Number: ____________________

Teacher: ____________________

KNOX GRAMMAR SCHOOL
2015
Preliminary Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Black pen is preferred
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Teachers:
Mr Sedgman
Mr Bradford
Ms Yun
Mr Mulray
Mr Johansen*
Ms Tran

Total marks – 70

Section I: Pages 1 – 4

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 5 – 10

60 marks
- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Write your full name and your teacher's name on the front cover of each writing booklet

This paper MUST NOT be removed from the examination room

Number of Students in Course: 84

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the solution of $5^n = 3$?

(A) $n = \dfrac{\log_{10} 3}{5}$

(B) $n = \dfrac{3}{\log_{10} 5}$

(C) $n = \dfrac{\log_{10} 3}{\log_{10} 5}$

(D) $n = \log_{10}\left(\dfrac{3}{5}\right)$

2 The solution to $2\sin^2\theta - \sin\theta = 0$ for $0^\circ \le \theta \le 180^\circ$ is:

(A) $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ$

(B) $\theta = 0^\circ, 30^\circ, 150^\circ, 180^\circ$

(C) $\theta = 60^\circ$ or 120°

(D) $\theta = 30^\circ$ or 150°

3 The acute angle between the lines $2x + 2y = 5$ and $y = 3x + 1$ is θ.

What is the value of $\tan\theta$?

(A) $\dfrac{1}{7}$

(B) $\dfrac{1}{2}$

(C) 1

(D) 2

4 The solution set for the inequation $\dfrac{4}{x-1} \le 3$ is given by:

(A) $x \le 1, x \ge \dfrac{7}{3}$

(B) $x \ge \dfrac{7}{3}, x \ne 1$

(C) $x \ge 1, x \le \dfrac{7}{3}$

(D) $x < 1, x \ge \dfrac{7}{3}$

5 The equation of the normal to the parabola $x^2 = 4ay$ at the variable point $P\left(2ap, ap^2\right)$ is given by $x + py = 2ap + ap^3$.

How many different values of p are there such that the normal passes through the focus of the parabola?

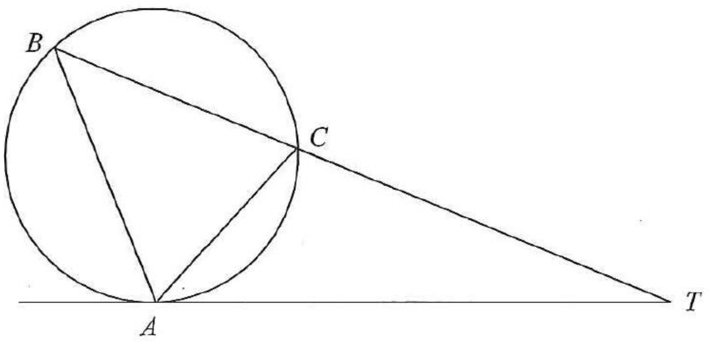
(A) 0

(B) 1

(C) 2

(D) 3

6



In the diagram, *ABC* is a triangle inscribed in a circle. The tangent to the circle at *A* meets *BC* produced at *T*. *TA* = 10 cm and *TC* = 8 cm. What is the length of *BC*?

(A) 3.5 cm

(B) 4 cm

(C) 4.5 cm

(D) 5 cm

7 A curve has parametric equations $x = at$ and $y = \frac{1}{2}\left(a + at^2\right)$. What is the Cartesian equation of the curve?

(A) $x^2 = 4ay$

(B) $x^2 = 2a\left(y - \frac{1}{2}a\right)$

(C) $x^2 = 2a(y - a)$

(D) $x^2 = 2y - a$

8 Which expression is a term of the geometric series $3x - 6x^2 + 12x^3 - \ldots$?

(A) $3072x^{10}$

(B) $-3072x^{10}$

(C) $3072x^{11}$

(D) $-3072x^{11}$

9 The radius r of a circular oil slick is increasing at a constant rate of 6 m/s. When the radius of the oil slick is 10 metres, what is the value of $\frac{dA}{dt}$, the rate of increase in the area A of the oil slick?

(A) $\frac{dA}{dt} = 240\pi$ m^2/s

(B) $\frac{dA}{dt} = 600\pi$ m^2/s

(C) $\frac{dA}{dt} = 60\pi$ m^2/s

(D) $\frac{dA}{dt} = 120\pi$ m^2/s

10 Which equation describes the locus of points (x, y) which are equidistant from the distinct points $(a+b, b-a)$ and $(a-b, b+a)$?

(A) $bx + ay = 0$

(B) $bx + ay = 2ab$

(C) $bx - ay = 0$

(D) $bx - ay = 2ab$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Write $\left(1+\sqrt{5}\right)^3$ in the form $a+b\sqrt{5}$, where a and b are integers. **2**

(b) Find the coordinates of the point $P(x,y)$ which divides the interval joining the points $A(-2,5)$ and $B(4,1)$ externally in the ratio $3:1$. **2**

(c) Solve $\left(x+\dfrac{2}{x}\right)^2-6\left(x+\dfrac{2}{x}\right)+9=0$. **3**

(d) Using the sum of two cubes, simplify: **2**

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}-1,$$

for $0^\circ<\theta<90^\circ$.

(e) For what values of b is the line $y=12x+b$ tangent to $y=x^3$? **3**

(f) $P\left(2ap,ap^2\right)$ and $Q\left(2aq,aq^2\right)$ are two points on the parabola $x^2=4ay$ with focus $(0,a)$.

(i) Show that the chord has equation $(p+q)x-2y-2apq=0$. **2**

(ii) Hence show that if $pq=-1$ then PQ is a focal chord. **1**

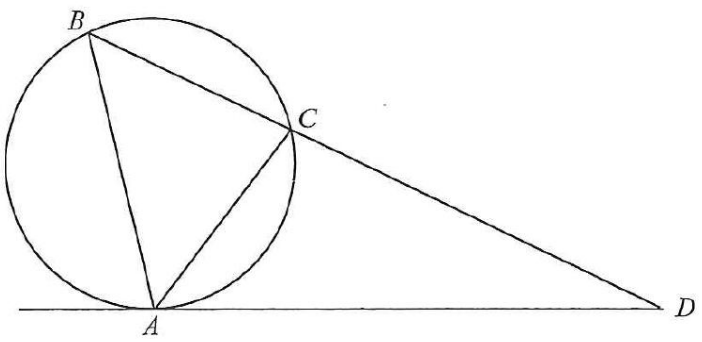
Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Use the definition of the derivative, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ when $f(x)=x^2+5x$. **2**

(b) (i) Find the horizontal asymptote of the graph $y=\dfrac{2x^2}{x^2+9}$. **1**

(ii) Without the use of calculus, sketch the graph $y=\dfrac{2x^2}{x^2+9}$, showing the asymptote found in part (i). **2**

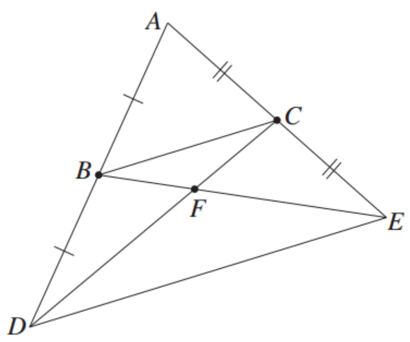
(c)



In the diagram, ABC is a triangle inscribed in a circle with $BC=AC$. The tangent to the circle at A meets BC produced at D. **3**

Show that AC bisects $\angle DAB$.

(d) The diagram shows ΔADE, where B is the midpoint of AD and C is the midpoint of AE. The intervals BE and CD meet at F.

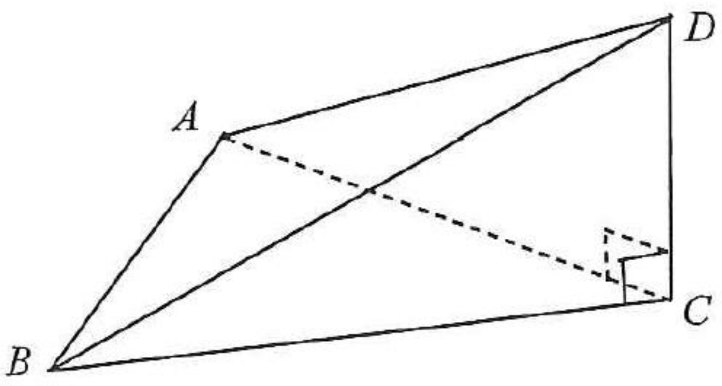


(i) Explain why ΔABC is similar to ΔADE. **1**

(ii) Hence, or otherwise, prove that the ratio $BF:FE=1:2$. **2**

Question 12 continues on page 7

(e)



In the diagram a vertical tower CD of height h metres stands with its bottom C on horizontal ground. A and B are two points on the ground such that $AB = 28$ metres. From B the bearing of the tower is 050° and the angle of elevation of the top D of the tower is 30°. From A the bearing of the tower is 110° and the angle of elevation of the top D of the tower is 60°. (*You may assume true north is up the page*.) **4**

Find the exact height of the tower.

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Prove that $\dfrac{1-\tan^2\beta}{1+\tan^2\beta}=\cos^2\beta-\sin^2\beta.$ **2**

(b) Find the coordinates of the focus, S, of the parabola $y=x^2+4$. **2**

(c) (i) When Mark started a new job, \$500 was deposited into his superannuation fund at the beginning of each month. The money was invested at 0.5% per month, compounded monthly. **2**

Let \$ P be the value of the investment after 240 months, when Mark retires.

Show that $P = 232\ 175.55$

(ii) After retirement, Mark withdraws \$2000 from the account at the end of each month, without making any further deposits. The account continues to earn interest at 0.5% per month with the interest calculated immediately before each monthly withdrawal of \$2000 . Let A_n be the amount in the account at the end of n months.

(1) Show that $A_n=\left(P-400000\right)\times1.005^n+400000.$ **3**

(2) For how many months after retirement will there be money left in the account? **2**

(d) The two points $P\left(2ap,ap^2\right)$ and $Q\left(2aq,aq^2\right)$ are on the parabola $x^2=4ay$.

(i) The equation of the tangent to $x^2=4ay$ at an arbitrary point $\left(2at,at^2\right)$ on the parabola is $y=tx-at^2$. (Do not prove this).

Show that the tangents at the points P and Q meet at R, where R is the point $\left(a(p+q),apq\right)$. **2**

(ii) As P varies, the point Q is always chosen so that $\angle POQ$ is a right angle, where O is the origin. **2**

Find the equation of the locus of R.

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch $y = \log_{10} x$ clearly indicating all relevant features. **1**

(ii) On the same sketch, find, graphically, the number of solutions of the equation **2**

$$\log_{10} x - x = -2$$

(b) Consider the geometric series $1 - \tan^2\theta + \tan^4\theta - \ldots$

(i) When the limiting sum exists, find its value in simplest form. **2**

(ii) For what values of θ in the interval $-90^\circ < \theta < 90^\circ$ does the limiting sum of the series exist? **2**

(c) (i) Rationalise the denominator in the expression **1**

$$\frac{1}{\sqrt{n} + \sqrt{n+1}},$$

where n is an integer and $n \geq 1$.

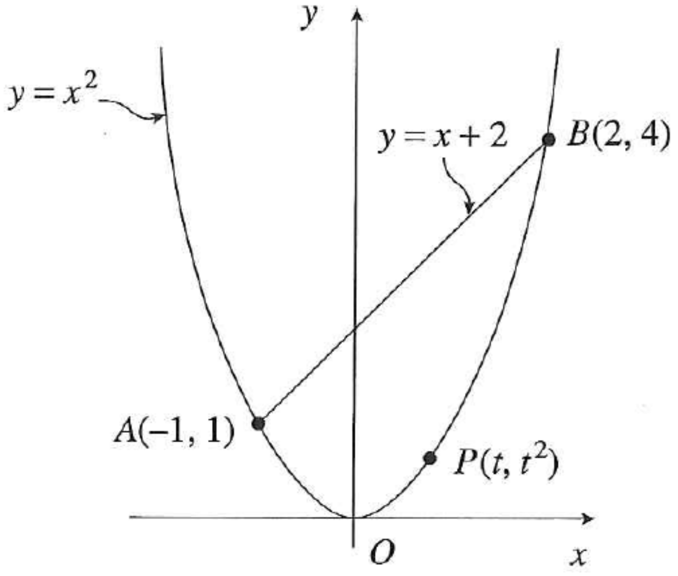
(ii) Using your result from part (i), or otherwise, find the value of the sum : **2**

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}.$$

Question 14 continues on page 10

Question 14 (continued)

(d)



In the diagram $A(-1,1)$ and $B(2,4)$ are the points of intersection of the parabola $y = x^2$ with the line $y = x + 2$. The point $P(t, t^2)$ is the variable point on the parabola below the line.

(i) Find the exact length of *AB*. **1**

(ii) Hence, or otherwise, find the maximum area of triangle *APB*. **4**

End of paper

2015 X1 Yearly Exam

Q11 → Sed
Q12 → Yus
Q13 → Mul
Q14 → Jhn

1. $5^n = 3$

$\log 5^n = \log 3$

$n \log 5 = \log 3$

$n = \frac{\log 3}{\log 5} \rightarrow \frac{\log_{10} 3}{\log_{10} 5} \rightarrow C$ ✓

2. $2\sin^2\theta - \sin\theta = 0$

$\sin\theta\left(2\sin\theta - 1\right) = 0$

$\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = 0, 180^\circ$ $\qquad \theta = 30^\circ, 150^\circ$

$\therefore \theta = 0, 30^\circ, 150^\circ, 180^\circ \rightarrow B$ ✓

3. $2x + 2y = 5$ $\qquad y = 3x+1$

$2y = -2x + 5$ $\qquad m_2 = 3$

$y = -x + \frac{5}{2}$

$m_1 = -1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-1-3}{1+(-1)(3)}\right|$

$= \left|\frac{-4}{-2}\right|$

$= 2 \rightarrow D$ ✓

4. $\frac{4}{x-1} \leq 3 \rightarrow x \neq 1$

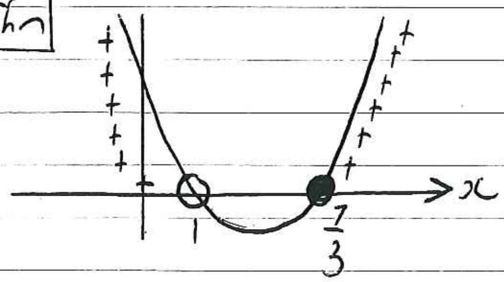
$4(x-1) \leq 3(x-1)^2$

$0 \leq 3(x-1)^2 - 4(x-1)$

$0 \leq (x-1)\left[3(x-1) - 4\right]$

$0 \leq (x-1)(3x-7)$

Multiple Choice ✓



$\therefore x < 1$ or $x \geq \frac{7}{3} \rightarrow D$ ✓

5. $x + py = 2ap + ap^3$

$x = 0, y = a \rightarrow ap = 2ap + ap^3$

$0 = ap + ap^3$

$0 = ap\left(1 + p^2\right)$

$\therefore p = 0$ is only soln $\rightarrow B$ ✓

6. $AT^2 = BT \times TC$

$10^2 = 8BT$

$100 = 8BT$

$BT = 12.5$

$\therefore BC = 12.5 - 8$

$= 4.5 \rightarrow C$ ✓

7. $x = at \rightarrow y = \frac{1}{2}\left(a + at^2\right)$

$t = \frac{x}{a}$

$= \frac{1}{2}a + \frac{1}{2}at^2$

$= \frac{1}{2}a + \frac{1}{2}a\left(\frac{x}{a}\right)^2$

$\therefore y = \frac{a}{2} + \frac{x^2}{2a}$

$2ay = a^2 + x^2$

$x^2 = 2ay - a^2$ ✓

$= 2a\left(y - \frac{1}{2}a\right) \rightarrow B$

8. $3x - 6x^2 + 12x^3 - \dots \rightarrow GP$ where $a = 3x$, $r = -2x$

$$T_n = ar^{n-1}$$
$$3072x^{11} = 3x(-2x)^{11-1}$$
$$= 3x(-2x)^{10}$$
$$= 3072x^{11} \rightarrow C$$ ✓

9. $$A = \pi r^2$$
$$\frac{dA}{dr} = 2\pi r$$
$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$
$$= 2\pi r \cdot 6$$
$$= 2\pi(10) \cdot 6$$
$$= 120\pi \ m^2/s \rightarrow D$$ ✓

10. Locus $\rightarrow PA^2 = PB^2$

$$[x-(a+b)]^2 + [y-(b-a)]^2 = [x-(a-b)]^2 + [y-(b+a)]^2$$
$$\cancel{x^2} - 2x(a+b) + \cancel{(a+b)^2} + \cancel{y^2} - 2y(b-a) + \cancel{(b-a)^2} = \cancel{x^2} - 2x(a-b) + \cancel{(a-b)^2} + \cancel{y^2} - 2y(b+a) + \cancel{(b+a)^2}$$
$$-2x(a+b) - 2y(b-a) = -2x(a-b) - 2y(b+a)$$
$$x(a+b) + y(b-a) = x(a-b) + y(b+a)$$
$$\cancel{xa} + xb + \cancel{by} - ay = \cancel{xa} - bx + \cancel{yb} + ya$$
$$bx - 2ay = 0$$
$$\therefore bx - ay = 0 \rightarrow C$$ ✓

Question 11

a) $(1+\sqrt{5})^3 = (1+\sqrt{5})(1+\sqrt{5})(1+\sqrt{5})$

$= (1+\sqrt{5})(1+2\sqrt{5}+5)$

$= (1+\sqrt{5})(6+2\sqrt{5})$ ✓

$= 6+2\sqrt{5}+6\sqrt{5}+10$

$= 16+8\sqrt{5}$ ✓

b) $A(-2,5)$ $B(4,1)$

$3 : -1$

$P = \left[\dfrac{3(4)+(-1)(-2)}{3+(-1)}, \dfrac{3(1)+(-1)(5)}{3+(-1)}\right]$ ✓

$= \left[\dfrac{12+2}{2}, \dfrac{3-5}{2}\right]$

$= (7,-1)$ ✓

c) Let $u = x+\dfrac{2}{x}$

$\therefore u^2-6u+9=0$

$(u-3)^2=0$

$u=3$ ✓

$x+\dfrac{2}{x}=3$

$x^2+2=3x$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\therefore x=2$ ✓ or $x=1$ ✓

d) $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}-1$ ✓

$= \dfrac{\cancel{(\sin\theta+\cos\theta)}(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{\cancel{\sin\theta+\cos\theta}}-1$

$= \sin^2\theta-\sin\theta\cos\theta+\cos^2\theta-1$ ✓

$= \cancel{(\sin^2\theta+\cos^2\theta)}-\sin\theta\cos\theta\cancel{-1}$

$= -\sin\theta\cos\theta$ ✓

e. $y=x^3$

$\dfrac{dy}{dx}=3x^2$

If $y=12x+b$ is a tangent,

then $3x^2=12$

$x^2=4$

$x=\pm 2$ ✓

When $x=\pm 2$, $y=\pm 8$

Sub $(2,8) \rightarrow y=12x+b$

$8=12(2)+b$

$b=-16$ ✓

Sub $(-2,-8) \rightarrow y=12x+b$

$-8=12(-2)+b$

$b=16$ ✓

$\therefore b=\pm 16$

Question 11 cont'd

) i. $m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a\cancel{(p-q)}(p+q)}{2a\cancel{(p-q)}}$

$= \frac{p+q}{2}$ ✓

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap(p+q)$

$2y - \cancel{2ap^2} = (p+q)x - \cancel{2ap^2} - 2apq$ ✓

$0 = (p+q)x - 2y - 2apq$

ii. $pq = -1 \rightarrow$ $0 = (p+q)x - 2y - 2apq$

$0 = (p+q)x - 2y + 2a$

$0 = (p+q)x - 2(y-a)$

$(p+q)x = 2(y-a)$ (1) ✓

Clearly $F(0,a)$ satisfies (1). Hence if $pq = -1$, PQ is a focal chord

Question 12

a) $$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - (x^2+5x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h - \cancel{x^2} - \cancel{5x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 5h}{h} \checkmark$$

$$= \lim_{h \to 0} 2x + h + 5$$

$$= 2x + 5 \checkmark$$

b) i. $$y = \lim_{x \to \pm\infty} \frac{2x^2}{\frac{x^2}{x^2} + \frac{9}{x^2}}$$

$$= \lim_{x \to \pm\infty} \frac{2}{1 + \frac{9}{x^2}}$$

$$= \lim_{x \to \pm\infty} \frac{2}{1+0}$$

$\therefore y = 2$ ✓ is the horizontal asymptote.

ii. $x^2 \geq 0$ for all x

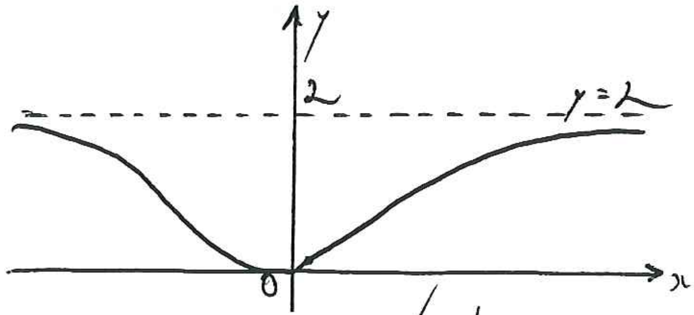
$$\therefore \frac{2x^2}{x^2+9} \geq 0 \text{ for all } x$$

If $f(x) = \dfrac{2x^2}{x^2+9}$,

$$f(0) = 0$$

Also $$f(-x) = \frac{2(-x)^2}{(-x)^2+9} = \frac{2x^2}{x^2+9} = f(x)$$

$\therefore y$ is even



✓ shape
✓ asymptote and origin

c) $\angle DAC = \angle ABC$ (angle between tangent and chord is equal to angle in alternate segment) ✓

But in isosceles ΔABC, ✱

$\angle ABC = \angle CAB$ (angles opposite equal sides are equal) ✓

$\therefore \angle DAC = \angle CAB$ ✓

Hence AC bisects $\angle DAB$

pto

✱ Alternate Segment Theorem OK well for reason

Question 12 cont'd

1) i. In Δ's ABC and ADE,

① $\angle A$ is common

② $\frac{AB}{BD} = \frac{AC}{AE}$ (given)

$\therefore \Delta ABC \parallel\!| \Delta ADE$ (2 sides in proportion and included angle equal)

ii. $\angle ABC = \angle ADE$ (corresponding angles of similar triangles)

$\therefore BC \parallel DE$ (corresponding angles equal)

$\therefore \angle CBF = \angle DEF$ (alternate angles, $BC \parallel DE$)

$\angle BCF = \angle EDF$ (alternate angles, $BC \parallel DE$)

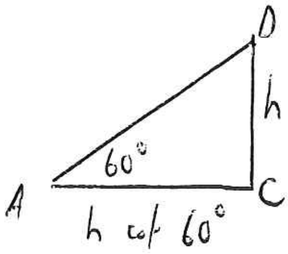
$\therefore \Delta BCF \parallel\!| \Delta EDF$ (equiangular)

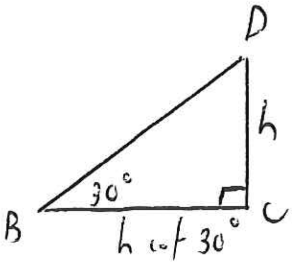
But $\frac{BC}{DE} = \frac{1}{2}$ (from similar triangles in part (i))

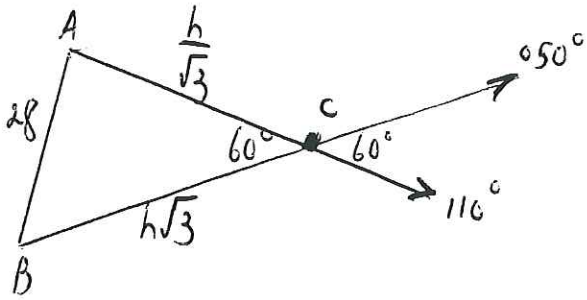
$\therefore \frac{BF}{FE} = \frac{1}{2}$ or $BF : FE = 1 : 2$ (corresponding sides of similar triangles are in proportion)

pto

Question 12 cont'd







$\therefore\ AC = \dfrac{h}{\sqrt{3}}$ ✓ $\qquad BC = h\sqrt{3}$

In ΔABC, $\quad 28^2 = 3h^2 + \frac{1}{3}h^2 - 2h^2\cos 60°$ ✓

$$784 = 3h^2 + \frac{1}{3}h^2 - h^2$$

$$784 = \frac{7}{3}h^2$$

$$h^2 = 336 \quad ✓ \rightarrow \sqrt{16} \times \sqrt{21}$$

$$h = 4\sqrt{21} \quad *$$

$\therefore$ Tower is $4\sqrt{21}$ metres tall ✓

* Also accept $28\sqrt{\frac{3}{7}}$

Question 13

a) $LHS = \dfrac{1-\tan^2\beta}{1+\tan^2\beta}$

$= \dfrac{1-\tan^2\beta}{\sec^2\beta}$ ✓ $\rightarrow 1+\tan^2\beta = \sec^2\beta$

$= \dfrac{1}{\sec^2\beta} - \dfrac{\tan^2\beta}{\sec^2\beta}$

$= \cos^2\beta - \dfrac{\sin^2\beta}{\cancel{\cos^2\beta}} \times \dfrac{\cancel{\cos^2\beta}}{1}$ ✓

$= \cos^2\beta - \sin^2\beta$ (or equivalent)

b) $y = x^2+4$

$\therefore x^2 = y-4 \rightarrow (x-h)^2 = 4a(y-k)$

$\therefore x^2 = 1(y-4) \rightarrow 4a = 1$

$a = \frac{1}{4}$ ✓

Vertex is $(0,4)$

$\therefore$ Focus, S, is $(0, 4\frac{1}{4})$ ✓

c) i) $P = 500 \times 1{\cdot}005^{240} + 500\times 1{\cdot}005^{239} + \dots + 500 \times 1{\cdot}005^{1}$

$= 500\left[1{\cdot}005 + 1{\cdot}005^2 + \dots + 1{\cdot}005^{240}\right]$ ✓

$a = 1{\cdot}005,\ r = 1{\cdot}005,\ n = 240$

$P = \dfrac{500 \times 1{\cdot}005\left(1{\cdot}005^{240}-1\right)}{1{\cdot}005-1}$

$= 232\,175{\cdot}5498$ ✓

$= 232\,175{\cdot}55$

ii) ① $A_1 = P \times 1{\cdot}005 - 2000$

$A_2 = A_1(1{\cdot}005) - 2000$

$= (P\times 1{\cdot}005 - 2000)1{\cdot}005 - 2000$

$= P(1{\cdot}005)^2 - 2000(1 + 1{\cdot}005)$ ✓

$\vdots$

$A_n = P(1{\cdot}005)^n - 2000\left(1 + 1{\cdot}005 + \dots + 1{\cdot}005^{n-1}\right)$

$a = 1$

$r = 1{\cdot}005$

$n = n$

$A_n = P(1{\cdot}005)^n - \cancel{2000}^{400000} \times 1\left(\dfrac{1{\cdot}005^n - 1}{\cancel{0{\cdot}005}}\right)$ ✓

$= P \times 1{\cdot}005^n - 400000\left(1{\cdot}005^n - 1\right)$

$= P \times 1{\cdot}005^n - 400000\left(1{\cdot}005^n\right) + 400000$ ✓

$= (P - 400000)\times 1{\cdot}005^n + 400000$

② Let $A_n = 0$

$(P - 400000)\times 1{\cdot}005^n = -400000$

$1{\cdot}005^n = \dfrac{-400000}{232\,175{\cdot}55 - 400000}$ ✓

$= 2{\cdot}383442937\dots$

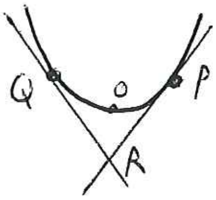
$\log 1{\cdot}005^n = \log 2{\cdot}38344\dots$

$n = \dfrac{\log 2{\cdot}38344\dots}{\log 1{\cdot}005}$ ✓

$= 174{\cdot}14312\dots$

$\therefore$ There will be money left for 175 months

Question 13 cont'd



d) i. Tangent at P

$y = px - ap^2$ (1)

Tangent at Q

$y = qx - aq^2$ (2)

Sub. (1) in (2) $\Rightarrow$ $px - ap^2 = qx - aq^2$

$px - qx = ap^2 - aq^2$

$x\cancel{(p-q)} = a\cancel{(p-q)}(p+q)$

$x = a(p+q)$

and $y = p[a(p+q)] - ap^2$

$= \cancel{ap^2} + apq \cancel{- ap^2}$

$= apq$

$\therefore R$ is $[a(p+q), apq]$

ii) If $\angle POQ = 90°$,

$m_{OP} \times m_{OQ} = -1$

$m_{OP} = \dfrac{ap^2 - 0}{2ap - 0}$

$= \dfrac{p}{2}$

and $m_{OQ} = \dfrac{q}{2}$

$\therefore \dfrac{p}{2} \times \dfrac{q}{2} = -1$

$pq = -4$

At R, $x = a(p+q)$

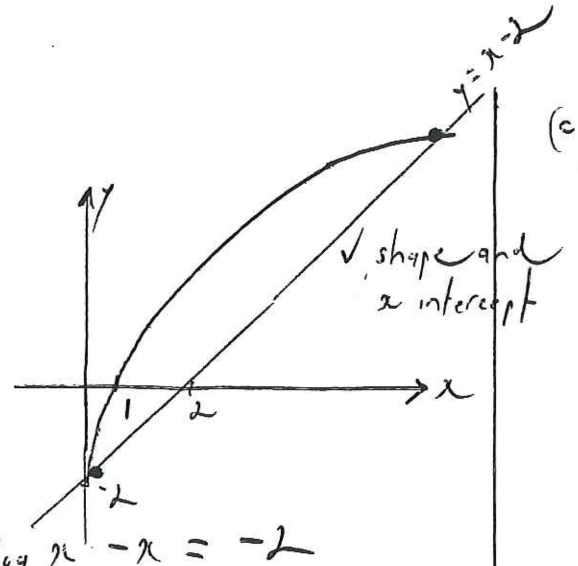
$y = apq$

$= -4a$ which is a constant

$\therefore$ Locus of R is $y = -4a$

Question 14

(a) i.



✓ shape and x intercept

✓ sketch of $y = x-2$

ii $\log_{10} x - x = -2$

$\therefore \log_{10} x = x - 2$

From the sketch, there are 2 solutions ✓

(b) i. $1 - \tan^2\theta + \tan^4\theta - \dots$

$a = 1, \quad r = -\tan^2\theta$

$S_\infty = \dfrac{1}{1-(-\tan^2\theta)}$ ✓

$= \dfrac{1}{1+\tan^2\theta}$

$= \dfrac{1}{\sec^2\theta}$

$= \cos^2\theta$ ✓

ii. Limiting sum $\rightarrow |r| < 1$

ie $|-\tan^2\theta| < 1$

$\therefore \tan^2\theta < 1$ ✓

$\therefore -1 < \tan\theta < 1$

and $-45° < \theta < 45°$ ✓

(c) i. $\dfrac{1}{\sqrt{n}+\sqrt{n+1}} \times \dfrac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$

$= \dfrac{\sqrt{n}-\sqrt{n+1}}{(\sqrt{n})^2 - (\sqrt{n+1})^2}$

$= \dfrac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$

$= \dfrac{\sqrt{n}-\sqrt{n+1}}{-1}$ ✓

$= \sqrt{n+1} - \sqrt{n}$

ii. $\dfrac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$

$n=1 \rightarrow \dfrac{1}{\sqrt{1}+\sqrt{2}} = \sqrt{2} - \sqrt{1}$

$n=2 \rightarrow \dfrac{1}{\sqrt{2}+\sqrt{3}} = \sqrt{3} - \sqrt{2}$

$n=3 \rightarrow \dfrac{1}{\sqrt{3}+\sqrt{4}} = \sqrt{4} - \sqrt{3}$

⋮ etc

$\therefore$ Series is $(\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \dots + (\sqrt{100}-\sqrt{99})$ ✓

$= -\sqrt{1} + \sqrt{100}$

$= -1 + 10$

$= 9$ ✓

pto

Question 14 cont'd

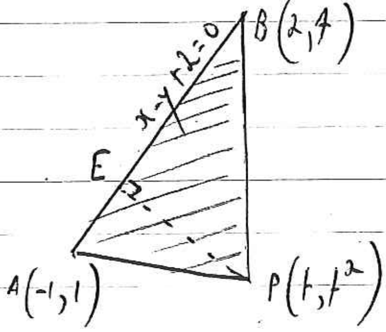
c) i. $AB = \sqrt{(2+1)^2 + (4-1)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$ ✓

ii. Area $\Delta = \frac{1}{2} \times AB \times PE$



$PE = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

x_1, y_1

(t, t^2)

$a = 1$

$b = -1$

$c = 2$

$= \left| \frac{1(t) - 1(t^2) + 2}{\sqrt{1^2 + (-1)^2}} \right|$

$= \left| \frac{t - t^2 + 2}{\sqrt{2}} \right|$

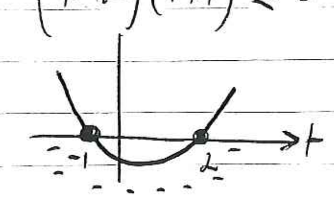
$= \frac{|t - t^2 + 2|}{\sqrt{2}}$ ✓

Hence $|t - t^2 + 2| = t - t^2 + 2$

when $t - t^2 + 2 \geq 0$,

$t^2 - t - 2 \leq 0$

$(t-2)(t+1) \leq 0$



$\therefore -1 \leq t \leq 2$ ✓

Area $\Delta APB = \frac{1}{2} \times 3\cancel{\sqrt{2}} \times \frac{t - t^2 + 2}{\cancel{\sqrt{2}}}$

$A = \frac{3}{2}(t - t^2 + 2) \quad (-1 \leq t \leq 2)$

$= -\frac{3}{2}t^2 + \frac{3t}{2} + 3$

$t = \frac{-b}{2a} \rightarrow a = \frac{3}{2}$ (sic), $b = \frac{-3}{2}$ (sic)

$= \frac{-\frac{3}{2}}{2\left(\frac{-3}{2}\right)}$

$\therefore t = \frac{1}{2}$ (Axis of symmetry) ✓

when $t = \frac{1}{2}$, $A = -\frac{3}{2}\left(\frac{1}{2}\right)^2 + \frac{3}{2}\left(\frac{1}{2}\right) + 3$

$= -\frac{3}{8} + \frac{3}{4} + 3$

$= \frac{27}{8}$ or $3\frac{3}{8}$ ✓

$\therefore$ Maximum Area $= 3\frac{3}{8}$ units^2

OR

$\frac{dA}{dt} = -3t + \frac{3}{2}$

$= -3\left(t - \frac{1}{2}\right)$

$= 0$ when $t = \frac{1}{2}$

$\frac{d^2A}{dt^2} = -3 < 0$

$\therefore$ At $t = \frac{1}{2}$ there is a maximum area

$\therefore t = \frac{1}{2}$, $A = -\frac{3}{2}\left(\frac{1}{2}\right)^2 + \frac{3}{2}\left(\frac{1}{2}\right) + 3$

$= \frac{27}{8}$ or $3\frac{3}{8}$ units^2

Marker Analysis

Y11 Ext 1 Yearly Examination 2015

Question 11 (SED)

(a) Too many boys tried to expand using a sum of two cubes. A few little pointers from Pascals triangle would have helped here. Nevertheless it was well done in the main. Expansion was the main problem.

(b) Too many boys did not know the formula. Confusion existed with whether to have a "-" ratio number and which formula to use.

(c) Mostly well done but too many did not realise that the use of a substitution v=(x+2/x) was necessary. Some tried to expand the original equation and got hopelessly lost.

(d) Incorrect expansion of the sum of two cubes was disturbing. The second bracket either had all "+" signs or had a "2" in the middle term or both errors. Far too many read the "-1" as "equals 1". They tried to form an equation and thought they had to solve for theta.

(e) Not well done. The derivative was found. Some had only one value of x. i.e. x = +2. This only gave one value of "b" Using x = -2 would give the (i)

(f) (i) Mostly well done

(ii) A little sloppy without proper explanation.

Question 12 (YUS)

(a) This was answered well by the majority of students. A few still need to work on the setting out.

(b) (i) Horizontal asymptote was generally done well. A few tried to find the vertical asymptotes, which didn't exist in this case. Typical error was $x = \pm 3$.

(ii) Some good responses came from students who drew table values. Many didn't recognise that it is an even function.

(c) A significant number of candidates did not copy the diagram. Candidates who used unclear language ran the risk of the marker being unable to interpret their meaning. Students are strongly recommended to write out their full reasoning rather than using abbreviations. Students who quoted the appropriate circle geometry theorem, e.g. 'The angle between the chord and tangent is equal to the angle in the alternate segment' (or equivalent) easily earned a mark.

(d) (i) It was 1 mark explaining question. Most students spent time proving the similar triangles.

(ii) Setting out (reasoning) could improve by many.

(e) Generally done well. Students managed to work out the expressions for the length *AC* and *BC*. Very few got the formula wrong.

Question 13 (MUL)

a) Large number of different solutions but usually well done.
b) Well done by most students who could write the fn in the correct form to find a=1/4.Those that drew a diagram were usually more successful in giving the coordinates of the focus correctly.
c) i) Well done by most students.
   Ii) part1. Common error was subtracting the monthly withdrawal before calculating the interest. Many students failed to give sufficient working to reach the given result for the amount in the account at the end of n months.
   Part2. Well done.
d) i) Well done
   ii) Varying degrees of success. Common error was p/2 times q/2 =-1 then pq = -2.

Question 14 (JHN)

a) Generally the two required graphs to demonstrate (2) solutions were drawn well; however, candidates are advised to use a ruler to draw axes and show **ALL** relevant features including intercepts.
b) Most boys were able to simplify the required limiting sum but finding the correct values in the designated interval was problematic.
c) (i) A number of candidates forfeited 1 mark for failing to recognise the -1 on the denominator once they had rationalised.

   (ii) Calculating the linked sum was difficult for a number of boys who wasted time using the sum to *n* terms formula when recognising a pattern was all that was required before summing and evaluating.

d) (i) The exact length *AB* using Pythagoras or the distance formula was completed well.
   (ii) A number of candidates failed to complete this question; however, more successful boys effectively used the Perpendicular Distance formula to assist with the required algebraic expression for the triangle's area and then generated the axis of symmetry and required maximum value. <u>All</u> candidates who were generally successful in this part forfeited 1 mark for failing to specify the correct values of *t* as it related to the perpendicular distance expression.