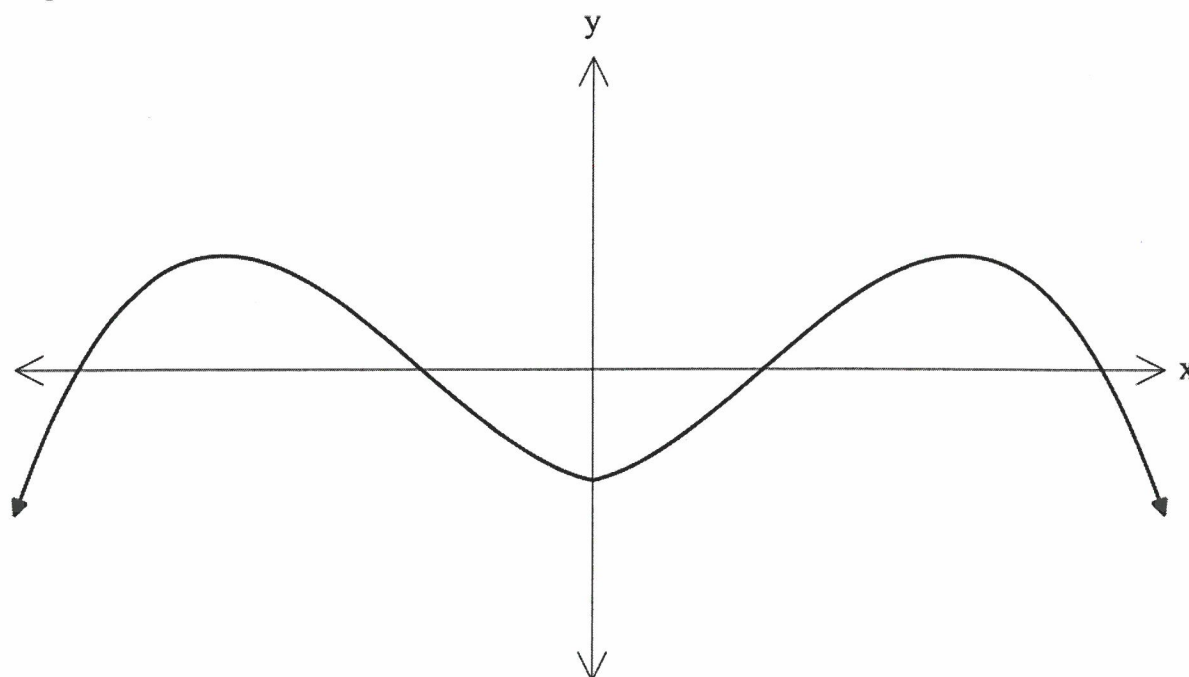
11 (a) (i)

Criteria	Marks
• Provides correct graph	1

Sample answer

11 (a) (ii)

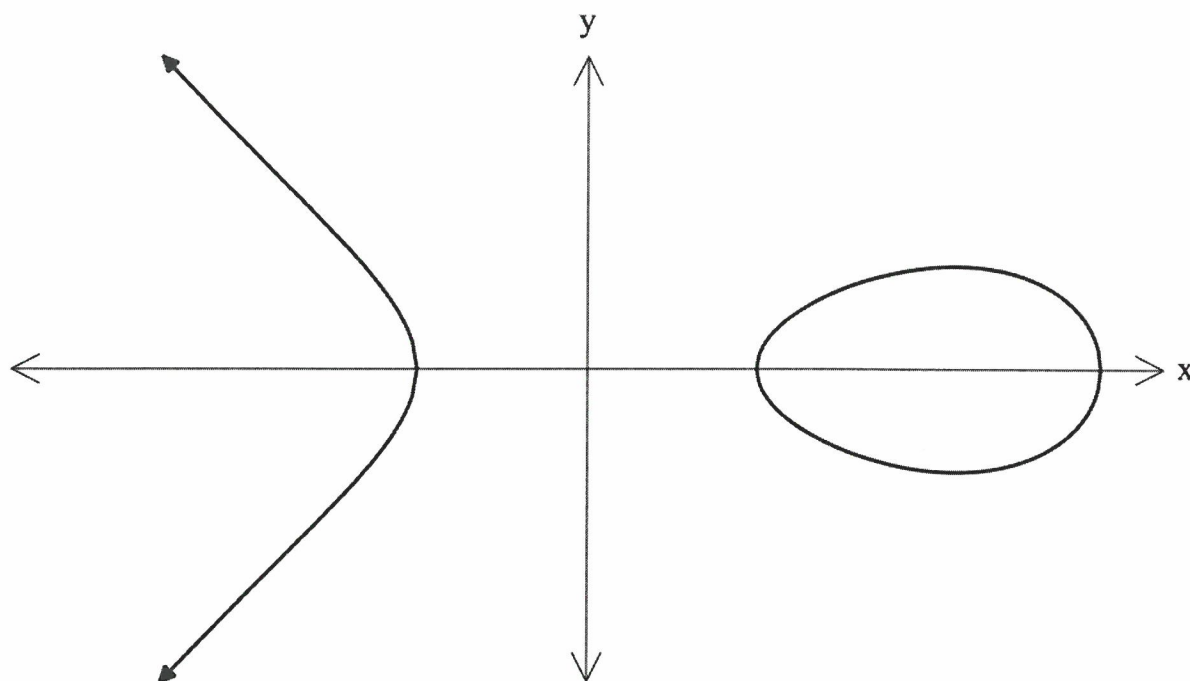
Criteria	Marks
• Provides correct graph	1

Sample answer

11 (a) (iii)

Criteria	Marks
• Provides correct graph	1

Sample answer



Question 11 (continued)

Marks

(b) Solve:

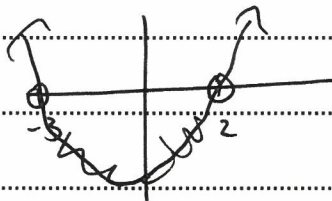
(i) $x^2 < 6 - x$

2

$$x^2 + x - 6 < 0$$

$$(x+3)(x-2) < 0$$

(1)



$$\therefore -3 < x < 2$$

(1)

(ii) $\frac{x+3}{1-x} \geq 1$

3

$$(x+3)(1-x) \geq (1-x)^2$$

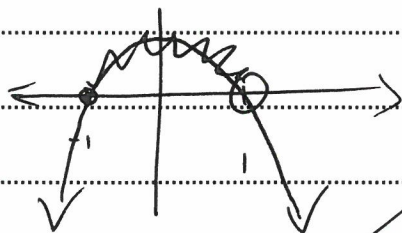
$$(x+3)(1-x) - (1-x)^2 \geq 0$$

$$(1-x)(x+3-1+x) \geq 0$$

$$(1-x)(2x+2) \geq 0$$

$$x \neq 1$$

(1)



$$-1 \leq x < 1$$

(1)

Question 11 continues on the next page

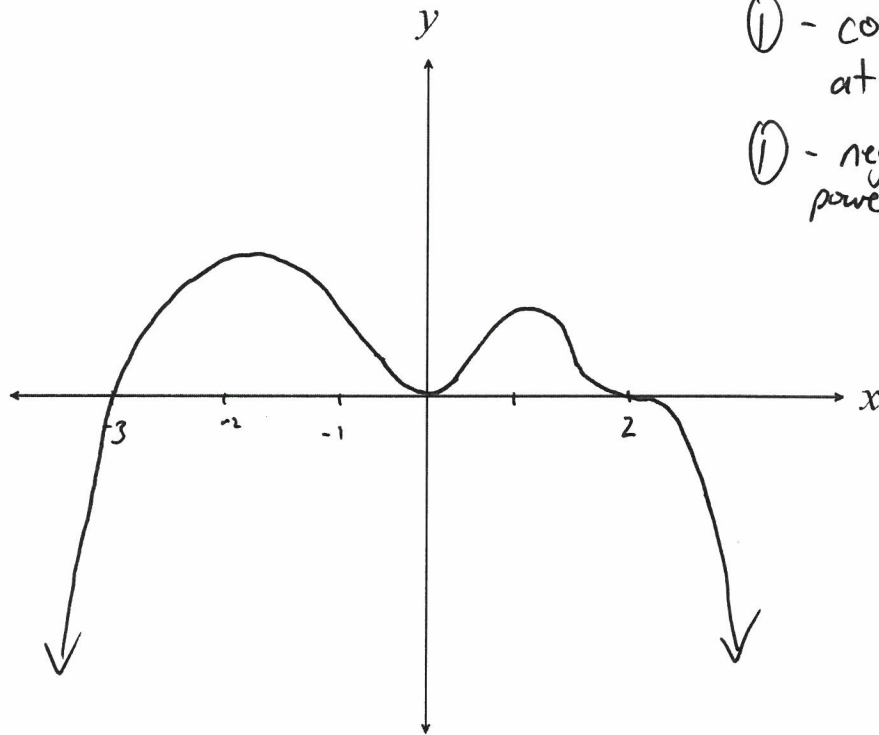
Question 11 (continued)

Marks

(c) For the polynomial function $P(x) = x^2(2 - x)^3(3 + x)$

(i) Sketch $y = P(x)$ on the number plane provided.

2



① - correct shape at roots.

① - negative even powered function.

(ii) Hence, solve $P(x) \leq 0$

2

$$x \leq -3, x = 0, x \geq 2$$

(-1 mark for each incorrect.)

Question 11 continues on the next page

11 (d)

Criteria	Marks
• Correct answer	3
• Significant progress towards solution (1 error)	2
• Correctly applies the Sine difference formula	1

Sample answer

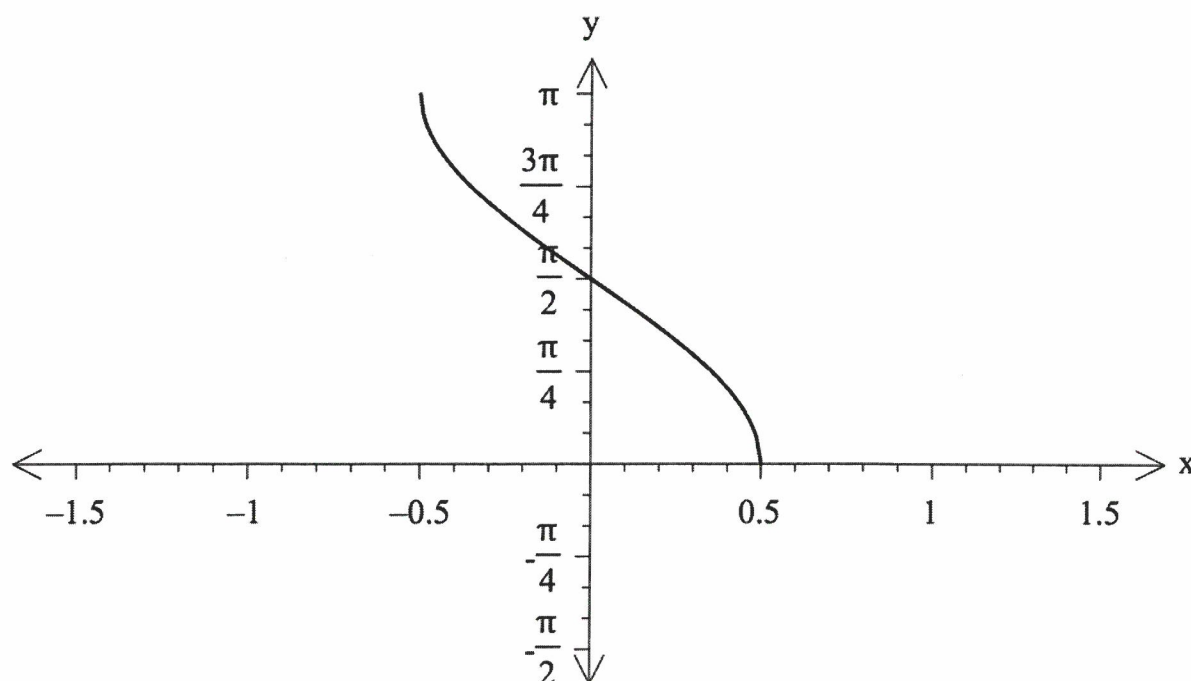
$$\begin{aligned}
 \sin 15^\circ &= \sin (45 - 30) \\
 &= \sin 45 \cdot \cos 30 - \cos 45 \cdot \sin 30 \\
 &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\
 &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \\
 &= \frac{\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

Question 12

12 (a)

Criteria	Marks
• Correct solution	2
• Correct shape but incorrect range and/or domain, or intercepts not labelled	1

Sample answer



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12 (b) (i)

Criteria	Marks
• Both equations correct	2
• One equation correct	1

Sample answer

$$\begin{aligned}
 x &= (t - 1)^3 \\
 \dot{x} &= 3(t - 1)^2 \\
 \ddot{x} &= 6(t - 1)
 \end{aligned}$$

12 (b) (ii)

Criteria	Marks
• Correct solution, with correct units.	1

Sample answer

at $t = 10$

$$\begin{aligned}
 \ddot{x} &= 6(10 - 1) \\
 &= 54 \text{ cm s}^{-2}
 \end{aligned}$$

12 (b) (iii)

Criteria	Marks
• Correct solution	1

Sample answer

for stationary points, $\dot{x} = 0$

$$\therefore t = 1 \text{ s}$$

12 (b) (iv)

Criteria	Marks
• Correct solution	1

Sample answer

$$\ddot{x} = 6(t - 1)$$

this is a linear function with a positive gradient

$\therefore$ minimum acceleration when $t = 0$.

$$\begin{aligned}
 \therefore \ddot{x} &= 6(0 - 1) \\
 &= -6 \text{ cm s}^{-2}
 \end{aligned}$$

12 (b) (v)

Criteria	Marks
Correct solution, including comment on location, velocity (stationary point) and acceleration.	2
- Partially correct solution including comment on two of the following: - Location - Velocity (stationary point) - Acceleration	1

Sample answer

The particle starts 1cm to the left of the origin. It slows down as it moves towards the origin and stops once it reaches it. It then accelerates as it moves the right, away from the origin.

12 (c)

Criteria	Marks
Correct solution	1

Sample answer

$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - t^2}{1 + t^2} = \cos 2\theta \quad (\text{when } t = \tan \theta)$$

12 (d)

Criteria	Marks
Correct solution	1

Sample answer

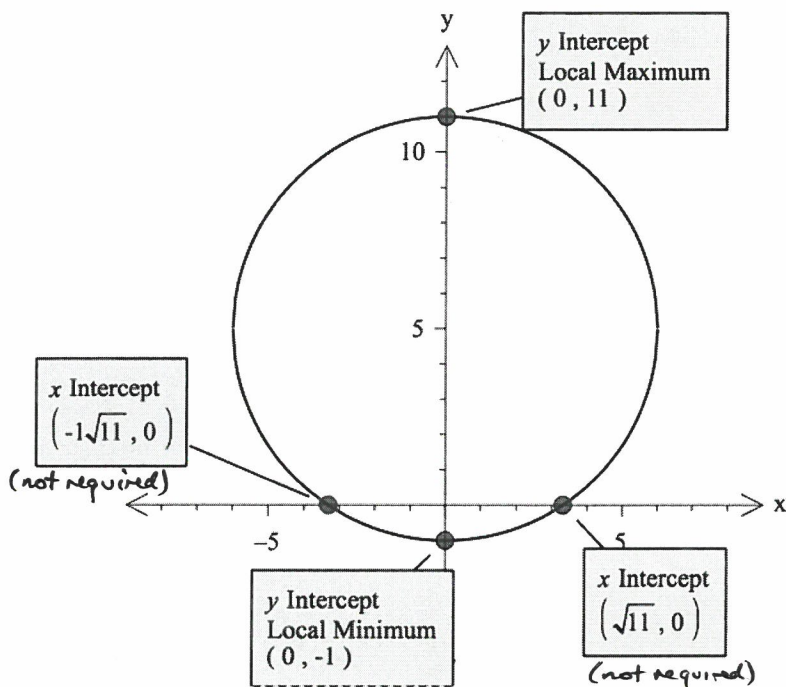
All real x ,

ie $(-\infty, \infty)$

12 (e) (i)

Criteria	Marks
• Correct graph, labelling centre and <u>y intercepts</u>	2
• Correct equation: $x^2 + (y-5)^2 = 36$, after completing the square	1

Sample answer



12 (e) (ii)

Criteria	Marks
• Correct solution	2
• Some progress towards solution	1

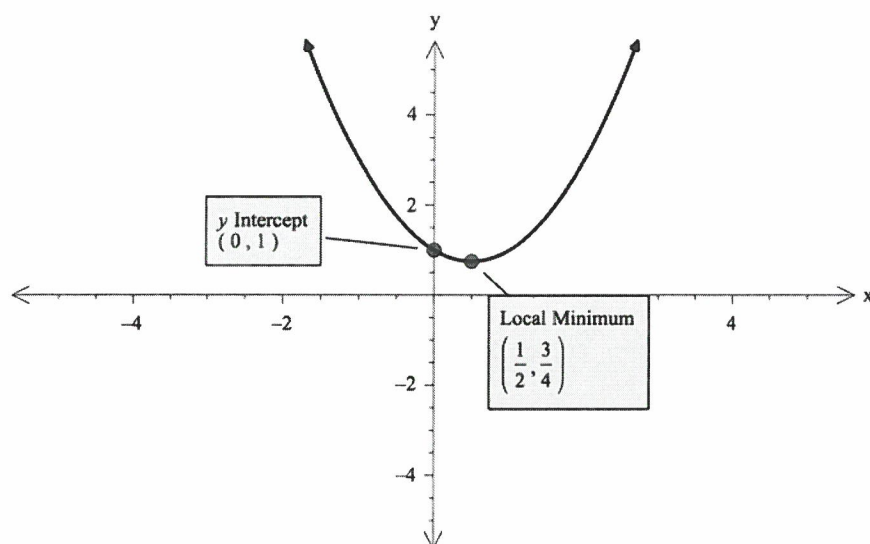
Sample answer

$$\begin{aligned}
 (6\sin\theta)^2 + (y-5)^2 &= 36 \\
 36\sin^2\theta + (y-5)^2 &= 36 & | \\
 (y-5)^2 &= 36(1 - \sin^2\theta) \\
 (y-5)^2 &= 36\cos^2\theta \\
 y-5 &= \pm 6\cos\theta \\
 y &= 5 \pm 6\cos\theta & |
 \end{aligned}$$

13 (a) (i)

Criteria	Marks
• Correct graph (critical point labels not required)	2
• Correct shape (concave up parabola)	1

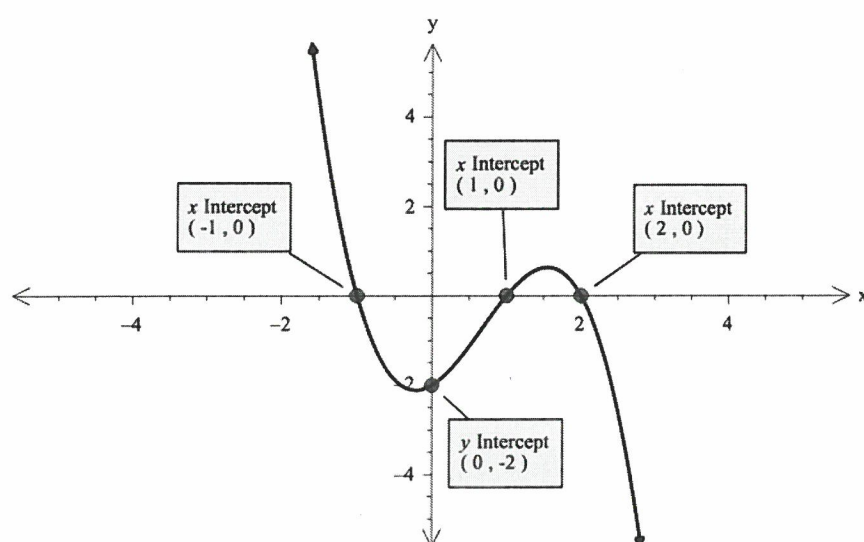
Sample answer



13 (a) (ii)

Criteria	Marks
• Correct graph with intercepts shown in correct positions (intercept values not required)	2
• Correct shape (negative cubic)	1

Sample answer



13 (b) (i)

Criteria	Marks
Correct solution	1

Sample answer

$$\begin{aligned}
 \text{LHS} &= \cos(A + B) + \cos(A - B) \\
 &= \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B \\
 &= 2 \cos A \cos B
 \end{aligned}$$

13 (b) (ii)

Criteria	Marks
Correct solution	3
Correct method with 1 error	2
Uses part i) to rewrite expression as $= 2 \cdot \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6}$	1

Sample answer

$$\begin{aligned}
 &\cos \frac{5\pi}{12} + \cos \frac{\pi}{12} \\
 &= \cos \left(\frac{\pi}{4} + \frac{\pi}{6} \right) + \cos \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \\
 &= 2 \cdot \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} \quad \text{using } i) \\
 &= 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{3}}{\sqrt{2}} \\
 &= \frac{\sqrt{6}}{2}
 \end{aligned}$$

Question 13 (continued)

Marks

(c) The number of bacteria in a culture is given by $N = N_0 e^{0.28t}$ where t is in hours.

- (i) If there are initially 100 bacteria, how many will there be after 10 hours? 1

$$N = 100 e^{0.28(10)}$$

$$= 1644 \text{ (nearest whole)}$$

- (ii) How many hours will it take for the population to reach 200 000? Answer to the nearest hour. 2

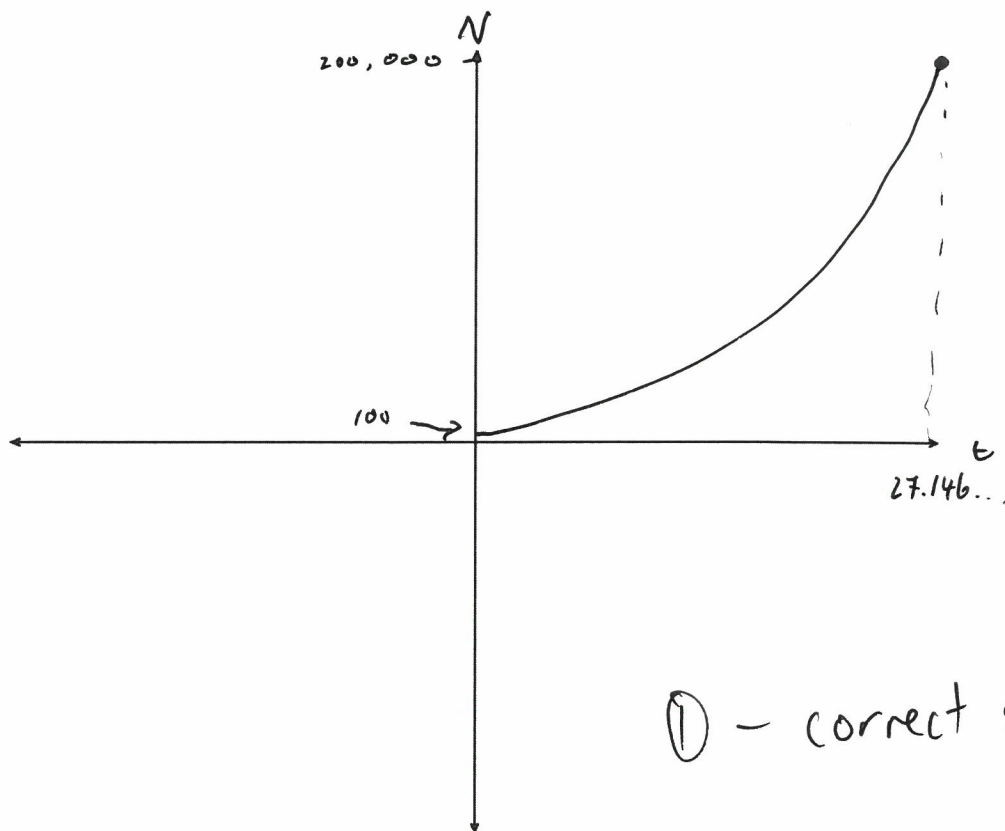
$$200000 = 100 e^{0.28t}$$

$$\frac{\ln 2000}{0.28} = t$$

$$t = 27.146 \dots$$

$$= 28 \text{ h (as when } t = 27, N < 200,000 \text{).}$$

- (iii) Graph the relationship between N and t for $N \in [100, 200\,000]$ 1



① - correct shape/location.

Question 13 continues on the next page

13 (d)

Criteria	Marks
• Correct solution, including explanation of what is happening.	3
• Correctly calculates the rate of change of the surface area when the radius is 100mm	2
• Correctly applies the chain rule	1

Sample answer

$$\frac{dr}{dt} = -0.2$$

$$SA = 4\pi r^2$$

$$\frac{dSA}{dr} = 8\pi r$$

$$\therefore \frac{dSA}{dt} = \frac{dSA}{dr} \times \frac{dr}{dt}$$

$$= -0.2 \times 8\pi r$$

$$= -\frac{8\pi r}{5}$$

when $r = 100$,

$$\frac{dSA}{dt} = -\frac{8\pi(100)}{5}$$

$$= -160\pi \text{ mms}^{-1}$$

$\therefore$ the surface area is decreasing by 160π mm per second.

Question 14

14 (a)

Criteria	Marks
• Correct solution	2
• Some progress towards solution	1

Sample answer

$$\text{let } y = \sin^{-1}(-x)$$

$$\therefore \sin y = -x, \text{ for } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$x = -\sin y$$

$$\therefore x = \sin(-y)$$

$$\therefore \sin^{-1}x = -y$$

$$y = -\sin^{-1}x$$

$$\therefore \sin^{-1}(-x) = -\sin^{-1}x$$

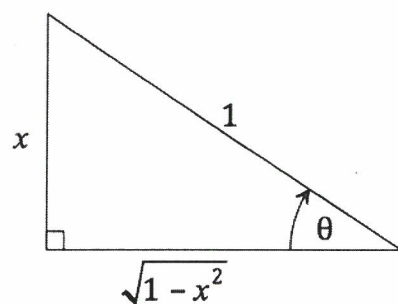
14 (b)

Criteria	Marks
• Correct solution	2
• Correctly constructs triangle	1

Sample answer

let $\theta = \sin^{-1} x$

$\therefore$ constructing triangle:



By Pythagoras' Thm, base = $\sqrt{1^2 - x^2}$

$\therefore$ using trigonometric ratios,

$$\tan \theta = \frac{x}{\sqrt{1-x^2}}$$

Question 14 (continued)

Marks

(c) Given $P(x) = 6x^3 - 11x^2 - 3x + 2$

(i) Show $x - 2$ is a factor of $P(x)$

1

$$\begin{aligned} P(2) &= 6(2)^3 - 11(2)^2 - 3(2) + 2 \\ &= 48 - 44 - 6 + 2 \\ &= 0. \end{aligned}$$

$\therefore x - 2$ is a factor of $P(x)$.

(ii) Hence, factorise $P(x)$ and find the roots of $P(x) = 0$

3

$$\begin{array}{r} 6x^2 + 7x - 1 \\ x - 2 \overline{) 6x^3 - 11x^2 - 3x + 2} \\ \underline{6x^3 - 12x^2} \\ 7x^2 - 3x + 2 \\ \underline{7x^2 - 14x} \\ 11x + 2 \\ \underline{11x - 22} \\ 24 \end{array}$$

$$\begin{aligned} \therefore P(x) &= (x - 2)(6x^2 + 7x - 1) \\ &= (x - 2)(3x - 1)(2x + 1) \end{aligned}$$

$\therefore$ for $P(x) = 0$.

$$x = -\frac{1}{2}, \frac{1}{3}, 2$$

Question 14 continues on the next page

Question 14 (continued)

Marks

- (d) Consider the polynomial $P(x) = (x - 2)(x - 5)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x - 2)$ the remainder is 4, and when it is divided by $(x - 5)$, the remainder is 13.

Find a and b

2

$$\begin{aligned} P(2) &= 4 & \therefore 2a + b &= 4 & \textcircled{1} \\ P(5) &= 13 & \therefore 5a + b &= 13 & \textcircled{2} \\ & & \textcircled{2} - \textcircled{1} & & \\ & & 3a &= 9 & \\ & \therefore a &= 3 & & \textcircled{1} \end{aligned}$$

Question 14 continues on the next page

$$\begin{aligned} \therefore 2(3) + b &= 4 \\ b &= -2 & & \textcircled{1} \end{aligned}$$

14 (e) (i)

Criteria	Marks
• Correct solution	2
• Correctly applies simultaneous equations	1

Sample answer

when $t = 0$ $T = 31.1$

$$\therefore 31.1 = P + Ae^{k \cdot 0}$$

$$\therefore 31.1 = P + A \quad (1)$$

when $t = 10$ $T = 30.7$

$$\therefore 30.7 = P + Ae^{10k} \quad (2)$$

$$(1) - (2)$$

$$0.4 = A - Ae^{10k}$$

$$Ae^{10k} = A - 0.4$$

$$e^{10k} = \frac{A - 0.4}{A}$$

$$10k = \ln \left(\frac{A - 0.4}{A} \right)$$

$$k = \frac{\ln \left(\frac{A - 0.4}{A} \right)}{10}$$

14 (e) (ii)

Criteria	Marks
• Correct solution	1

Sample answer

when $t = 0$ $T = 31.1$

$$\therefore 31.1 = 26 + Ae^{k \cdot 0}$$

$$\therefore A = 5.1$$

$\therefore$ using $i)$

$$k = \frac{\ln \left(\frac{5.1 - 0.4}{5.1} \right)}{10}$$

$$k = -0.00816....$$

$$k = -0.008 \text{ (3 d.p.)}$$

14 (e) (iii)

Criteria	Marks
• Correct solution	2
• Correctly calculates t	1

Sample answer

when $T = 38.6$

$$38.6 = 26 + 5.1 \cdot e^{-0.008t}$$

$$12.6 = 5.1 \cdot e^{-0.008t}$$

$$\frac{12.6}{5.1} = e^{-0.008t}$$

$$\ln\left(\frac{12.6}{5.1}\right) = -0.008t$$

$$t = -\frac{\ln\left(\frac{12.6}{5.1}\right)}{0.008}$$

$$t = -113.057... \text{ minutes}$$

$\therefore$ time of death = 8 pm - 113.057... minutes
= 6:07 pm