KNOX GRAMMAR SCHOOL

Name: ____________________

Teacher: ____________________

2020

Year 11 Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I: 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II: 60 marks (pages 7 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Teachers:
Mr Cheah
Mr Lemaire
Mr Meli
Mr Willcocks
Ms Yun

Setter: YUS

This paper MUST NOT be removed from the examination room

Write your full name and your teacher's name on the front cover of each writing booklet

Number of Students in Course: 107

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 It is given that $\log_a 8 = 1.893$, correct to 3 decimal places.

What is the value of $\log_a 4$, correct to 2 decimal places?

A. 0.95

B. 1.26

C. 1.53

D. 2.84

2 What is the domain of the function $f(x) = \sqrt{x^2 - 3x}$?

A. $x \geq 3$

B. $x > 3$

C. $x \leq 0$ or $x \geq 3$

D. $x < 0$ or $x > 3$

3 A polynomial equation has roots α, β and γ where

$$\alpha + \beta + \gamma = -2\,,\ \alpha\beta + \alpha\gamma + \beta\gamma = 3 \text{ and } \alpha\beta\gamma = 1.$$

Which polynomial equation has the roots α, β and γ?

A. $x^3 + 2x^2 + 3x + 1 = 0$

B. $x^3 + 2x^2 + 3x - 1 = 0$

C. $x^3 - 2x^2 + 3x + 1 = 0$

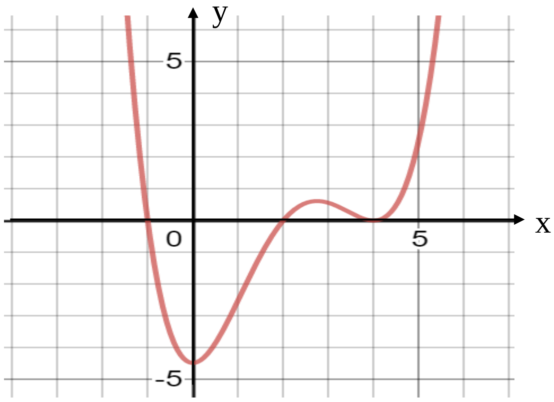
D. $x^3 - 2x^2 + 3x - 1 = 0$

4 Which expression is the equivalent to $\sin 2x \cos 2x$?

A. $\frac{1}{2}\sin 4x$ B. $2\sin 4x$

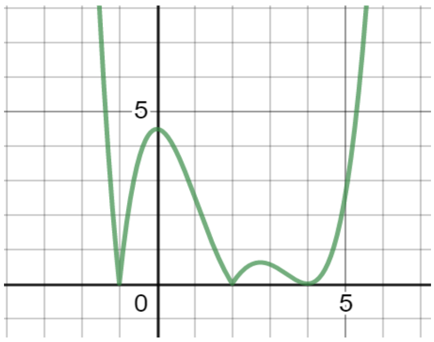
C. $\sin 4x$ D. $\sin 2x$

5 The diagram shows the graph of the function $y = f(x)$.

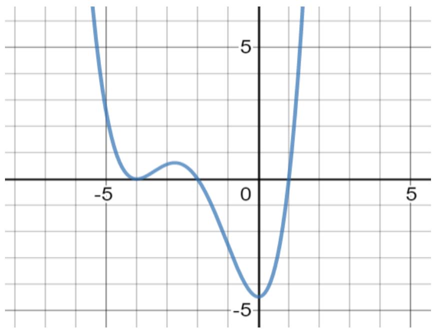


Which graph represents $y = f(|x|)$?

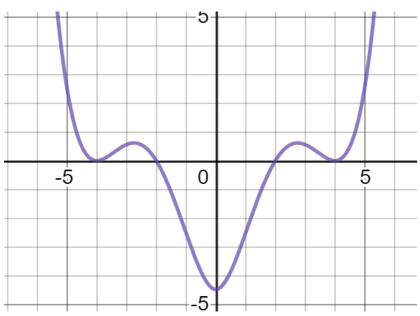
A.



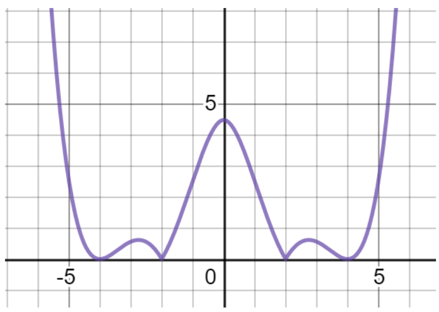
B.



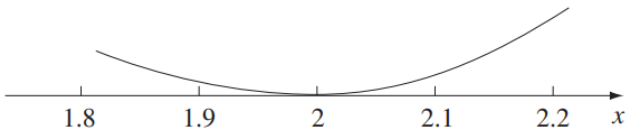
C.



D.



6 Part of the graph of $y = P(x)$, where $P(x)$ is a polynomial of degree four, is shown below.



Which of the following could be the polynomial of $P(x)$?

A. $P(x) = x^2(x+2)^2$

B. $P(x) = (x+2)^4$

C. $P(x) = x(x-2)^3$

D. $P(x) = (x-1)^2(x-2)^2$

7 How many arrangements of the letters of the word OLYMPIC are possible if the M and C are to be together in any order?

A. $5!$

B. $6!$

C. $2\times 5!$

D. $2\times 6!$

8 When the polynomial $P(x)$ is divided by $(x+1)(x-3)$, the remainder is $2x+7$.

What is the remainder when $P(x)$ is divided by $(x-3)$?

A. 13

B. 9

C. 7

D. 1

9 A circle's circumference is given by $C = \pi D$. Its area is given by $A = \pi r^2$, where D is the diameter and r is the radius.

The expression for $\dfrac{dA}{dC}$ is given by:

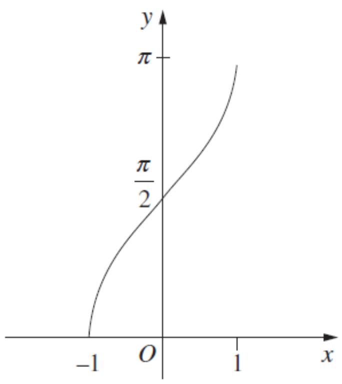
A. $\dfrac{1}{r}$

B. r

C. $\dfrac{1}{2\pi}$

D. 2π

10 The diagram shows the graph of a function.



Which function does the graph represent?

A. $y = \cos^{-1} x$

B. $y = -\cos^{-1} x$

C. $y = \dfrac{\pi}{2} + \sin^{-1} x$

D. $y = -\dfrac{\pi}{2} - \sin^{-1} x$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $|2x-6| \geq 8$. **2**

(b) Write $\dfrac{1}{n!} - \dfrac{1}{(n+1)!}$ as a single fraction. **1**

(c) Forty-five balls, numbered 1 to 45, are placed in a barrel, and one ball is drawn at random. What is the probability that the number on the ball drawn is odd? **1**

(d) In how many ways can a committee of 4 men and 5 women be selected from a group of 8 men and 9 women? **1**

(e) The polynomial $P(x) = x^2 + ax + b$ has a zero at $x = 2$. When $P(x)$ is divided by $x+1$, the remainder is 18. **3**

Find the value of a and b.

(f) A function $f(x)$ is given by $x^2 - 4x + 3$.

(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq 2$. **2**

(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (ii). **2**

(iv) Sketch the curve $y = f^{-1}(x)$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

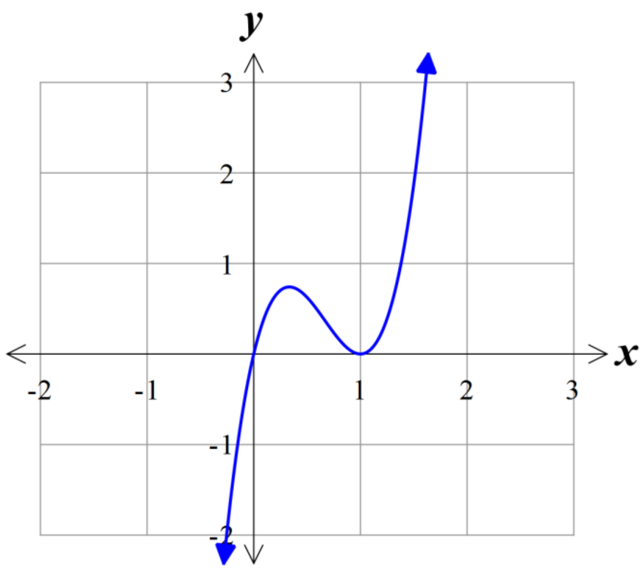
(a) (i) Find the domain and range of the function $y = 2\sin^{-1}(x-1)$. **2**

(ii) Sketch the graph of the function. **1**

(b) Find the exact value of $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right)$. **1**

(c) The polynomial $P(x) = x^3 - 12x + k$, has a double zero. **3**
Find the two possible values of *k*.

(d) The diagram shows the graph of $y = f(x)$.



By using the graph $y = f(x)$ on the answer sheet provided, sketch the following functions. Your sketches should show all asymptotes and intercepts.

(i) $y = \dfrac{1}{f(x)}$ **3**

(ii) $y = \dfrac{1}{|f(x)|}$ **1**

Question 12 continues on page 9

(e) A refrigerator has a constant temperature of $3°\mathrm{C}$. A can of drink with temperature $30°\mathrm{C}$ is placed in the refrigerator.

After being in the refrigerator for 15 minutes, the temperature of the can of drink is $28°\mathrm{C}$.

The change in the temperature of the can of drink can be modelled by $\frac{dT}{dt} = k(T-3)$, where T is the temperature of the can of drink, t is the time in minutes after the can is placed in the refrigerator and k is a constant.

(i) Show that $T = 3 + Ae^{kt}$, where A is a constant, satisfies $\frac{dT}{dt} = k(T-3)$. **1**

(ii) After 60 minutes, at what rate is the temperature of the can of drink changing? **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) A discrete random variable X has the probability distribution table shown. **3**

$X = x$	11	12	13	14
$P(x)$	0.2	0.3	m	0.4

By finding the value of m, calculate the expected value and the variance of X.

(b) Show, without a calculator, that $2\cos 75° \cos 15° = \frac{1}{2}$. **2**

(c) Sixteen bananas are shared amongst five monkeys. Show that there is at least one monkey who receives at least four bananas. **1**

(d) If one zero of the cubic $f(x) = ax^3 + bx^2 + cx + d$ is the opposite of another, prove that $ad = bc$. **3**

(e) A 10-metre ladder is leaning against a wall, and the base is sliding away from the wall at 7 cm/s. Find the rate at which the height is changing when the base of the ladder is 1.8 m from the wall, to 3 decimal places. **3**

(f) By using $t = \tan\frac{\theta}{2}$, solve the equation $\sqrt{3}\sin\theta + \cos\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

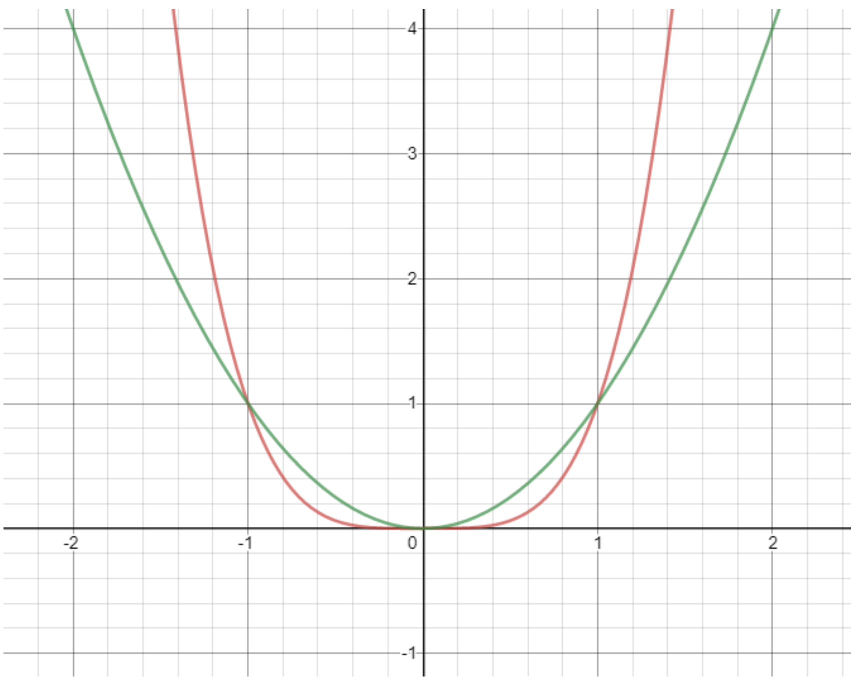
(a) Solve the inequation $\dfrac{6-x^2}{x} \geq 1$. **3**

(b) (i) Prove the trigonometric identity $\sin 3\theta = -4\sin^3\theta + 3\sin\theta$. **3**

(ii) Hence find expressions for the exact values of the solutions to the equation $8x^3 - 6x = -1$. **4**

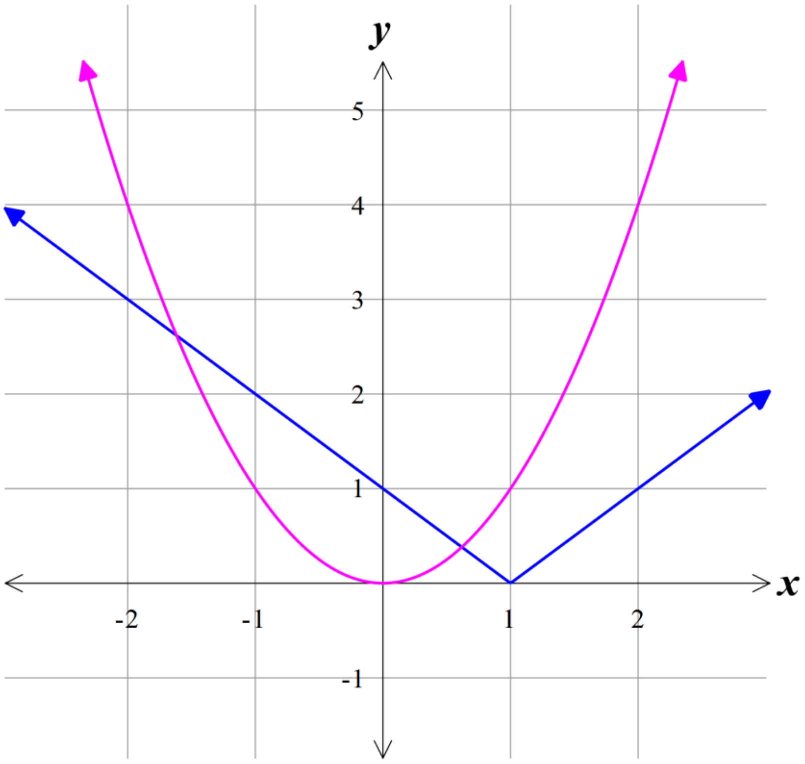
(c) The diagram shows the graph of $f(x) = x^4$ and $g(x) = x^2$. **2**

On the answer sheet provided, sketch $y = f(x) - g(x)$.



Question 14 continues on page 12

(d) An absolute value function $y = f(x)$, and a parabolic function, $y = g(x)$ are shown below. **3**



Use the answer sheet provided to sketch the graph of $y = f(x) \times g(x)$.

End of Examination

2020 Year 11 Mathematics Extension 1 Yearly SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

SECTION I

1 $\log_a 2^3 = 1.893$ (B)

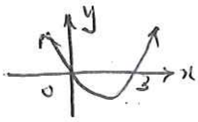
ie) $3\log_a 2 = 1.893$

So, $\log_a 2 = 0.631$

$\therefore \log_a 4 = 1.262$

2. $x^2 - 3x \geq 0$

$x(x-3) \geq 0$



$x \leq 0$ or $x \geq 3$ (C)

3. For $ax^3 + bx^2 + cx + d$

$a = 1$

$\alpha + \beta + \gamma = -\frac{b}{a}$

ie) $-2 = -b$

$\boxed{b = 2}$ (B)

$\alpha\beta\gamma = -\frac{d}{a}$

ie) $1 = -d$

$\boxed{d = -1}$

4) $\sin 4x = 2\sin 2x \cos 2x$

ie) $\sin 2x \cos 2x = \frac{1}{2}\sin 4x$

(A)

5) (C)

6. (D) includes $(x-2)^2$

7. $\boxed{CO}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

$\uparrow$

2ways (M & C together)

$6! \times 2$ (D)

8. $P(x) = (x+1)(x-3)Q(x) + 2x + 7$

$P(3) = (3+1)(3-3)Q(3) + 2(3) + 7$

$= 13$ (A)

9. $C = \pi D$ $\quad A = \pi r^2$

$C = 2\pi r$ $\quad \frac{dA}{dr} = 2\pi r$

$\frac{dC}{dr} = 2\pi$

$\therefore \frac{dA}{dC} = \frac{dA}{dr} \cdot \frac{dr}{dC}$ (B)

$= 2\pi r \times \frac{1}{2\pi}$

$= r$

10. $\sin^{-1}x$ move up by $\frac{\pi}{2}$.

(C)

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

SECTION II

Question 11

(a) $2x-6 \geq 8$ $\quad -2x+6 \geq 8$

$2x \geq 14$ $\quad -2x \geq 2$

$x \geq 7$ or $x \leq -1$

(b) $\dfrac{(n+1)-1}{n!(n+1)}$

$= \dfrac{n}{(n+1)!}$

Also accept $\dfrac{n}{(n+1)n!}$ or $\dfrac{1}{(n+1)(n-1)!}$

c) $\dfrac{23}{45}$

d) ${}^8C_4\,{}^9C_5 = 8820$

e) $P(x) = x^2 + ax + b$

$P(2) = 0$

ie) $2^2 + a(2) + b = 0$

$2a + b = -4 \quad$ — ①

$P(-1) = 18$

ie) $(-1)^2 + a(-1) + b = 18$

$-a + b = 17$ — ②

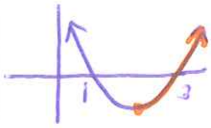
Solve ① & ② simultaneously

$3a = -21$

$\boxed{\therefore a = -7}$ $\quad \boxed{b = 10}$

(f) $f(x) = x^2 - 4x + 3$

$= (x-3)(x-1)$



• It fails horizontal line test.

• not one-to-one function

(ii) $f^{-1}(x)$: $x = y^2 - 4y + 3$

$x = y^2 - 4y + 4 - 1$

$x + 1 = (y-2)^2$ } ①

$y - 2 = \pm\sqrt{x+1}$

$y = 2 \pm \sqrt{x+1}$ $\quad y \geq 2$

So $\boxed{f^{-1}(x) : y = 2 + \sqrt{x+1}}$ ①

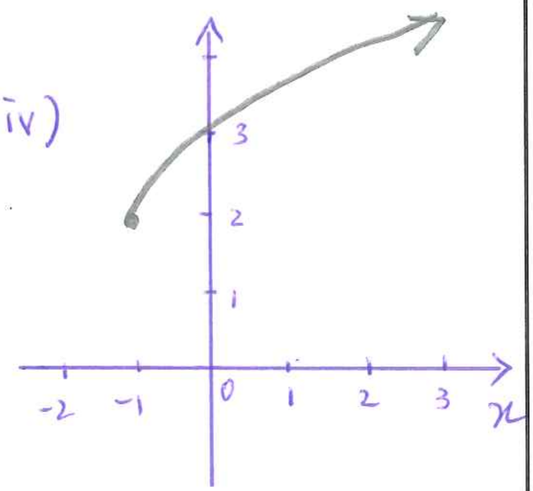
iii) Domain $x \geq -1$

Range $y \geq 2$

Domain of $f(x)$ $x \geq 2$

Range of $f(x)$ $y \geq -1$

iv)



2020 Year 11 Mathematics Extension 1 Yearly SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 12

a) (i) $\frac{y}{2} = \sin^{-1}(x-1)$

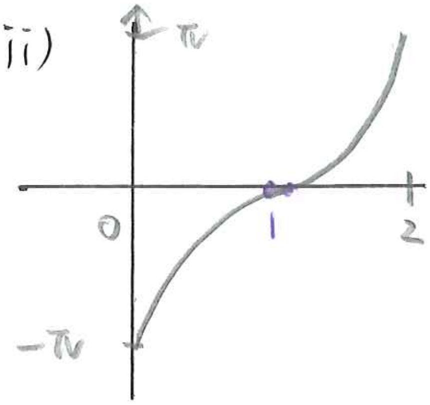
Domain $-1 \le x-1 \le 1$

ie) $0 \le x \le 2$

Range $-\frac{\pi}{2} \le \frac{y}{2} \le \frac{\pi}{2}$

ie) $-\pi \le y \le \pi$

(ii)



V shape

(e.c) from part(i)

(b) $\cos(-\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

Let $\alpha = \cos^{-1}(\frac{1}{\sqrt{2}})$

$\cos\alpha = \frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{4}$

So $\cos^{-1}(\cos(-\frac{\pi}{4})) = \frac{\pi}{4}$

(c) $P(x) = x^3 - 12x + k$

$P'(x) = 3x^2 - 12$

Let the root be α

$P'(\alpha) = 0$

ie) $3\alpha^2 - 12 = 0$

$\alpha^2 = 4$

$\therefore \alpha = \pm 2$ 1

3

(c) cont. $P(2) = 0$

$P(2) \Rightarrow 2^3 - 12(2) + k = 0$

ie) $\boxed{k = 16}$ 1

$P(-2) \Rightarrow (-2)^3 - 12(-2) + k = 0$

$\boxed{k = -16}$ 1

(d) see separate diagram

(e)(i) $T = 3 + Ae^{kt}$

$\frac{dT}{dt} = k.Ae^{kt}$

but $Ae^{kt} = T - 3$

$\therefore \frac{dT}{dt} = k(T-3)$

(ii) $t = 60$ $\frac{dT}{dt} = ?$

$t = 0$ $T = 30$

ie) $T = 3 + Ae^{kt}$

$30 = 3 + Ae^0$

$\therefore A = 27$ — 1 for A

$t = 15$ $T = 28$

ie) $28 = 3 + 27e^{15k}$ — $k = -0.00513$

$\frac{25}{27} = e^{15k}$

$k = \frac{1}{15}\ln(\frac{25}{27})$ — 1 for k

$t = 60$, $T = 3 + 27e^{k.60}$

$= 22.8458...$

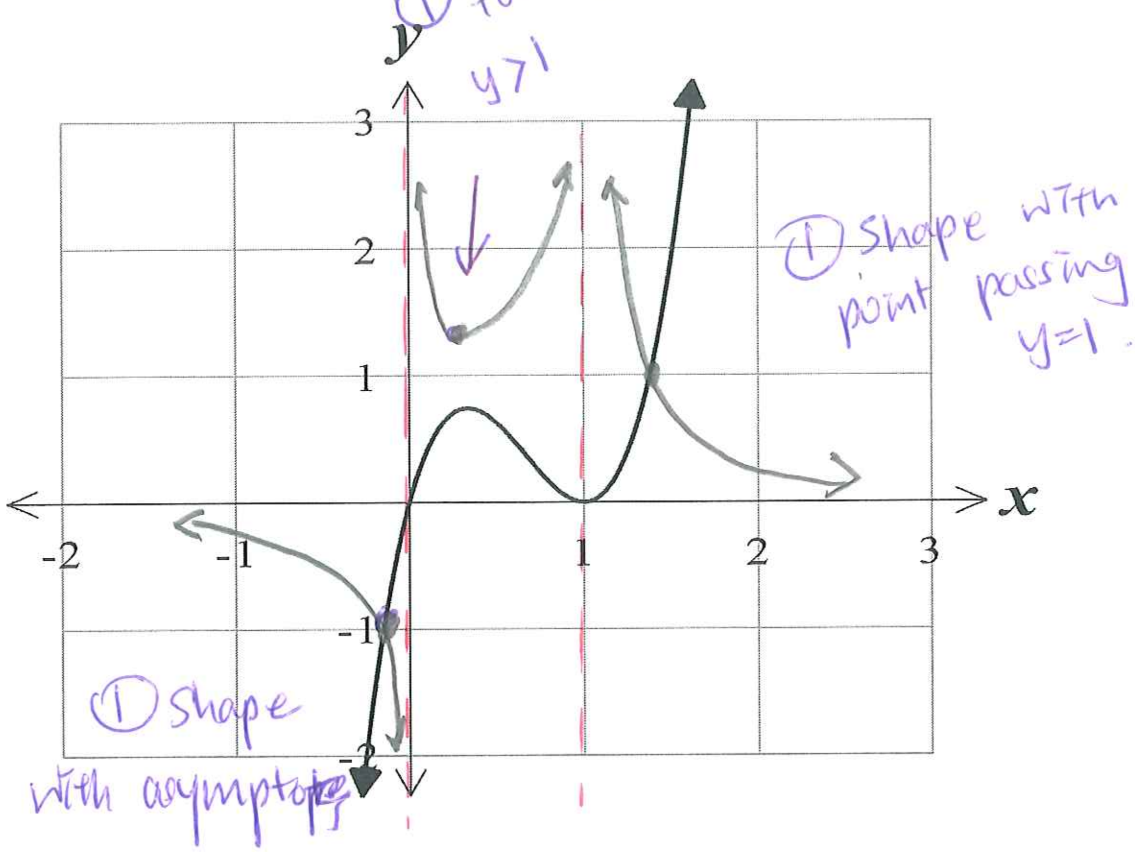
So $\frac{dT}{dt} = \frac{1}{15}\ln(\frac{25}{27})(22.8458 - 3)$

$= -0.1018$ 1

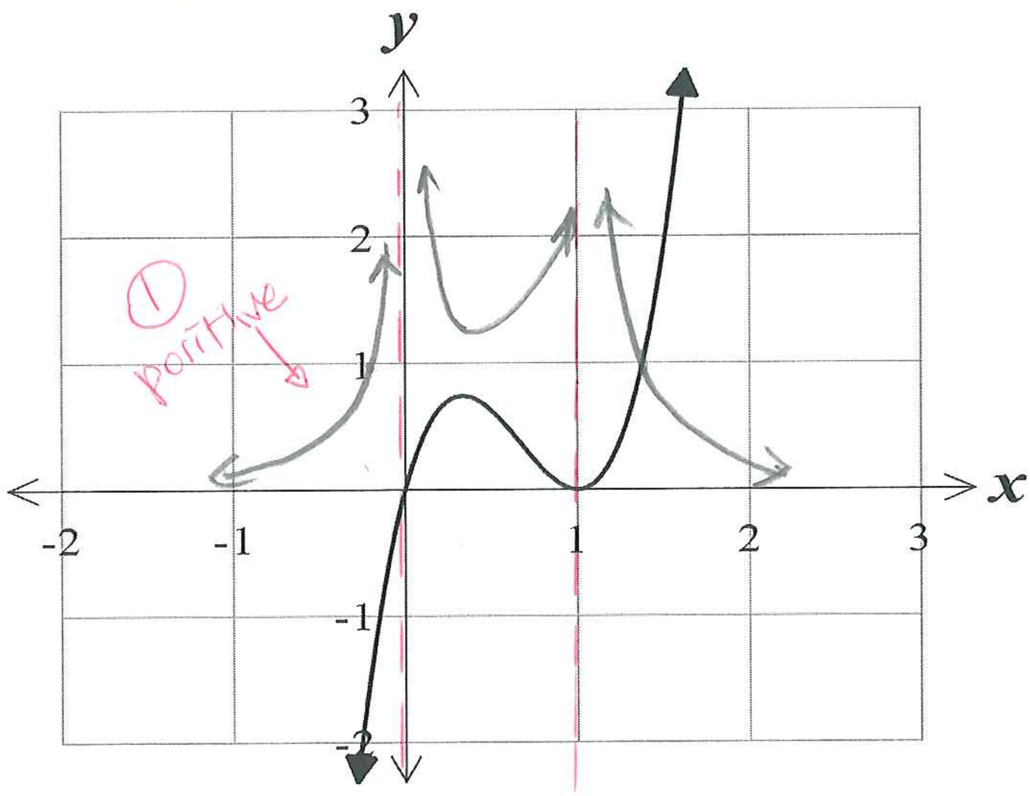
Temp. is decreasing at $0.102°/min$

Answer sheets for Question 12 (d)

(i)



(ii)



④

2020 Year 11 Mathematics Extension 1 Yearly SOLUTIONS

Suggested Solution (s)	Comments

Question 13

(a)

x	11	12	13	14	Sum
$P(x)$	0.2	0.3	0.1	0.4	1
$E(x)$	2.2	3.6	1.3	5.6	12.7
$E(x^2)$	24.2	43.2	16.9	78.4	162.7

$m = 0.1$ ✓

$E(X) = 12.7$ ✓

$Var(x) = E(x^2) - \mu^2$

$= 162.7 - (12.7)^2$

$= 1.41$ ✓

(b) R.T.P $2\cos 75 \cos 15 = \frac{1}{2}$

LHS $= 2\cos 75 \cos 15$

$= \cos(75+15) + \cos(75-15)$

$= \cos 90 + \cos 60$

$= 0 + \frac{1}{2}$ ✓

$= \frac{1}{2}$

$=$ RHS

(c) There are 16 bananas & 5 monkeys.

ie) $\frac{16}{5} = 3\frac{1}{5}$ ✓ $(x \geq 3\frac{1}{5} = 4)$

By the pigeonhole principle, there are at least one monkey receives at least 4 bananas.

(d) Let the roots be $\alpha, -\alpha$ & β.

So, $\alpha + (-\alpha) + \beta = -\frac{b}{a}$

ie) $\boxed{\beta = -\frac{b}{a}}$ — ① ✓

$-\alpha^2 + \alpha\beta - \alpha\beta = \frac{c}{a}$

$-\alpha^2 = \frac{c}{a}$

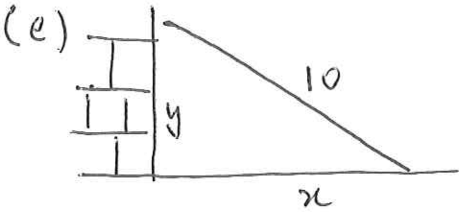
ie) $\boxed{\alpha^2 = \frac{-c}{a}}$ — ② ✓

$\alpha(-\alpha)\beta = -\frac{d}{a}$

ie) $\boxed{-\alpha^2\beta = -\frac{d}{a}}$ — ③

$\therefore -\left(-\frac{c}{a}\right)\left(-\frac{b}{\cancel{a}}\right) = \frac{-d}{\cancel{a}}$ ✓

$\therefore bc = da$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Q13 cont. (e)  $\frac{dx}{dt} = 7\text{cm/s}$ $= 0.07 \text{ m/s}$ $\frac{dy}{dt} = ?$ when $x = 1.8\text{m}$ Now $x^2+y^2=10^2$ ie) $y^2 = 100 - x^2$ $y = (100-x^2)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(100-x^2)^{-\frac{1}{2}} \times -2x$ $= \frac{-x}{\sqrt{100-x^2}}$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ @ $x = 1.8\text{m}$ $= \frac{-1.8}{\sqrt{100-1.8^2}} \times 0.07$ $= -0.0128092\ldots$ $\doteqdot -0.013 \text{ m/s}$ OR decreasing at 1.28 cm/s	1 1 1 sign & unit included	(f) Let $t = \tan\frac{\theta}{2}$ $\sqrt{3}\sin\theta + \cos\theta = 1$ ie) $\frac{2\sqrt{3}t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$ $\cancel{1} + 2\sqrt{3}t - t^2 = \cancel{1} + t^2$ $2t^2 - 2\sqrt{3}t = 0$ $2t(t-\sqrt{3}) = 0$ $t = 0$ or $t = \sqrt{3}$ $\tan\frac{x}{2} = 0$ $\tan\frac{x}{2} = \sqrt{3}$ $\frac{x}{2} = 0, \pi$ $\frac{x}{2} = \frac{\pi}{3}$ $\therefore x = \underbrace{0, 2\pi}_{1}$ or $\underbrace{\frac{2\pi}{3}}_{1}$.	1

2020 Year 11 Mathematics Extension 1 Yearly SOLUTIONS

Suggested Solution (s)	Comments

Question 14

a) $x^2 \times \dfrac{6-x^2}{x} \geq 1 \times x^2$ note $x \neq 0$

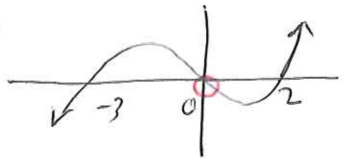
$x(6-x^2) \geq x^2$ (1)

$x(6-x^2) - x^2 \geq 0$

$-x\left[-6+x^2+x\right] \geq 0$

$-x(x+3)(x-2) \geq 0$ (1)

ie $x(x+3)(x-2) \leq 0$



$\therefore \; x \leq -3$ or $0 < x \leq 2$. (1) $x \neq 0$

(b)(i) $\sin(2\theta+\theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ (1)

$= (2\sin\theta\cos\theta)\cos\theta + (2\cos^2\theta - 1)\sin\theta$ (1)

$= 2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$

$= 4\sin\theta(1-\sin^2\theta) - \sin\theta$ (1)

$= 4\sin\theta - 4\sin^3\theta - \sin\theta$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(b)(ii) $8x^3 - 6x = -1$

$-2(3x - 4x^3) = -1$

$3x - 4x^3 = \frac{1}{2}$

Let $x = \sin\theta$

$4\sin^3\theta - 3\sin\theta = -\sin 3\theta$

$8\sin^3\theta - 6\sin\theta = -2\sin 3\theta$

$2\sin 3\theta = 1$

$\sin 3\theta = \frac{1}{2}$ (1)

$3\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, \ldots$ (1)

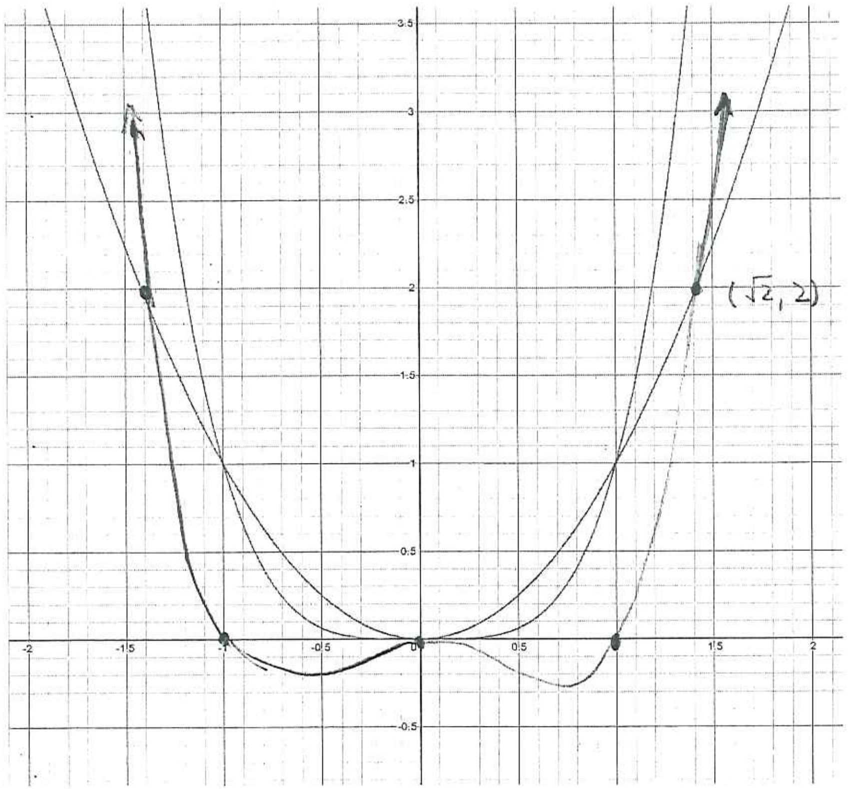
$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}, \ldots$ (1)

$\therefore x = \sin\left(\frac{\pi}{18}\right), \sin\left(\frac{5\pi}{18}\right), \sin\left(\frac{25\pi}{18}\right)$, (1) for 3 distinct solutions.

or $-\sin\frac{7\pi}{18}$

(7)

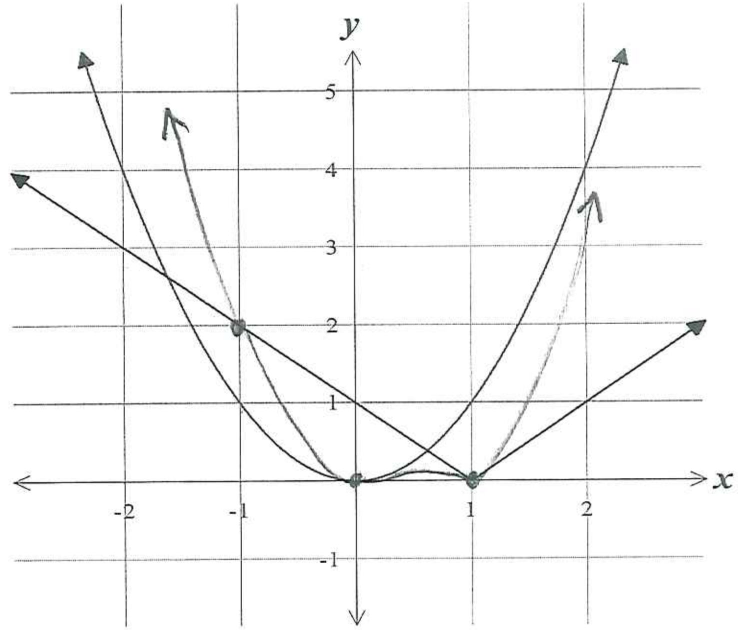
Answer sheet Q 14 (c)



① 3 x intercepts

① shape including behaviour as $x \to \infty$

Answer sheet Q 14 (d)



✓ 2 x-intercepts

✓ passing $(-1, 2)$ (or a point)

✓ shape