KNOX GRAMMAR SCHOOL

2021 Preliminary HSC Assessment Task 3 – Online

Year 11 Mathematics Extension 1

General Instructions

- Total Time Allowed: 70 minutes
  - 5 minutes reading time
  - 60 minutes writing time
  - 5 minutes scan and upload time
- You may be penalised if you submit your assessment after the closing time
- Write using black or blue pen only
- Write on your own single-side lined paper
- NESA approved calculators may be used
- This examination is open-book, and you may use your own textbook and notes
- You are not permitted to use any electronic device to research answers
- For Questions 1 - 10 in Section I, write down only the letter of your chosen answer
- For Questions 11 - 13 in Section II, show relevant mathematical reasoning and/or calculations
- Setter: DEL

Total Marks

40

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 30 marks (pages 5 – 8)

- Attempt Questions 11 – 13
- Allow about 45 minutes for this section

MC	Q11	Q12	Q13	TOTAL
/10	/10	/10	/10	**/40**

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Write down only the letter of your chosen answer for Questions 1 – 10.

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1. What is the solution to the equation $4^x - 9(2^x) + 8 = 0$?

(A) $x = 1$ or $x = 3$ (B) $x = 1$ or $x = 4$

(C) $x = 0$ or $x = 3$ (D) $x = 0$ or $x = 4$

2. What is the derivative of $(5x+4)^3(x+1)^4$?

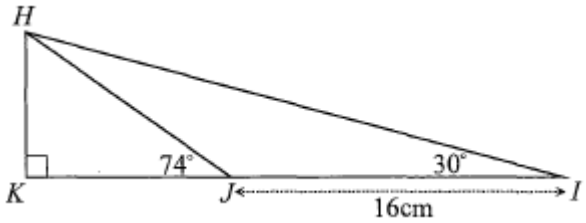
(A) $(5x+4)^2(x+1)^3(6x+5)$ (B) $(5x+4)^2(x+1)^3(10x+9)$

(C) $1212(5x+4)^2(x+1)^3$ (D) $(5x+4)^2(x+1)^3(35x+31)$

3. Given that $t = \tan\frac{x}{2}$, which of the following is an expression for $1 + secx$?

(A) $\frac{2t}{1+t^2}$ (B) $\frac{2t}{1-t^2}$ (C) $\frac{2}{1+t^2}$ (D) $\frac{2}{1-t^2}$

4. In the diagram, find the length of *HK*, correct to two decimal places.



(A) 8.00 cm (B) 11.07 cm (C) 11.52 cm (D) 55.80 cm

5. If $ln3a = lnb - 2lnc$ where $a, b, c > 0$, which of the following is true?

(A) $a = \frac{b - c^2}{3}$ (B) $a = \frac{b}{3c^2}$

(C) $ln3a = \frac{b}{c^2}$ (D) $ln3a = \frac{lnb}{lnc^2}$

6. A spherical hailstone is forming in a cloud with its radius increasing by 2mm/s. At what rate, in mm^3/s, is the volume increasing when the radius is 5 mm?

(A) $\frac{500\pi}{3}$ (B) 200π (C) 100π (D) 50π

7. The number of followers of an Instagram account, N grows according to the formula $N = 80 + 20e^{kt}$, where t is the time in hours and $k = 0.2$.

How long will it take for the Instagram account to have 1000 more followers than it started with?

(A) 19 hours 8 minutes (B) 19 hours 34 minutes

(C) 19 hours 40 minutes (D) 20 hours 6 minutes

8. What is the x-coordinate of the point on the curve $y = e^{2x}$ where the tangent is parallel to the line $y = 4x - 1$?

(A) $x = \frac{1}{2}ln2$ (B) $x = ln2$

(C) $x = -\frac{1}{2}ln2$ (D) $x = 2$

9. Which expression is equivalent to $(sin2x + cos2x)^2$?

(A) 1 (B) $1 + sin2x$

(C) $1 + \frac{1}{2}sin4x$ (D) $1 + sin4x$

10. The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $ax + 3$.

What is the value of *a*?

(A) – 14 (B) –11 (C) –2 (D) 5

End of Section I

Section II

30 marks
Attempt Questions 11 – 13
Allow about 45 minutes for this section

Instructions

Answer the questions on your own **single-side lined paper**.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) (Start a new page) **Marks**

(a) A discrete random variable X has the probability distribution table shown. **3**

$X = x$	20	21	22	23
$P(x)$	0.24	0.2	m	0.4

By first finding the value of m, calculate the **variance** of X.

(b) A six-sided die with faces labelled 1, 2, 3, 4, 5 and 6 is constructed such that the probability of rolling a six is larger than the probability of rolling a one. For some constant a, the probability distribution table is as follows.

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6} - a$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6} + a$

It is known that the expected value of rolling this die is 4. Find the value of a. **2**

(c) The probability distribution of a discrete random variable X is given by the table below.

X	0	1	2	3	4
$P(X = x)$	0.2	$0.6p^2$	0.1	$1 - p$	0.1

(i) Show that $p = \frac{2}{3}$ or $p = 1$. **1**

(ii) Taking $p = \frac{2}{3}$, determine $P(X \geq E(X))$ **2**

(d) A bunch of flowers is formed using single flowers from five different types: daffodils, tulips, roses, lilies and petunias.

What is the minimum number of flowers in a bunch required to ensure that it contains at least four flowers of any one type? **1**

(e) A certain multiple-choice quiz contains ten questions, each with three possible answers: *A*, *B* or *C*. **1**

The students have been told that there are three questions with the correct answer '*A*', five questions with the correct answer '*B*', and two questions with the correct answer '*C*'.

Nathaniel decides to allocate his answers randomly, ensuring that he puts down three *A*s, five *B*s and two *C*s. What is the probability he gets every question correct?

End of Question 11

Question 12 (10 marks) **(Start a new page)**

(a) Eight friends are seated in a row at the cinema. The seats on each end of the row are aisle seats. In how many ways can they be seated if Alice and Bob insist on sitting in an aisle seat and Corinne sits the same number of seats away from Douglas as she does from Ellen? **2**

(b) From a group of 6 men and 7 women, a committee of 5 people is to be formed.

(i) How many committees can be formed if there are more men than women? **1**

(ii) What is the probability that if the committee contains three men and two women, that only one of Anne or Harold is definitely in the committee? **1**

(c) 12 people need to be split up into teams for a quiz.

(i) Explain why the number of ways of splitting them into two groups of the **same** size is

$$\frac{1}{2}\binom{12}{6}$$ **1**

(ii) How many ways are there of splitting them into two groups of any size (but there must be at least one person in each group)? **2**

(d) A family of eight is seated randomly around a circular table. **1**

What is the probability that the two youngest members of the family sit directly opposite each other?

(e) A standard pack of 52 cards consists of 13 cards of four suits : hearts, diamonds, clubs and spades. **2**

What is the probability that if five cards are selected without replacement, that **at least** four of the cards are the same suit? Leave your answer as a fraction in simplest form.

End of Question 12

Question 13 (10 marks) (Start a new page) **Marks**

(a) Express $\frac{{}^{10}C_k}{{}^{9}C_k}$ in simplest form. **1**

(b) (i) Expand $(x+\frac{2}{x})^5$, giving your answer in simplest form. **1**

(ii) Hence simplify $(x+\frac{2}{x})^5+(x-\frac{2}{x})^5$, again leaving your answer in simplest form. **1**

(c) Find the coefficient of the term containing x^3 in the expansion of $(2x-\frac{3}{x})^{11}$, leaving your answer in index form. **2**

(d) In the expansion of $(ax+b)^6$ where *a* and *b* are constants, the coefficient of x^4 is 2160 and the coefficient of x^5 is –576. Find *a* and *b.* **3**

(e) We recall from our text: "A set *A* is a *subset* of a set *S*, written as $A \subset S$, if every element of *A* is also an element of *S.*"

Additionally, "the symbol $|S|$ is used to mean the number of members of *S.*"

So if $S=\{1,2,3,4,5,6,7,8,9\}$ then $|S|=9$.

Use this definition and the *Pigeonhole Principle* to show that any subset *A* from the set *S* listed above, where $|A|=6$, must contain two elements whose sum is 10. **2**

End of Examination

2021 Year 11 Mathematics Extension 1 Task 3 SOLUTIONS

Suggested Solution (s)	Comments

SECTION I (MULTIPLE CHOICE)

1. $4^x - 9(2^x) + 8 = 0$

Let $u = 2^x$

$\therefore u^2 - 9u + 8 = 0$

$(u-8)(u-1) = 0$

$u = 8$ or $u = 1$

$2^x = 8$ $\quad 2^x = 1$

$x = 3$ or $x = 0$ **(C)**

2. $y = (5x+4)^3(x+1)^4$

$\frac{dy}{dx} = uv' + vu'$

$= 4(x+1)^3(5x+4)^3 + 3(5x+4)^2 \cdot 5 \cdot (x+1)^4$

$= (x+1)^3(5x+4)^2[4(5x+4) + 15(x+1)]$

$= (x+1)^3(5x+4)^2(35x+31)$ **(D)**

3. $1 + \sec x$ (letting $t = \tan\frac{x}{2}$)

$= 1 + \frac{1}{\cos x}$

$= 1 + \frac{1+t^2}{1-t^2}$

$= \frac{1-t^2+1+t^2}{1-t^2}$

$= \frac{2}{1-t^2}$ **(D)**

4. In ΔHIJ, using Sine Rule

$\frac{HJ}{\sin 30^\circ} = \frac{16}{\sin 44^\circ}$

$\therefore HJ = \frac{8}{\sin 44^\circ}$

In ΔHKJ, $\sin 74^\circ = \frac{HK}{\frac{8}{\sin 44^\circ}}$

$\therefore HK = \frac{8\sin 74^\circ}{\sin 44^\circ}$

$= 11.070\ldots$

$= 11.07$ (to 2dp)

Suggested Solution (s)	Comments

5. $\ln 3a = \ln b - 2\ln c$

$= \ln b - \ln c^2$

$= \ln \frac{b}{c^2}$

$\therefore 3a = \frac{b}{c^2}$

$\therefore a = \frac{b}{3c^2}$ **(B)**

6. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $\quad V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$= 4\pi r^2 \times 2$

when $r = 5$, $\frac{dV}{dt} = 4\pi \times 5^2 \times 2$ **(B)**

$= 200\pi$ mm^3/s

7. $N = 80 + 20e^{kt}$

when $t = 0$, $N = 80 + 20e^0$

$= 100$

$\therefore 1100 = 80 + 20e^{0.2t}$

$51 = e^{0.2t}$

$t = \frac{\ln 51}{0.2}$ **(C)**

$= 19.65\ldots$ i.e. 19h 40 min

8. $y = 4x - 1$

$m = 4 \therefore m_\perp = 4$.

$\frac{dy}{dx} = 2e^{2x}$

$2e^{2x} = 4$

$e^{2x} = 2$

$x = \frac{\ln 2}{2}$ **(A)**

9. $(\sin 2x + \cos 2x)^2 = \sin^2 2x + 2\sin 2x \cos 2x + \cos^2 2x$

$= 1 + 2\sin 2x \cos 2x$

$= 1 + \sin 4x$ **(D)**

10. $P(x) = x^4 - 8x^3 - 7x^2 + 3$

$\equiv x(x+1)Q(x) + ax + 3$

$\therefore P(0) = 3$ and $P(-1) = (-1)^4 - 8(-1)^3 - 7(-1)^2 + 3$

$= 5$

$\therefore 5 = 3 - a$

$a = 2$ **(C)**

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 11

(a) $0.24+0.2+m+0.4=1$

$\therefore m=0.16$ ✓

$E(X)=20\times0.24+21\times0.2+22\times0.16+23\times0.4$

$=21.72$

$E(X^2)=20^2\times0.24+21^2\times0.2+22^2\times0.16+23^2\times0.4$

$=473.24$

} one of $E(X)$ or $E(X^2)$

$Var(X)=E(X^2)-E(X)^2$

$=473.24-21.72^2$ ✓

$=1.4816$

(b) $E(X)=\frac{1}{6}-a+2(\frac{1}{6})+3(\frac{1}{6})+4(\frac{1}{6})+5(\frac{1}{6})+6(\frac{1}{6}+a)$

$\therefore 4=\frac{1}{6}-a+\frac{1}{3}+\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+1+6a$ ✓

$4=\frac{7}{2}+5a$

$5a=\frac{1}{2}$ (2)

$a=\frac{1}{10}$ ✓

(c) (i) $0.2+0.6p^2+0.1+1-p+0.1=1$

$0.6p^2-p+0.4=0$

$3p^2-5p+2=0$ (1)

$(3p-2)(p-1)=0$

$\therefore p=\frac{2}{3}$ or $p=1$ as req'd

(ii) $E(X)=0+0.6\times(\frac{2}{3})^2+2\times0.1+3(1-\frac{2}{3})+4\times0.1$

$=\frac{28}{15}$ ✓

$P(X\geqslant E(X))=P(X\geqslant\frac{28}{15})$

$=P(2,3,4)$

$=6.1+1-\frac{2}{3}+0.1$ ✓

$=\frac{8}{15}$

(d) minimum number $=5\times3+1$

$=16$ (1)

(e) Number of arrangements

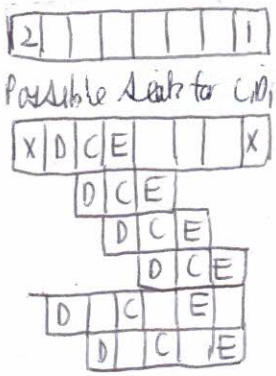
$=\frac{10!}{3!5!2!}$

$=2520$

$\therefore P(\text{correct})=\frac{1}{2520}$ (1)

OR. $\frac{1}{{}^{10}C_3\times{}^{7}C_5\times{}^{2}C_2}$

$=\frac{1}{2520}$

Suggested Solution (s)	Comments
QUESTION 12	
(a)  ✓ significant progress Sitting together $4 \times 2!$ one seat apart $2 \times 2!$ ∴ number of ways $= 2 \times (4 \times 2! + 2 \times 2!) \times 3!$ $= 144$ ✓	
(b) 6M 7W committee of 5	
(i) more men than women ${}^6C_3 \times {}^7C_2 + {}^6C_4 \times {}^7C_1 + {}^6C_5$ $= 531$	
(ii) P(only one of H or A) $= \dfrac{{}^5C_3 \times {}^6C_1 + {}^5C_2 \times {}^6C_2}{{}^6C_3 \times {}^7C_2}$ $= \frac{1}{2}$ ✓	(no mark for bald answer)
(c) (i) The two groups of six are indistinguishable in size ∴ Number of groups $= \dfrac{{}^{12}C_6 \times {}^6C_6}{2!}$ $= \frac{1}{2} \times {}^{12}C_6$	
OR: Each group of 6 chosen automatically chooses another group of 6 ∴ ${}^{12}C_6$ counts twice the number of groups ∴ Number of ways $= \frac{1}{2} \times {}^{12}C_6$ (1)	

Suggested Solution (s)	Comments
(ii) number of groups $= \frac{1}{2} \times {}^{12}C_6 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_4 + {}^{12}C_5$ ✓ $= 2047$ ✓ (2)	
(d) P(two youngest opp each other) $= \frac{6!}{7!}$ $= \frac{1}{7}$ (1) 	
(e) P(at least four of same suit) $= 4 \times \dfrac{{}^{13}C_4 \times {}^{39}C_1 + 4 \times {}^{13}C_5}{{}^{52}C_5}$ ✓ $= \frac{11}{245}$ ✓ (2)	

Suggested Solution (s)	Comments

QUESTION B

(a) $\frac{{}^{10}C_k}{{}^{9}C_k} = \frac{10!}{k!(10-k)!} \times \frac{k!(9-k)!}{9!}$

$= \frac{10}{10-k}$.

(b) i) $(x+\frac{2}{x})^5$

$= x^5 + 5 \cdot x^4(\frac{2}{x}) + {}^5C_2 x^3(\frac{2}{x})^2 + {}^5C_3 x^2(\frac{2}{x})^3$

$+ {}^5C_4 x \cdot (\frac{2}{x})^4 + (\frac{2}{x})^5$ (1)

$= x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$

ii) $(x+\frac{2}{x})^5 + (x-\frac{2}{x})^5$

$= 2x^3 + 2 \times 40x + 2 \times \frac{80}{x^3}$ (1)

$= 2x^5 + 80x + \frac{160}{x^3}$

(c) $T_{k+1} = {}^nC_k a^{n-k} b^k$

$(2x - \frac{3}{x})^{11}$: $T_{k+1} = {}^{11}C_k (2x)^{11-k} (\frac{-3}{x})^k$

$= {}^{11}C_k 2^{11-k} x^{11-k} (-3)^k x^{-k}$

$= {}^{11}C_k 2^{11-k} \cdot (-3)^k x^{11-2k}$ ✓

for term in x^3 : $11-2k = 3$

$2k = 8$

$k = 4$. (2)

$\therefore T_5 = {}^{11}C_4 2^7 (-3)^4$

$= {}^{11}C_4 2^7 3^4$. ✓

Suggested Solution (s)	Comments

(d) $(ax+b)^6$

$T_{k+1} = {}^6C_k (ax)^{6-k} b^k$

x^4 : $6-k=4 \therefore k=2$

x^5 : $6-k=5 \therefore k=1$ ✓

$\therefore {}^6C_2 a^4 b^2 = 2160$

${}^6C_1 a^5 b = -576$ (3)

$\therefore b = \frac{-576}{6a^5}$

$= \frac{-96}{a^5}$ ✓

$\therefore {}^6C_2 a^4 (\frac{-96}{a^5})^2 = 2160$

$\frac{{}^6C_2 \times 96^2}{2160} = a^6 = 64$

$\therefore a = 2, b = \frac{-96}{2^5}$ ✓

$= 3$

(e) Dividing S into 5 subsets (of two elements whose sum is 10)

$A = \{1, 9\}$

$B = \{2, 8\}$ 5 'pidgeonholes'

$C = \{3, 7\}$ ✓

$D = \{4, 6\}$

$E = \{5\}$ (2)

'pidgeons' are the six members of subset A

$\therefore$ by the pidgeonhole principle, ✓ at least one of the five subsets must contain two elements whose sum is 10