VIRILE AGITUR

KNOX GRAMMAR SCHOOL

2016

Year 11 Yearly Examination

Name: ____________________

Teacher: ____________________

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- The official BOSTES Reference sheet is provided
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Teachers:
Ms Ruff
Mrs Dempsey (Examiner)
Ms Tran
Mr Wilcocks

This paper MUST NOT be removed from the examination room

Total marks – 70

Section I: Pages 4 – 6

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 7 – 11

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Write your full name and your teacher's name on the front cover of each writing booklet

Number of Students in Course: 91

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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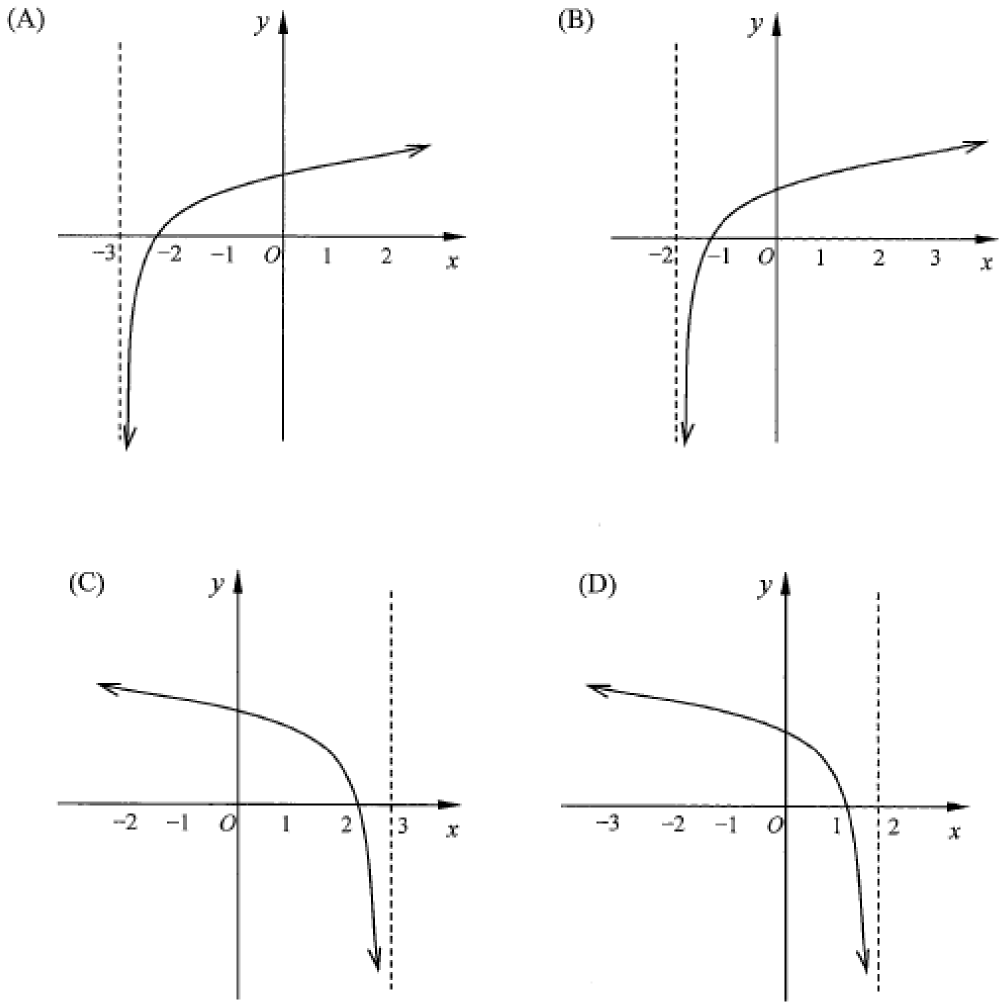
1 Find the value of $\log_5 200 - 3\log_5 2$.

(A) 1.4 (B) 2.0 (C) 3.2 (D) 2.5

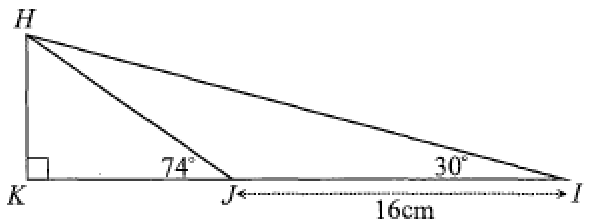
2 Let α and β be the roots of $2x^2 - 5x - 9 = 0$. Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$?

(A) $-\dfrac{9}{2}$ (B) $-\dfrac{9}{5}$ (C) $-\dfrac{5}{9}$ (D) $\dfrac{5}{2}$

3 Which diagram shows the graph of $y = 2\log_{10}(x+3)$?



4 In the diagram, find the length of *HK*, correct to two decimal places.



(A) 8.00 cm (B) 11.07 cm (C) 11.52 cm (D) 55.80 cm

5 What is the limiting sum of the geometric series

$$x^2 - 2x^3 + 4x^4 - 8x^5 + \ldots \text{ where } |x| < 1 \quad ?$$

(A) $\dfrac{x^2}{1+2x}$ (B) $\dfrac{x^2}{1-2x}$ (C) $\dfrac{-2x}{1-x^2}$ (D) $\dfrac{x^2\left[1-(-2x)^n\right]}{1+2x}$

6 Find $\lim\limits_{x \to \infty} \dfrac{3\sqrt{x}}{x-2}$.

(A) $\sqrt{x}$ (B) 3 (C) $\dfrac{3}{x}$ (D) 0

7 The solution of $|3x+7| < 3$ is :

(A) $-3\frac{1}{3} < x < -1\frac{1}{3}$ (B) $x < -3\frac{1}{3}$ *or* $x > -1\frac{1}{3}$

(C) $x < -1\frac{1}{3}$ (D) *No solution*

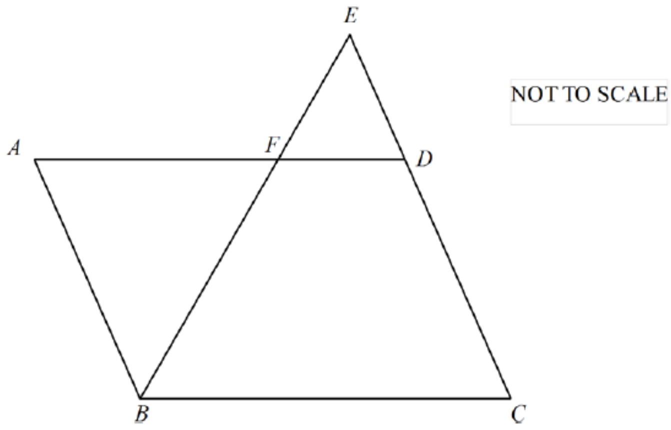
8 The second term of an arithmetic series is 39 and the sixth term is 19.
What is the sum of the first ten terms?

(A) 64 (B) 215 (C) 290 (D) 400

9 Determine the coordinates of the vertex of the parabola $x^2 - 4x - 12 = 8y$?

(A) (-2, 2) (B) (2, -2) (C) (0, 2) (D) (2, 0)

10 In the diagram below, $ABCD$ is a parallelogram. F is a point lying on AD. BF produced and CD produced meet at E. If CD: DE = 2 : 1, then what is the ratio of AF: BC?



(A) 8 : 9 (B) 2 : 3 (C) 3 : 4 (D) 1 : 2

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) Solve these simultaneous equations algebraically : **3**

$$2x - 5y = 3$$
$$x^2 - 2y^2 - 3x = 2$$

(b) Simplify

$(\sec^2\theta - 1) \times \tan(90^\circ - \theta)$ **1**

(c) Evaluate $\lim\limits_{x \to 2}\left(\dfrac{x^3 - 8}{x - 2}\right)$ **2**

(d) Determine the inverse of $f(x) = \dfrac{3x}{x+2}$ **2**

(e) Find the coordinates of the point P that divides the interval joining the points $A(-1,4)$ and $B(5,-5)$ externally in the ratio 1:2. **2**

(f) Solve $2^{x+1} = 3$ correct to two decimal places. **2**

(g) Determine the domain for which the function $y = \sqrt{1 + \dfrac{1}{1+x}}$ is defined. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) Sketch the graph $y = 2(x-1) + |x-1|$. **2**

(b) Find the gradient of the normal to the curve $x + 7y^3 = 1$ at the point where it crosses the x - axis. **3**

(c) Solve $8\cos^2 x = 2\sin x + 7$ for $0^\circ \le x \le 360^\circ$. **3**

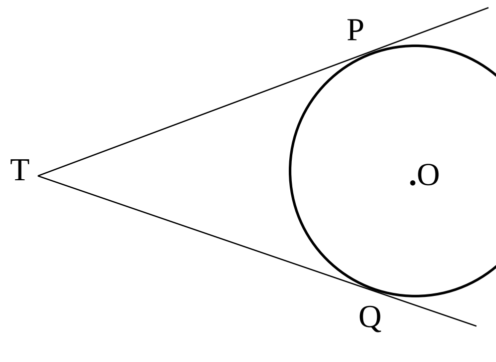
(d) Consider the circle $x^2 + y^2 - 2x - 14y + 25 = 0$.

(i) Show that if the line $y = mx$ does not intersect the circle, then **2**

$$(1+7m)^2 - 25(1+m^2) < 0.$$

(ii) For what values of m does the line $y = mx$ not intersect the circle? **2**

(e) TP and TQ are tangents to a circle centre O. **3**
Prove that $\angle PTQ = 2\angle PQO$.



End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) A spherical balloon is being inflated at a constant rate of 100 L per minute. At what rate is its surface area increasing when the radius is 2 metres? **3**

(b) (i) Show that $y = \dfrac{x^2 - x + 1}{x - 1}$ can be written as $y = x + \dfrac{1}{x-1}$. **1**

(ii) Find the y-intercept of the function and show that there are no x- intercepts. **2**

(iii) State the asymptotes of its graph. **2**

(iv) Sketch the graph of the function. **2**

(c) $P(4p, 2p^2)$ is any point on the parabola $x^2 = 8y$. S is the focus. The tangent to the parabola at P meets the y-axis at A. The perpendicular from S to the tangent PA meets the tangent at B.

(i) Find the coordinates of A and B. You may assume the equation of the tangent at P is given by $y = px - 2p^2$. **3**

(ii) Find the coordinates of the midpoint M of interval AB. **1**

(iii) Determine the Cartesian equation of the locus of the point M as P varies. **1**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) The angle of elevation of a tower PQ of height, h metres at a point A due east is 12°. From another point, B, the bearing of the tower is $051^{\circ}T$ and the angle of elevation is 11°. The points A and B are 1000 m apart and on the same level as the base Q of the tower.

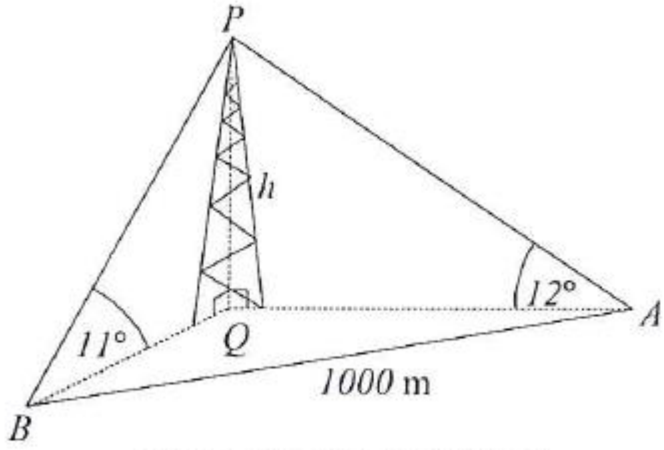


FIGURE NOT TO SCALE

(i) Show that $\angle AQB = 141^{\circ}$. **1**

(ii) Determine the height of the tower, giving your final answer correct to the nearest metre. **3**

(b) Lou takes out a loan of \$560 000. The loan is to be repaid in equal monthly repayments of \$$M$ at the end of each month, over 25 years (300 months). Reducible interest is charged at 6% per annum, calculated monthly.

Let $\$A_n$ be the amount owing after the n th repayment.

(i) Write down an expression in terms of M, for the amount owing after two months, that is, $\$A_2$. **1**

(ii) Find the value of M that pays off the loan in 300 months. **3**

(iii) After how many months will the value of $\$A_n$ become less than half of the amount borrowed? **2**

Question 14 continued on the following page

(c) A "rational" number is defined as any number that can be expressed in the form $\frac{p}{q}$ where p and $q \neq 0$ are integers, with no common factor other than 1. An "irrational" number is thus defined as a number that cannot be expressed in this form.

(i) Prove that $\sqrt{6}$ is irrational. **3**

(ii) Hence prove that $\sqrt{2}+\sqrt{3}$ is also irrational. **2**

End of Examination

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

SECTION I

1. $\log_5 200 - 3\log_5 2$
$= \log_5 200 - \log_5 2^3$
$= \log_5 \frac{200}{8}$
$= \log_5 25$
$= 2$ $\boxed{B}$

2. $2x^2 - 5x - 9 = 0$
$\alpha + \beta = \frac{5}{2}$
$\alpha\beta = -\frac{9}{2}$
$\frac{1}{\alpha} + \frac{1}{\beta}$
$= \frac{\beta + \alpha}{\alpha\beta}$
$= \frac{5}{2} \div -\frac{9}{2}$
$= \frac{5}{2} \times -\frac{2}{9}$
$= -\frac{5}{9}$ $\boxed{C}$

3. $y = 2\log_e(x+3)$
D: $x+3 > 0$ $\quad$ x-int: $y = 0$
$x > -3$ $\quad$ $0 = 2\log_e(x+3$
$0 = \log_e(x+3)$
$1 = x+3$
$\therefore x = -2$
$\boxed{A}$

4. $\tan 74° = \frac{HK}{JK}$ $\quad$ $\tan 30° = \frac{HK}{JK+16}$
$JK = \frac{HK}{\tan 74°}$ $\quad$ $JK + 16 = \frac{HK}{\tan 30°}$
$JK = \frac{HK}{\tan 30°} - 16$
$\therefore \frac{HK}{\tan 74°} = \frac{HK}{\tan 30°} - 16$
$\frac{HK}{\tan 30°} - \frac{HK}{\tan 74°} = 16$
$HK\left(\frac{1}{\tan 30°} - \frac{1}{\tan 74°}\right) = 16$

$\therefore HK = \frac{16}{\frac{1}{\tan 30°} - \frac{1}{\tan 74°}}$
$= 11.0703\ldots$
$= 11.07$ (to 2dp) $\boxed{B}$

5. $x^2 - 2x^3 + 4x^4 - 8x^5 + \ldots$
$r = \frac{-2x^3}{x^2}$ $\quad$ $S_\infty = \frac{a}{1-r}$
$= -2x$ $\quad$ $= \frac{x^2}{1-(-2x)}$
$= \frac{x^2}{1+2x}$ $\boxed{A}$

6. $\lim_{x\to\infty} \frac{3\sqrt{x}}{x-2}$
$= \lim_{x\to\infty} \frac{\frac{3\sqrt{x}}{x}}{\frac{x}{x} - \frac{2}{x}}$
$= \lim_{x\to\infty} \frac{\frac{3}{\sqrt{x}}}{1 - \frac{2}{x}}$
$= 0$ $\boxed{D}$

7. $|3x+7| < 3$
$-3 < 3x+7 < 3$
$-10 < 3x < -4$
$-\frac{10}{3} < x < -\frac{4}{3}$ $\boxed{A}$

8. $T_2 = 39$, $T_6 = 19$ $\quad$ AP
$\therefore$ $a + d = 39$
$a + 5d = 19$
$4d = -20$
$d = -5$, $\quad$ $a - 5 = 39$
$\therefore a = 44$
$S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{10} = \frac{10}{2}(2(44) + 9(-5))$
$= 215$ $\boxed{B}$

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

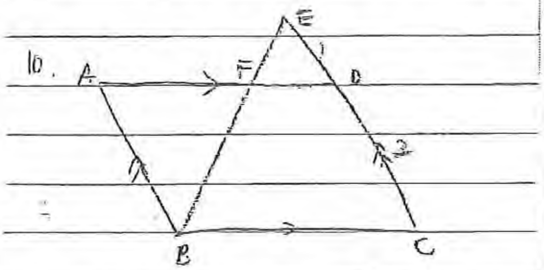
9. $x^2-4x-12=8y$

$x^2-4x+4=8y+12+4$

$(x-2)^2=8(y+2)$

vertex $(2,-2)$ $\boxed{B}$

10.



$CD:DE=2:1$

$\therefore BF:FE=2:1$ (ratio of intercepts theorem)

In Δs AFB, DFE

$\angle BAF=\angle FDE$ (alternate $\angle$s, $AB\parallel EC$)

$\angle AFB=\angle EFD$ (vertically opp angles)

$\therefore \Delta AFB \parallel\!\mid \Delta DFE$

$\therefore \dfrac{AF}{DF}=\dfrac{AB}{DE}=\dfrac{FB}{FE}$ (matching sides of $\parallel\!\mid$ Δs in same ratio)

$\therefore \dfrac{AF}{DF}=\dfrac{2}{1}$

$\therefore \dfrac{AF}{AD}=\dfrac{2}{3}$

$\therefore AF:BC=2:3$ $\boxed{B}$

(since $AD=BC$ opp sides of parallelogram)

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s) | **Comments**

SECTION II

QUESTION 11

(a) $2x - 5y = 3 \quad —①$

$x^2 - 2y^2 - 3x = 2 \quad —②$

Rearranging ① $\Rightarrow \quad x = \frac{3+5y}{2}$

Sub into ② $\Rightarrow \left(\frac{3+5y}{2}\right)^2 - 2y^2 - 3\left(\frac{3+5y}{2}\right) = 2$

$(3+5y)^2 - 8y^2 - 6(3+5y) = 8$

$9 + 30y + 25y^2 - 8y^2 - 18 - 30y = 8$

$17y^2 = 17$

$y^2 = 1$

$y = 1 \quad$ or $\quad y = -1$

$2x - 5 = 3 \qquad 2x + 5 = 3$

$x = 4 \qquad x = -1$

$\therefore x = 4, y = 1 \quad$ or $\quad x = -1, y = -1$

(3)

(b) $(\sec^2\theta - 1) \times \tan(90° - \theta)$

$= \tan^2\theta \times \frac{1}{\tan\theta}$

$= \tan\theta$

(1)

(c) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$= \lim_{x \to 2} (x^2 + 2x + 4)$

$= 2^2 + 2(2) + 4$

$= 12.$

(2)

(d) $f(x) = \frac{3x}{x+2}$

Let $y = \frac{3x}{x+2}$

$f^{-1}: \quad x = \frac{3y}{y+2}$

$x(y+2) = 3y$

$xy + 2x = 3y$

$y(3 - x) = 2x$

$y = \frac{2x}{3-x}$

$\therefore f^{-1}(x) = \frac{2x}{3-x}$

(2)

(e) $A(-1, 4) \quad B(5, -5)$

$A(-1,4)$ $\quad -1$

$B(5,-5)$ $\quad 2$

$P = \left(\frac{2(-1) - 1(5)}{-1+2}, \frac{2(4) - 1(-5)}{-1+2}\right)$

$= (-7, 13)$

(2)

(f) $2^{x+1} = 3$

$x + 1 = \log_2 3$

$x = \frac{\log_{10} 3}{\log_{10} 2} - 1$

$= 0.5849\ldots$

$= 0.58$ (to 2dp)

(2)

(g) $y = \sqrt{1 + \frac{1}{1+x}}$

D: $1 + \frac{1}{1+x} \geq 0$

$\frac{1+x+1}{1+x} \geq 0$

$\frac{2+x}{1+x} \geq 0$

$(1+x)(2+x) \geq 0$

$x \leq -2$ or $x > -1$

(3)

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

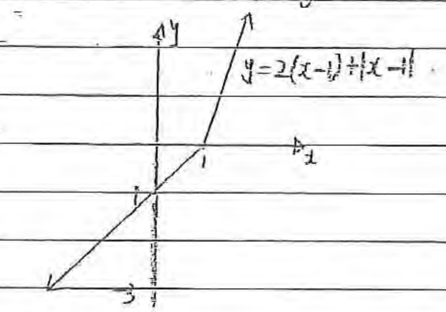
QUESTION 12.

(a) $y = 2(x-1) + |x-1|$

for $x-1 > 0$ ie. $x > 1$ $\quad y = 2(x-1)+x-1$
$= 3x-3$

for $x<1$ $\quad y = 2(x-1)+1-x$
$= x-1$

for $x=1$, $y=0$.



(2)

(b) $x+7y^3=1$

x-int: $y=0$
$\therefore x=1$. ✓

$7y^3 = 1-x$
$y = \sqrt[3]{\frac{1-x}{7}}$
$= \left(\frac{1-x}{7}\right)^{\frac{1}{3}}$

gradient fn, $\frac{dy}{dx} = \frac{1}{3}(1-x)^{-\frac{2}{3}} \times \frac{-1}{\sqrt[3]{7}}$
$= \frac{-1}{3\cdot\sqrt[3]{7}\left(\sqrt[3]{1-x}\right)^2}$ ✓ (3)

when $x=1$, gradient is undefined ✓
$\therefore$ gradient of normal $= 0$.

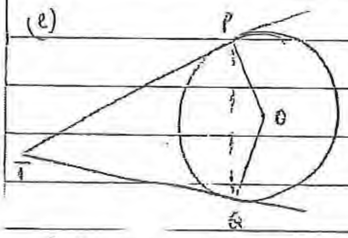
(c) $8\cos^2 x = 2\sin x + 7 \quad 0 \le x \le 360°$
$8(1-\sin^2 x) = 2\sin x + 7$
$8\sin^2 x + 2\sin x - 1 = 0$. ✓
$(4\sin x - 1)(2\sin x + 1) = 0$.
$\sin x = \frac{1}{4}$ or $\sin x = -\frac{1}{2}$ ✓ (3)
$x = 14°29', 165°31'$ $\quad x = 180°+30°, 360°-30°$
(to nearest min) $= 210°, 330°$ ✓

(d) $x^2+y^2-2x-14y+25=0$. $\quad y=mx$

(i) At points of intersection,
$x^2+(mx)^2-2x-14mx+25=0$
$x^2(1+m^2)-2x(1+7m)+25=0$ ✓
If $y=mx$ does not intersect the circle, then $b^2-4ac<0$
$(-2(1+7m))^2 - 4(25)(1+m^2)<0$ ✓
$4(1+7m)^2-100(1+m^2)<0$ (2)
$(1+7m)^2-25(1+m^2)<0$ as req'd

(ii) $(1+7m)^2-25(1+m^2)<0$
$1+14m+49m^2-25-25m^2<0$
$24m^2+14m-24<0$ ✓ (2)
$12m^2+7m-12<0$
$(4m-3)(3m+4)<0$
$-\frac{4}{3}<m<\frac{3}{4}$ ✓

(e)



Construction: join OP, OQ, PQ
Proof: Let $\angle PQO = x°$
$\therefore \angle OPQ = x°$ (base angles isos $\triangle OPQ$
equal radii) ✓
$\angle POQ + 2x = 180°$ (angle sum isos $\triangle POQ$)
$\therefore \angle POQ = 180° - 2x$
Now, $\angle OPQ = \angle OQT$ (90°, tangent ⊥ radius at
pt of contact) ✓ (3)
$\therefore \angle PTQ + 2\times 90° + 180° - 2x = 360°$
(angle sum quad OPTQ) ✓
$\angle PTQ = 2x°$
$= 2\angle PQO$ as required.

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 13

(a) $\frac{dV}{dt} = 100\text{ L/min}$

$= 0.1\text{ m}^3/\text{min}$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$= 4\pi r^2 \times \frac{dr}{dt}$

$\therefore \frac{dV}{dr} = 4\pi r^2$ ✓

$\therefore 0.1 = 4\pi \times 2^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{160\pi}$

$S = 4\pi r^2$

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$\frac{dS}{dr} = 8\pi r$

$= 8\pi r \times \frac{1}{160\pi}$

$= 8\pi(2) \times \frac{1}{160\pi}$ ✓

$= \frac{1}{10}$ ✓

$\therefore$ surface area increasing at $0.1\text{ m}^2/\text{min}$ ✓

Comment: (3)

(b) (i) $y = \frac{x^2 - x + 1}{x - 1}$

$= \frac{x(x-1)+1}{x-1}$

$= x + \frac{1}{x-1}$ as req'd

Comment: (1)

(ii) y-int : $x = 0$

$\therefore y = 0 \div 1$

$= \div 1$ ✓

x-ints : $y = 0$

$0 = x^2 - x + 1$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(1)$

$= -3$ ✓

< 0

$\therefore$ no sol'ns

$\therefore$ no x-ints

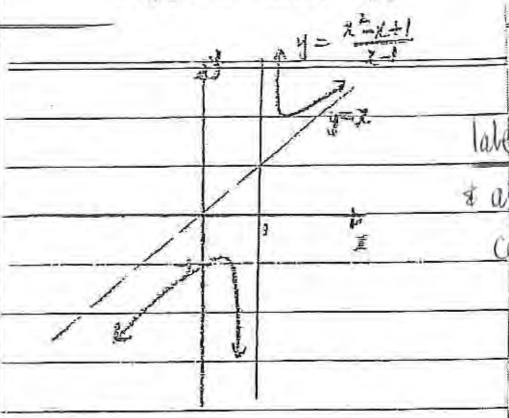
Comment: (2)

(iii) asymptotes :

vertical asymptote : $x = 1$ ✓

oblique asymptote : $y = x$ ✓

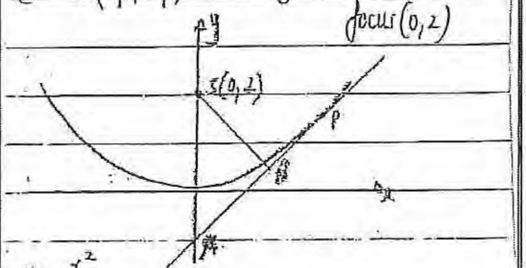
Comment: (2)



Comment: label y-int & asymptote ✓ correct position and shape ✓

(c) $P(4p, 2p^2)$ $x^2 = 8y$ $a = 2$

focus $(0, 2)$



$y = \frac{x^2}{8}$

$\frac{dy}{dx} = \frac{2x}{8}$

$= \frac{x}{4}$

when $x = 4p$, $m = \frac{4p}{4}$

$= p$

tangent eqn : $y - 2p^2 = p(x - 4p)$

$y = px - 2p^2$

at A, $x = 0 \therefore y = -2p^2$

$\therefore A(0, -2p^2)$ ✓

at B : $m = \frac{-1}{p}$

eqn SB : $y = -\frac{1}{p}x + 2$ ✓

$\therefore$ at B : $px - 2p^2 = -\frac{1}{p}x + 2$

$p^2x - 2p^3 = -x + 2p$

$x(p^2+1) = 2p^3 + 2p$

$x = \frac{2p(p^2+1)}{p^2+1}$

$= 2p$

$y = -\frac{1}{p}(2p) + 2$

$= 0$ ✓

$\therefore B(2p, 0)$

Comment: (3)

(ii) $M_{AB} = \left(\frac{0+2p}{2}, \frac{-2p^2+0}{2}\right)$

$= (p, -p^2)$ (1)

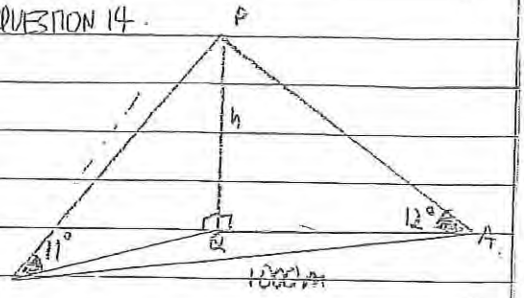
(iii) locus of midpt

$y = -x^2$ (1)

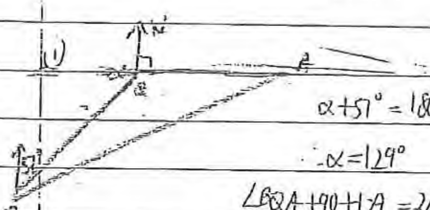
2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 14.



(i)



$\alpha + 51° = 180°$ (cointerior $\angle$s in // lines)

$\therefore \alpha = 129°$

$\angle BQA + 90 + 129 = 360°$ (angles at a point)

$\therefore \angle BQA = 141°$ (1)

(ii) $\tan 11° = \frac{h}{BQ}$ $\quad$ $\tan 12° = \frac{h}{AQ}$

$\therefore BQ = h\cot 11°$ $\quad$ $AQ = h\cot 12°$

Using Cosine Rule,

$q^2 = a^2 + b^2 - 2ab\cos Q$

$1000^2 = h^2\cot^2 11° + h^2\cot^2 12° - 2h\cot 11° \times h\cot 12° \cos 141°$

$1000\,000 = h^2(\cot^2 11° + \cot^2 12° - 2\cot 11° \cot 12° \cos 141°)$

$h^2 = \dfrac{1000\,000}{\cot^2 11° + \cot^2 12° - 2\cot 11° \cot 12° \cos 141°}$

$= 901.815\ldots$

$\therefore h = \sqrt{901.815\ldots}$

$= 30.030\ldots$

$= 30$ m (to nearest m) (3)

(b) Let A_n be the amount owing after the nth repayment and M be the monthly repayment $\quad r = \frac{0.06}{12} = 0.005$

(i) $A_1 = 560000(1.005) - M$

$\therefore A_2 = A_1(1.005) - M$

$= (560000(1.005) - M)1.005 - M$

$= 560000(1.005)^2 - M(1 + 1.005)$ (1)

(ii) $A_3 = 560000(1.005)^3 - M(1 + 1.005 + 1.005^2)$

$A_n = 560000(1.005)^n - M(1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1})$

$\therefore A_{300} = 560000(1.005)^{300} - M(1 + 1.005 + 1.005^2 + \ldots + 1.005^{299})$

GP. $a = 1$, $r = 1.005$, $n = 300$

$A_{300} = 0$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore 0 = 560000(1.005)^{300} - M\dfrac{(1.005^{300} - 1)}{1.005 - 1}$

$\dfrac{M(1.005^{300} - 1)}{0.005} = 560000(1.005)^{300}$

$M = \dfrac{560000 \times 0.005(1.005)^{300}}{1.005^{300} - 1}$

$= \$3608.0878\ldots$

$= \$3608.09$ (to nearest c) (3)

(iii) $A_n < \frac{1}{2}$

$560000(1.005)^n - \dfrac{3608.09(1.005^n - 1)}{0.005} < 280000$

$560000(1.005)^n - 721618(1.005^n - 1) < 280000$

$1.005^n(560000 - 721618) + 721618 < 280000$

$1.005^n > \dfrac{280000 - 721618}{560000 - 721618}$

$n \log 1.005 > \log 2.732\ldots$

$n > \dfrac{\log 2.7324\ldots}{\log 1.005}$

$n > 201.544\ldots$

$\therefore$ after 202 months. (2)

2016 Year 11 Mathematics Extension 1 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
e) (i) RTP $\sqrt{6}$ is irrational Suppose that $\sqrt{6}$ were rational $\therefore \sqrt{6}$ could be written as. $\sqrt{6} = \frac{a}{b}$ where a, b are integers with no common factor, $b \geq 1$. Multiplying both sides by b $\Rightarrow \sqrt{6}\, b = a$ Squaring $\Rightarrow \therefore a^2 = 6b^2$ Since $6b^2$ is even then a^2 must also be even $\therefore a$ must be even (if a were odd, a^2 would be odd) So $a = 2k$ for some integer k. So $a^2 = 4k^2$ is divisible by 4 So the right-hand side $6b^2$ is divisible by 4 $\therefore b^2$ is even $\therefore b$ must be even (if b were odd, b^2 would be odd) $\therefore$ Now both a and b are even so have common factor 2 This is a contradiction since a, b have no common factors $\therefore \sqrt{6}$ must be irrational (b) RTP $\sqrt{2}+\sqrt{3}$ is irrational Proof: Suppose $x = \sqrt{2}+\sqrt{3}$ were rational then $(\sqrt{2}+\sqrt{3})^2 = 2 + 2\sqrt{6} + 3$ $= 5 + 2\sqrt{6}$ from (a) $\sqrt{6}$ is irrational $\therefore 5+2\sqrt{6}$ is irrational $\therefore \sqrt{5+2\sqrt{6}} = \sqrt{2}+\sqrt{3}$ is irrational.	✓ 3 ✓ ✓ ✓ 2 ✓		

Marker Analysis

Y11 Ext 1 Yearly Examination 2016

Question 11 (DEL)

(a) Handled well by many students though problems arose when attempting to square $\frac{3+5y}{2}$ or $\frac{2x-3}{5}$

(b) Many students revealed they need to learn their trigonometric identities!

(c) Students need to note use of notation (write $\lim_{x\to 2}\frac{x^3-8}{x-2}$ rather than write $\lim_{x\to 2} =$)

(d) Please refer to solutions for notation. Students needed to show the interchange of x and y, then solve for y.

(e) This was a straightforward external division of an interval that a number of students made a mess of. Needed to use a formula or use the 'cross' as shown in the solutions.

(f) Done well by pretty much all candidates. Important to write the calculator display value as an incorrectly rounded value with no previously calculated result scores zero.

(g) This question caused a lot of problems but the first line needed to be D : $1+\frac{1}{1+x}\geq 0$. From there the inequality needed to be solved. Many were unsure how to do this and others ignored the fact that x could not assume the value of -1.

Question 12 (TRA)

a) Done reasonably well. Most students wrote the x and y intercepts on their graph but got the actual graph wrong. Many did not use a ruler.

b) Students need to do more revision on questions involving differentiation with fractional indices. If the student did not show their working for the gradient of the tangent as being undefined and therefore the gradient of the normal is equal to 0, then they were **not** awarded the mark. Some students wrote the gradient of the tangent = 0, therefore the gradient of the normal is also equal to 0. A carry through mark was only awarded if they did the working for the gradient of the tangent and made an error in their next step.

c) Done well.

di) Many students correctly substituted y=mx. A common error occurred when applying the discriminant and omitting the -2 in $[-2(1+7m)]^2$.

ii) Most students were awarded the 1st mark for $24m^2 + 14m - 24 < 0$ and most got the final inequality. Errors were made with factorising.

e) Very poorly done. Students need to learn their circle geometry reasons. They need to avoid abbreviating the reasons and using acronyms. To gain the full marks they needed to have the correct reason for their steps of working. Many did not provide reasons at all even though this was a "proof" question.

Question 13 (WIL)

(a) Students struggled with this part of Q13. Many scored 0. One mark was given for conversion of units from capacity to volume(either cm or m). One mark was given for dV/dt and dSA/dt and some sort of statement correctly relating these rates. One for the answer of 0.1 m^2/min or 1000 cm^2/min.

(b)(i) Most students were able to show this.

(ii) Some students struggled to show no roots using the discriminant etc. Correct reasoning was needed here.

(iii) Most identified x=1 but many were not able to identify y=x as an oblique asymptote.

(iv) Error was carried here in the sketch depending on their answer in part (iii).

(c)(i) Most students found the coordinates of A but most struggled with B.

(ii) I carried through the error with the midpoint which was a mess but if error was made in (i) the locus in part (iii) became inaccessible for most.

(iii) Only completed by a few.

Question 14 (RUE)

a. (i) A number of students did not attempt to show the size of angle BQA. Writing 90 + 51 = 141 was not sufficient for the mark to be awarded. Adequate geometrical reasoning and/or a diagram clearly supporting their working steps was required.

(ii) The majority of students were able to start this question correctly. Some mistakes were made with substitution into the cosine rule and many students need to take greater care with their notation (a number misread their own writing). Note that the final answer should be 108 m (not 30 m). Students who used AQ = tan78 and BQ = tan79 tended not to have mistakes with their final calculations.

b. Lack of time appears to have been an issue quite a few students did not attempt/finish this part of the question.

(i) Most students who attempted this question were able to determine the correct expression for the value at the end of the second month. Some students used an incorrect monthly interest rate. Some made the repayment prior to dealing with the interest calculation.

(ii) This was done reasonably well by students who attempted it. Some students need to show more working. Errors made with summing the series or due to incorrect monthly rate could only awarded marks for a carried error if working could be properly followed. Again poor writing was an issue and a few students went from writing 560000 to 500000 due to poorly formed 6s.

(iii) Done fairly well by those students who made a serious attempt at it. Some students did not realise they needed to use their answer from part (ii) which prevented them from determining the answer. A number of students incorrectly expanded $560000 \times 1.005^n - 721618(1.005^n - 1)$ (or equivalent) to get $560000 \times 1.005^n - 721618 \times 1.005^n - 17216181$.

c. Only 1 student was able to make any significant attempt at providing a solution to this part.

FINIS