Teacher's Name________________

Student Number

Knox Grammar School

2013

Preliminary
Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Board approved calculators may be used
- All necessary working should be shown in every question

Total Marks – 70

Section I Pages 4-8

- Attempt Questions 1 – 10.
- Use answer sheet attached.
- Allow 15 minutes for this section.

Section II Pages 9-13

- Answer each question in a separate writing booklet.
- Attempt questions 11- 14.
- All questions are of equal value.
- Allow 1 hour and 45 minutes for this section.

Subject Teachers
Bradford
Harnwell
Yamaner*
Sedgman
Tran

This paper MUST NOT be removed from the examination room

Number of students in the course: 107

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use this multiple choice answer sheet for questions 1 – 10.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D)

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ● B ~~●~~ C ○ D ○

Student Number ____________________________

Knox Grammar School

Preliminary Examination

Extension 1 Mathematics 2013

Multiple Choice Answer Sheet

Teacher's Name ______________________________

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Section I

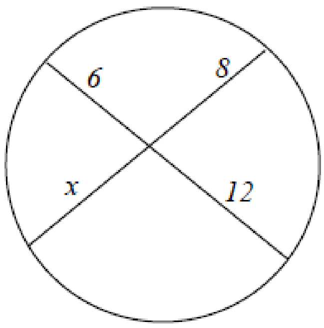
10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for questions 1 – 10.

1.



NOT TO SCALE

The value of x is:

(A) 1

(B) 4

(C) 5.5

(D) 9

2. The limiting sum of the series 81, 27, 9, 3, … is:

(A) $\frac{243}{2}$

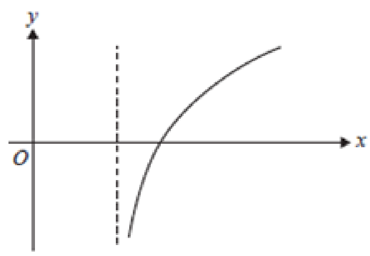
(B) $-\frac{81}{2}$

(C) 120

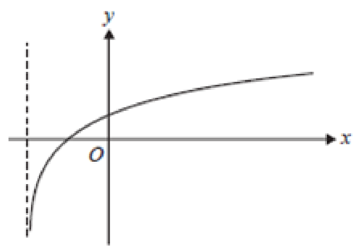
(D) $\frac{81}{2}$

3. Which of the following could be a graph of $y = a\log_2(x-b)$, where $a<0$ and $b>0$?

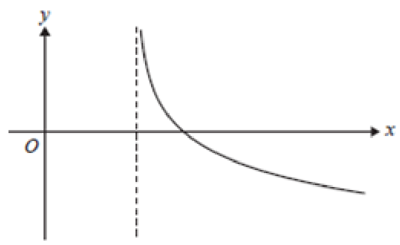
(A)



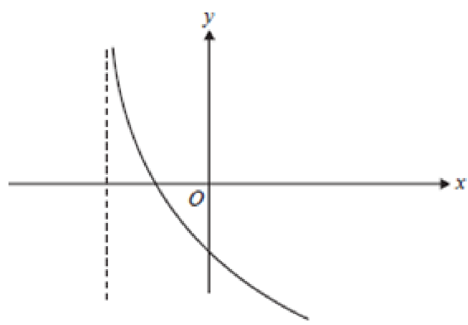
(B)



(C)



(D)



4. The variable point $P(\cos t, \sin t)$, where t is the parameter, describes which Cartesian equation.

(A) $x^2 = 4ay$

(B) $x^2 + y^2 = 1$

(C) $y = \dfrac{1}{x}$

(D) $y = \tan x$

5. Find the coordinates of the focus of the parabola $x^2 = -4(y+2)$.

(A) $(0, -3)$

(B) $(0, 1)$

(C) $(0, 0)$

(D) $(0, -1)$

6. A curve has parametric equations $x = t + 1$ and $y = 2t^2$. What is the Cartesian equation of this curve?

(A) $y = 2\sqrt{(x-1)}$

(B) $y = 2\sqrt{(x+1)}$

(C) $y = 2(x-1)^2$

(D) $y = 2(x+1)^2$

7. The x-coordinate of the point which divides the interval from $A(3,1)$ to $B(-1, 5)$ externally in the ratio $4:3$ is calculated by:

(A) $\dfrac{4(-1)+(3)(3)}{4+3}$

(B) $\dfrac{4(3)+(3)(-1)}{4+3}$

(C) $\dfrac{4(-1)+(-3)(3)}{4+(-3)}$

(D) $\dfrac{4(3)+(-3)(-1)}{4+(-3)}$

8. The acute angle θ between the lines $2x-y=4$ and $x+y=2$ is given by:

(A) $\tan\theta=\left|\dfrac{2+(-1)}{1-(2)(-1)}\right|$

(B) $\tan\theta=\left|\dfrac{2-(-1)}{1+(2)(-1)}\right|$

(C) $\tan\theta=\left|\dfrac{2+(-1)}{1+(2)(-1)}\right|$

(D) $\tan\theta=\left|\dfrac{2-(-1)}{1-(2)(-1)}\right|$

9. Given the arithmetic sequence $1,\ 2+x,\ldots$, the <u>tenth term</u> of the sequence will be:

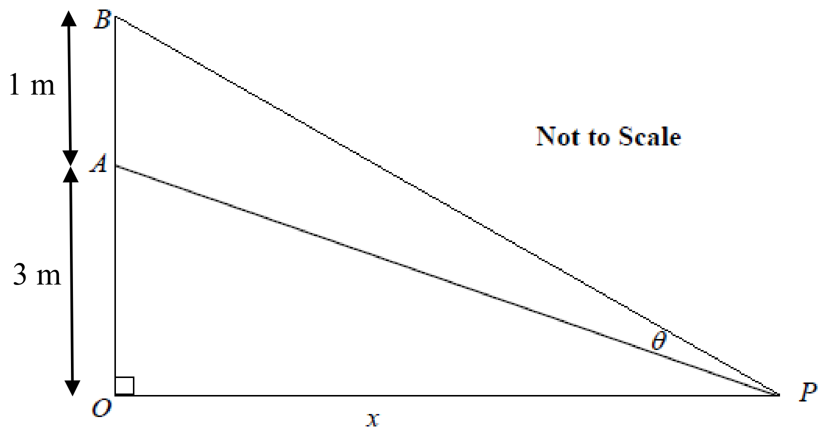
(A) $(2+x)^9$

(B) $(2+x)^{10}$

(C) $10+x$

(D) $10+9x$

10. In the diagram below, $\theta = \angle APB$, $OA = 3\,\text{m}$ and $AB = 1\,\text{m}$.



What is the correct expression for $\cos\theta$?

(A) $\dfrac{x^2 + 12}{\sqrt{9+x^2}\sqrt{16+x^2}}$

(B) $\dfrac{\sqrt{9+x^2}}{\sqrt{16+x^2}}$

(C) $\dfrac{\sqrt{9+x^2}}{\sqrt{16+x^2}\sqrt{12+x^2}}$

(D) $\dfrac{x}{\sqrt{16+x^2}}$

Section II 60 marks

Attempt Questions 11-14.
Allow about 1 hour 45 minutes for this section
Answer each question in the appropriate writing booklet. Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Simplify $2\log_3 6 - \log_3 4$. **2**

(b) Differentiate:

(i) $y = \dfrac{x^3 + 7x}{x}$ **1**

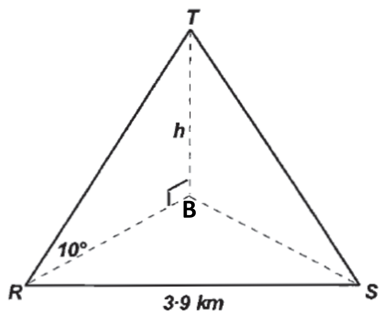
(ii) $y = \dfrac{1}{\left(x^2 - 1\right)^3}$ **2**

(c) Solve the equation $2\cos x = \sqrt{3}$ in the domain $0° \le x \le 360°$. **2**

(d) Solve the inequality $\dfrac{x^2 - 9}{x} \le 8$. **4**

(e) Two boats, Rascal and Sirocco, are sailing near Ball's Pyramid, a giant pillar of rock that rises from the sea.

The boats are 3.9 km apart, and the angle of elevation of the top T of Ball's Pyramid from Rascal, R, is 10°. $\angle TRS$ and $\angle TSR$ are 65° and 48° respectively.



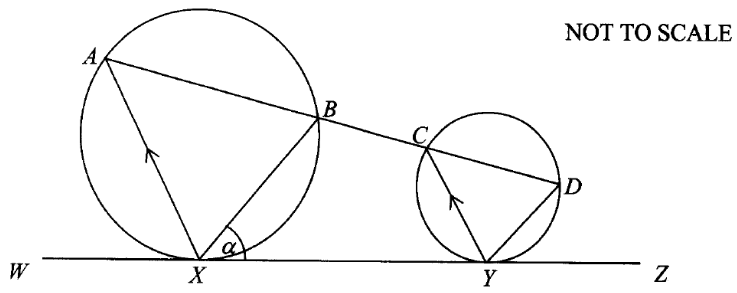
(i) Show $TR = \dfrac{3.9\sin 48°}{\sin 67°}$. **2**

(ii) Hence find the height h of Ball's Pyramid to the nearest **metre**. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the tangent to the curve $y = 2x^3 - 7$ at the point where $x = 1$. **3**

(b) In the diagram below, WZ is a common tangent to the two circles and AX is parallel to CY. AD is a straight line through B and C on the circles as shown.



Let $\angle BXY = \alpha$.

Copy the diagram into your Writing Booklet.

(i) Prove that $\angle DCY = \alpha$. **2**

(ii) Show that $BCYX$ is a cyclic quadrilateral. **1**

(iii) Explain why BX is parallel to DY. **1**

(c) Consider the function $f(x) = \dfrac{x}{x+2}$.

(i) State the domain. **1**

(ii) What is the equation of the vertical asymptote? **1**

(iii) Find the equation of the horizontal asymptote of $y = f(x)$. **1**

(iv) Without using any further calculus, sketch the graph of $y = f(x)$. **2**

(v) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(vi) Find an expression for $f^{-1}(x)$. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Rewrite the sum without sigma notation and evaluate $\sum_{n=1}^{16}(13-5n)$. **2**

(b) Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta-\sin^2\theta$. **2**

(c) Given that α and β are the roots of the equation $2x^2-6x-7=0$, find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha^2+\beta^2$ **2**

(iii) $\alpha^2-3\alpha$ **1**

(d) Ken and Barbie take out a home loan of \$460000 over 30 years at 9% p.a. compounded monthly. Let \$$M$ represent the monthly repayment and n represents the number of monthly payments made.

(i) Write an expression for A_1, the amount owing after one month. **1**

(ii) Show that the amount owing after 2 months is: **1**

$$A_2=460000(1.0075)^2-M(1+1.0075)\ .$$

(iii) Hence write an expression for A_n. **1**

(iv) Find Ken and Barbie's monthly instalment \$$M$, correct to the nearest cent. **2**

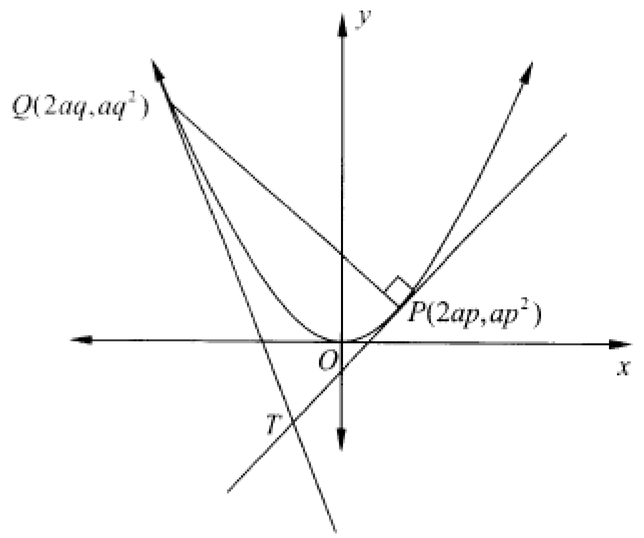
(v) Ken and Barbie decide to pay \$5000 a month. Find the amount owing after 10 years. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Given that $m^2 + n^2 = 14mn$, expand and <u>simplify</u> $\left(\frac{(m+n)}{4}\right)^2$. **2**

(ii) Hence show that $\log\left(\frac{m+n}{4}\right) = \frac{1}{2}\log(mn)$. **1**

(b) The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$. The equation of the tangent at the point P is $y = px - ap^2$ and the gradient of the chord PQ is $\frac{p+q}{2}$. The point T is the intersection of the tangent P and Q.



NOT TO SCALE

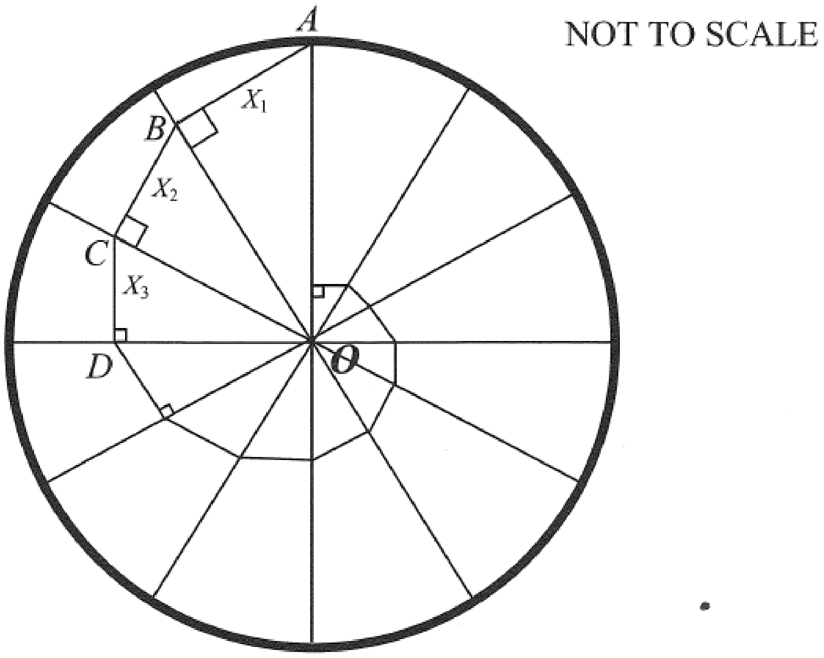
(i) Show that the coordinates of T are $\left(a\left(p+q\right), apq\right)$. **2**

(ii) Given that the chord PQ is also the normal at P, show that $p + q + \frac{2}{p} = 0$. **2**

Question 14 continues on page 13

Question 14 (continued) **Marks**

(c)



The diagram above shows a circular cartwheel with radius OA= 4cm. The cartwheel is divided into 12 equal sectors.

Perpendiculars AB, BC, CD etc., have been drawn and have respective lengths X_1, X_2, X_3 etc., as shown. Collectively, these perpendiculars form a spiral.

(i) By considering the triangle OAB, show that the distance $X_1 = 2\text{cm}$. **1**

(ii) Show that the distances $X_1, X_2, X_3, \ldots$ form a geometric sequence. **3**

(iii) Find the value of X_{12} leaving your answer in exact form. **1**

(iv) Find the length of the spiral, that is, calculate $\left(X_1 + X_2 + X_3 + \ldots + X_{12}\right)$. Give your answer to 2 decimal places. **2**

(v) If the spiral is continued indefinitely, show that its total length will not exceed 15 cm. **1**

End of Assessment

2013 Year 11 Mathematics Extension 1 YEARLY SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

MC

1. D 2. A 3. C 4. B 5. A

6. C 7. C 8. B 9. D 10. A

Question 11

a) $\log_3 6^2 - \log_3 4$ ✓

$\log_3 9$

$=2$ ✓

bi) $2x$ ✓

ii) $-3(x^2-1)^{-4}\cdot 2x$ ✓

$-6x(x^2-1)^{-4}$ ✓

c) $\cos x = \frac{\sqrt{3}}{2}$

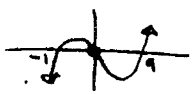
$x = 30°$ ✓

$\therefore x = 30°, 330°$ ✓

d) $x(x^2-9) \le 8x^2$ ✓ $x \ne 0$

$x(x^2-9-8x) \le 0$

$x(x-9)(x+1) \le 0$ ✓



$\therefore x \le -1$ or $0 < x \le 9$ ✓ ✓

e) $\angle RTS = 180° - 65° - 48° = 67°$ ✓

$\therefore \frac{TR}{\sin 48} = \frac{3.9}{\sin 67}$ ✓

hence $TR = \frac{3.9 \sin 48}{\sin 67}$

eii) $\frac{h}{TR} = \sin 10$

$h = \sin 10 \times TR$ ✓

$= \sin 10\left(\frac{3.9\sin 48}{\sin 67}\right)$

$= 547m$ ✓

Question 12

a) $y = 2x^3 - 7$

$y' = 6x^2$

$\therefore m = 6$ ✓ $(1,-5)$

$y-(-5) = 6(x-1)$ ✓

$y = 6x - 11$ ✓

b)i) $\angle BAX = \alpha$ (alternate segment theorem) ✓

$\therefore \angle DCY = \alpha$ (corresponding ∠'s in ∥ lines, $AX \parallel CY$) ✓

bii) $\angle DCY = \angle BXY = \alpha$ ✓

(exterior ∠ = to interior opposite ∠) then BCYX is cyclic ✓

biii) $\angle BAX = \alpha$ (alt. seg. th), $\angle DCY = \alpha$ (corr. ∠s, $AB \parallel CY$)

$\angle DYZ = \alpha$ (alt. segment)

$\therefore \angle DYZ = \angle BXZ = \alpha$

$\therefore BX \parallel DY$

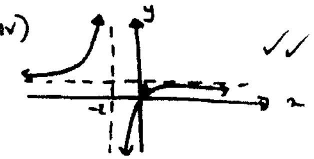
(corresponding ∠'s in ∥ lines)

ci) $x \in \mathbb{R}, x \ne -2$ ✓

ii) $x = -2$ ✓

iii) $y = 1$ ✓

iv) ✓✓



v) one to one function i.e. for each value of y in the range of f, there is exactly one x-value.

vi) $y = \frac{x}{x+2}$ ✓

$yx + 2y = x$

$x(y-1) = -2y$

$x = \frac{-2y}{y-1}$

$\therefore f^{-1}(x) = \frac{-2x}{x-1}$ ✓

2013 Year 11 Mathematics Extension 1 YEARLY SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 13

a) $8+3+(-2)+\dots+(-67)$ ✓

$S_n = \frac{n}{2}(a+l)$

$= \frac{16}{2}(8+-67)$

$= -472$

or $S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{16}{2}(2\times 8 + 15\times -5)$

$= -472$ ✓

b) LHS: $\frac{1-\tan^2\theta}{\sec^2\theta}$ ✓ (as $1+\tan^2\theta = \sec^2\theta$)

$\frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$

$\cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$

$\left(\frac{1}{\sec^2\theta} = \cos^2\theta,\ \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right)$

meaningful progress

$= \cos^2\theta - \sin^2\theta$

$=$ RHS

ci) $\alpha + \beta = -b/a = +3$ ✓

ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 3^2 - 2\left(-\frac{7}{2}\right)$

$= 16$ ✓ ✓

iii) as α is a root

$\therefore 2\alpha^2 - 6\alpha - 7 = 0$

$2\alpha^2 - 6\alpha = 7$

$\alpha^2 - 3\alpha = 7/2$

$= 3\frac{1}{2}$ ✓ $(7/2)$

di) $A_1 = 460000(1+0.0075) - M$ ✓

ii) $A_2 = A_1(1.0075) - M$

$= [460000(1.0075) - M](1.0075) - M$

$= 460000(1.0075)^2 - M(1.0075) - M$

$= 460000(1.0075)^2 - M(1.0075+1)$ ✓

iii) $A_n = 460000(1.0075)^n - M(1 + 1.0075 + \dots + 1.0075^{n-1})$ ✓

iv) $A_{360} = 0$

$460000(1.0075)^{360} - M(1 + 1.0075 + \dots + 1.0075^{359}) = 0$

$460000(1.0075)^{360} = M\left(\frac{1.0075^{360}-1}{0.0075}\right)$ ✓

$\therefore M = \$3701.26$ ✓ (no penalty if answer is not to the nearest cent i.e. $\$3701$ ok)

v) $A_{120} = 460000(1.0075)^{120} - 5000\left(\frac{1.0075^{120}-1}{0.0075}\right)$ ✓

$= \$160\,052.87$ ✓

(also accept $\$160\,053$)

2013 Year 11 Mathematics Extension 1 YEARLY SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 14

a i) $\left(\frac{m+n}{4}\right)^2 = \frac{m^2+2mn+n^2}{16}$ ✓

$= \frac{m^2+n^2+2mn}{16}$

$= \frac{14mn+2mn}{16}$

$= \frac{16mn}{16}$

$= mn$ ✓

ii) $\log\left(\frac{m+n}{4}\right)^2 = \log(mn)$ ✓

$2\log\left(\frac{m+n}{4}\right) = \log(mn)$ ✓

$\therefore \log\frac{m+n}{4} = \frac{1}{2}\log(mn)$

or $\log\sqrt{mn}$ ✓

$= \log mn^{\frac{1}{2}}$ ✓

$= \frac{1}{2}\log mn$

b i) $y = px - ap^2$ — (1)

$y = qx - aq^2$ — (2)

(1) = (2)

$px - ap^2 = qx - aq^2$ ✓

$px - qx = ap^2 - aq^2$

$x(p-q) = a(p+q)(p-q)$ ✓

$x = a(p+q)$

$y = p(a(p+q)) - ap^2$

$= ap^2 + apq - ap^2$ ✓

$= apq$

ii) $m_N \times m_T = -1$

$\frac{p+q}{2} \times p = -1$ ✓ (equating gradients) or $\frac{p+q}{2} = -\frac{1}{p}$ etc

$p^2 + pq = -2$ ✓

$\therefore p + q + \frac{2}{p} = 0$

c i) angle of each sector is $360° \div 12 = 30°$

In $\triangle AOB$, $\sin 30° = \frac{x_1}{4}$

$\frac{1}{2} = \frac{x_1}{4}$

$x_1 = 4 \times \frac{1}{2}$ ✓

$= 2$cm

ii) In $\triangle OBA$ $OB^2 = 4^2 - 2^2 = 12$ ✓

$OB = 2\sqrt{3}$ cm

In $\triangle OCB$, $\sin 30 = \frac{x_2}{OB} = \frac{x_2}{2\sqrt{3}}$

$\therefore \frac{1}{2} = \frac{x_2}{2\sqrt{3}}$ ✓

$x_2 = \sqrt{3}$

Similarly $x_3 = 3/2$

$\therefore x_1, x_2, x_3 \ldots = 2, \sqrt{3}, 3/2 \ldots$

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{\sqrt{3}}{2}$ $\therefore$ GP ✓

iii) $x_{12} = ar^{11}$

$= 2 \times (\sqrt{3}/2)^{11}$ ✓

iv) $S_{12} = \frac{a(1-r^n)}{1-r} = \frac{2\left[1-(\sqrt{3}/2)^{12}\right]}{1-\sqrt{3}/2}$ ✓

$= 12.27$ ✓

(no penalty if answer is to 2dp)

v) $S_\infty = \frac{a}{1-r}$

$= \frac{2}{1-\sqrt{3}/2}$

$= 14.9$ ✓

$\therefore$ The total length will not exceed 15cm