Moriah College

בית ספר הר המוריה

2021

YEAR 11 Extension 1 Preliminary Examination

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour 30 minutes
- Write using black or blue pen
- Board-approved calculators may be used

Total Marks – 62

Section I Pages 2 – 4

5 marks

- Attempt Questions 1 – 5

Section II Pages 5 – 12

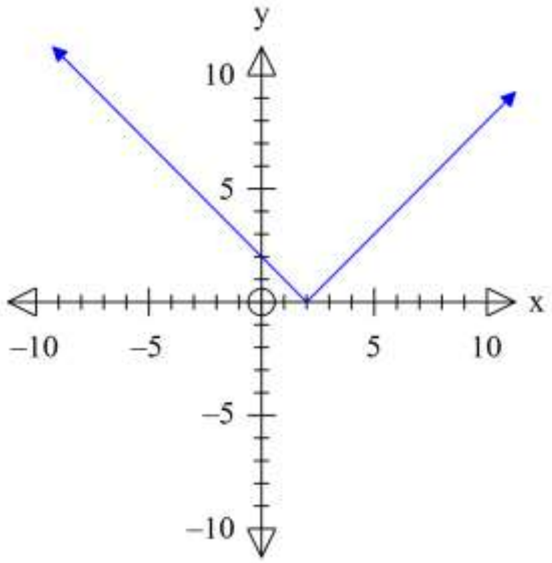
57 marks

- Attempt Questions 6 – 9

Section I

5 marks

Attempt Questions 1 – 5

1. 

The curve shown has equation $y = |x - 2|$.

By considering this graph and the graph of $y = 2x + 1$, how many solutions does the equation $|x - 2| = 2x + 1$ have?

(A) 0

(B) 1

(C) 2

(D) 3

2. Consider the polynomial $P(x) = 3x^3 + 3x + k$.

If $(x - 2)$ is a factor of $P(x)$, what is the value of k?

(A) -30

(B) -18

(C) 18

(D) 30

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3. What is the exact value of $\tan(\frac{\pi}{3} - \frac{\pi}{4})$

(A) $\sqrt{3} - 1$

(B) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(C) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(D) -1

4. Which of the following is equivalent to the expression $\dfrac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1}$?

(A) $\cot\theta$

(B) $\sec\theta$

(C) $\sin\theta$

(D) $\tan\theta$

5. It is known that two of the roots of the equation $3x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. What is the value of k?

(A) -2

(B) 6

(C) 7

(D) 17

Section II (57 marks)

Attempt Questions 6 – 9

Question 6 (15 marks) **Start this on a new page**

(a) The number of ducks N in a community increases over time t.

The rate of increase is proportional to the difference between the number at any time N and the number 150.

This can be represented by the equation $\frac{dN}{dt} = k\,(N - 150)$

where t is measured in years.

(i) Show that the function $N = 150 + Ae^{kt}$ satisfies the equation above **2**

(ii) The initial population is 750 ducks **1**

Find the value of A

(iii) After two years the population has grown to 900 ducks. **2**

Find the value of k in exact form.

(iv) Find the time when the population has grown to 1050 ducks, giving your answer correct in years to 2 decimal places **2**

(v) Find the population when the rate of growth is twice the initial rate of growth. **2**

(b) Solve the equation $log_2(x-6) + log_2(x-10) = 5$ **3**

(c) You are given the following information:

- $\cos\theta = \frac{1}{2}$ and $\cos\beta = \frac{3}{5}$
- θ and β are both acute angles

(i) Find the exact value of $\cos(\theta + \beta)$ **2**

(ii) If neither θ or β are acute angles, then the answer in part (i) is unchanged. **1**

Briefly explain why this is so.

Question 7 (15 marks) **Start this on a new page**

(a) The roots of the cubic equation $x^3 - 3x^2 + 4x + 2 = 0$ are α, β, γ. **2**

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(b) Consider the polynomial $P(x) = x^3 - 6x^2 + 10x - 3$.

(i) Show that $P(3) = 0$ **1**

(ii) Find all solutions to the polynomial equation $P(x) = 0$, leaving answers in exact form **3**

(c) (i) Find the derivative of the function $y = 3e^{-2x}$ **1**

(ii) Sketch the curve $y = 3e^{-2x}$ **1**

(iii) The horizontal line $y = 12$ is drawn to intersect the curve $y = 3e^{-2x}$ **2**
Find the coordinates of the point of intersection, leaving your answer in exact form

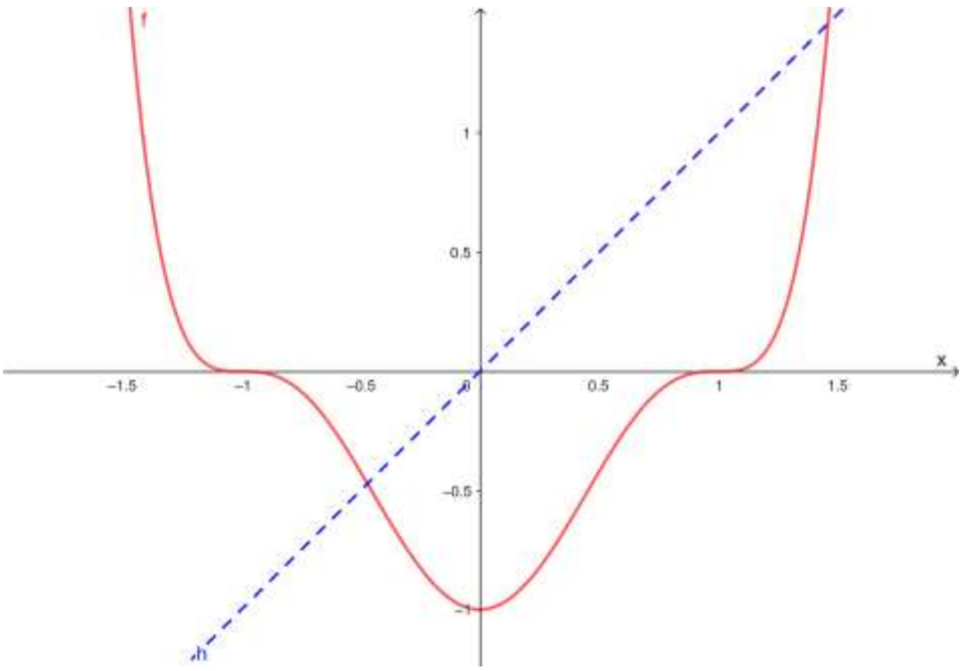
(iv) A particle is moving along the curve $y = 3e^{-2x}$ so that its x-coordinate is changing at a constant rate $\frac{dx}{dt} = 4$ units/second

Use an appropriate chain rule to find the rate of change $\frac{dy}{dt}$ of its y-coordinate when $x = 2$ (give your answer correct to 2 decimal places) **3**

(v) Find the rate of change $\frac{dy}{dt}$ when $y = 12$ (give your answer in exact form) **2**

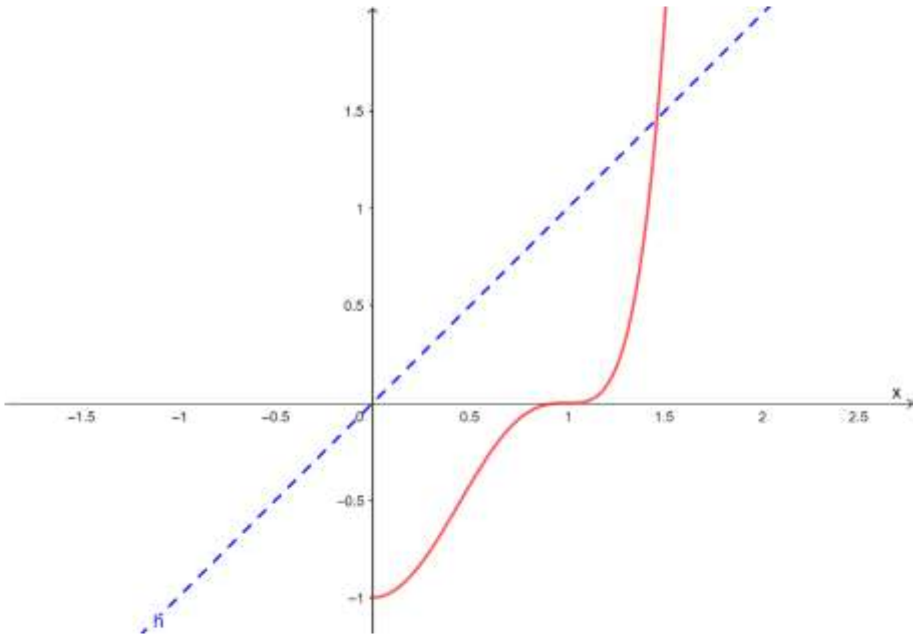
Question 8 (15 marks) **Start this on a new page**

(a) The graph shows the function $y = f(x) = (x^2 - 1)^3$, and the line $y = x$.



Question continues over the page

The domain of the function is restricted to $x \geq 0$ and the diagram below shows the restricted function.



(i) Carefully copy this diagram onto your working paper. **2**

Now carefully draw the inverse function $y = f^{-1}(x)$ on your number plane

(ii) State the domain and range of $y = f^{-1}(x)$ **1**

(iii) Find the inverse function $f^{-1}(x)$ **2**

(b) Gemma sketched the parabola $f(x) = x^2 - 4$

On the same axes Taube then sketched the curve $y = \sqrt{f(x)}$

(i) Find the x-coordinates of all points where Gemma's parabola meets Taube's curve. **3**

(ii) Draw the two curves on the **same** half-page number plane **2**

(c) Consider the function $y = x\sqrt{4 - x^2}$

(i) Show that $\frac{dy}{dx} = \frac{4-2x^2}{\sqrt{4-x^2}}$ **3**

(ii) The curve $y = x\sqrt{4 - x^2}$ passes through the origin. **2**
Find the angle that the tangent at $(0 , 0)$ makes with the x-axis, giving your answer in radians, correct to 2 decimal places.

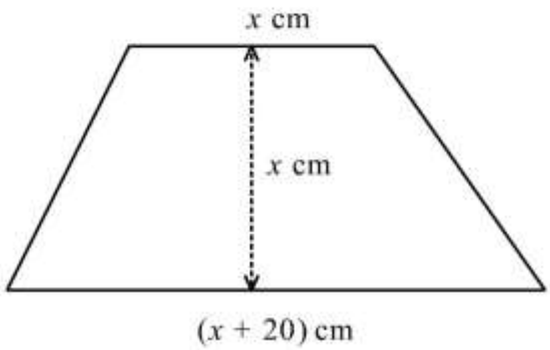
Question 9 (12 marks) **Start this on a new page**

(a) (i) Solve the equation $x + 2 = \frac{4}{x-1}$ **2**

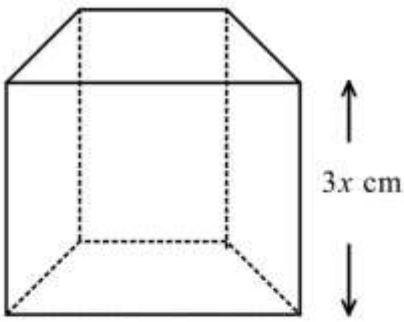
(ii) Hence, or otherwise, solve the inequality $x + 2 \leq \frac{4}{x-1}$ **2**

(b) A prism has a trapezoidal base with parallel sides of length *x* cm and (*x* + 20) cm.
The perpendicular distance between the parallel sides is also *x* cm.
The **height** of the **prism** is 3*x* cm, as shown in the diagram on the right.

BASE



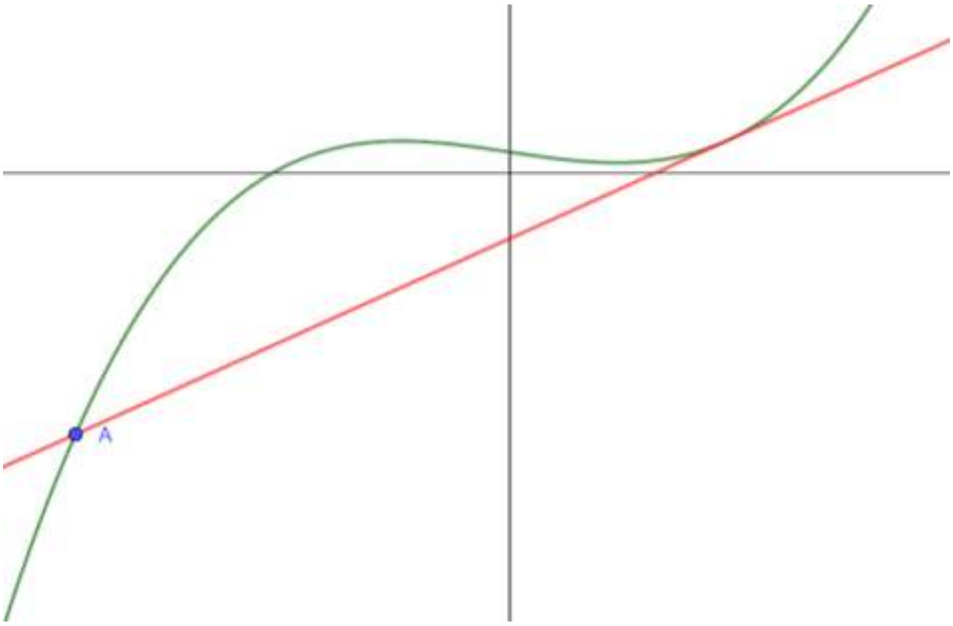
PRISM



(i) Show that the volume of the prism is given by $V = 3x^3 + 30x^2$ **1**

(ii) The height of the prism is increasing at 6 cm/minute. **2**
Find the rate at which the volume of the prism is increasing when the height of the prism is 3 centimetres.

(c)



The curve shown has equation $y = x^3 - 3x + 4$

The point *A* is shown on the curve with coordinates $(-4, -48)$

A line is drawn to cut through the curve at point *A*.

This line is tangential to the curve at another point, as shown.

The aim of this question is to find the gradient of this tangent.

A small amount of scaffolding is given to you below but you have to do the rest of the work yourself

Scaffolding

Let *m* be the required gradient

(i) Show that the equation of the line through *A* with gradient *m* is given by: **1**

$$y = mx + 4m - 48$$

(ii) Find the value of *m*. **4**

2021

YEAR 11 Extension 1 Preliminary Examination

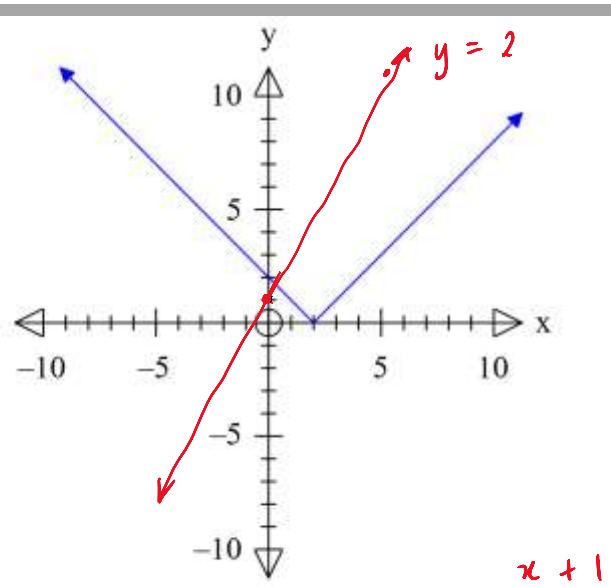
Solutions

Section I

5 marks

Attempt Questions 1 – 5

1.



The curve shown has equation $y = |x - 2|$.

By considering this graph and the graph of $y = 2x + 1$, how many solutions does the equation $|x - 2| = 2x + 1$ have?

(A) 0

(B) 1

(C) 2

(D) 3

2. Consider the polynomial $P(x) = 3x^3 + 3x + k$.

If $(x-2)$ is a factor of $P(x)$, what is the value of k?

(A) -30

(B) -18

(C) 18

(D) 30

$$P(2) = 30 + k = 0$$

$$k = -30$$

---

3. What is the exact value of $\tan(\frac{\pi}{3} - \frac{\pi}{4})$

(A) $\sqrt{3} - 1$

(B) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

(C) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(D) -1

$$\tan(\frac{\pi}{3} - \frac{\pi}{4}) = \frac{tan\frac{\pi}{3} - tan\frac{\pi}{4}}{1 + tan\frac{\pi}{3}tan\frac{\pi}{4}}$$

4. Which of the following is equivalent to the expression $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1}$?

(A) $\cot\theta$

(B) $\sec\theta$

(C) $\sin\theta$

(D) $tan\theta$

$$\frac{2sin\theta cos\theta + sin\theta}{2cos^2\theta - 1 + cos\theta + 1}$$

$$= \frac{sin\theta(2cos\theta + 1)}{cos\theta(2cos\theta + 1)}$$

5. It is known that two of the roots of the equation $3x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. What is the value of k?

(A) -2

(B) 6

(C) 7

(D) 17

Let the roots be $\alpha, \frac{1}{\alpha}\ \beta$

Product of roots = $\frac{-d}{a}$

Hence $\beta = -2$

$$P(-2) = -14 + 2k = 0$$

$$k = 7$$

Question 6 (15 marks) **Start this on a new page**

(a) The number of ducks N in a community increases over time t.

The rate of increase is proportional to the difference between the number at any time N and the number 150.

This can be represented by the equation $\frac{dN}{dt} = k\,(N - 150)$

where t is measured in years.

(i) Show that the function $N = 150 + Ae^{kt}$ satisfies the equation above **2**

LHS $=$ $\frac{dN}{dt} = \frac{d(150+Ae^{kt})}{dt} = kAe^{kt}$

RHS $=$ $k\,(N - 150) = k(150 + Ae^{kt} - 150) = kAe^{kt}$

(ii) The initial population is 750 ducks **1**

Find the value of A

$$t = 0 \rightarrow 750 = 150 + Ae^{0}$$

$$A = 600$$

(iii) After two years the population has grown to 900 ducks. **2**

Find the value of k in exact form.

$$t = 2 \rightarrow 900 = 150 + 600e^{2k}$$

$$e^{2k} = \frac{5}{4}$$

$$k = \frac{1}{2} ln\left(\frac{5}{4}\right)$$

(iv) Find the time when the population has grown to 1050 ducks, giving your answer correct in years to 2 decimal places **2**

$$1050 = 150 + 600e^{kt}$$

$$e^{kt} = \frac{3}{2}$$

$$kt = ln\frac{3}{2}$$

$$t = \frac{2ln\left(\frac{3}{2}\right)}{ln\left(\frac{5}{4}\right)}$$

t =3.63 years

(v) Find the population when the rate of growth is twice the initial rate of growth. **2**

$$when\ N = 750 \rightarrow \frac{dN}{dt} = k\,(750 - 150)$$

Initial rate of growth is $\frac{dN}{dt} = 600k$

We want $\frac{dN}{dt} = 1200k \rightarrow \ k\,(N - 150) = 1200k$

$$N = 1350$$

(b) Solve the equation $log_2(x-6) + log_2(x-10) = 5$ **3**

$$log_2(x^2 - 16x + 60)$$

$$(x^2 - 16x + 60) = 32$$

$$x^2 - 16x + 28 = 0$$

$$Giving\ x = 14\ or\ x = 2$$

$$Only\ x = 14\ solves\ the\ equation$$

(c) You are given the following information:

- $\cos\theta = \frac{1}{2}$ and $\cos\beta = \frac{3}{5}$
- θ and β are both acute angles

(i) Find the exact value of $\cos(\theta + \beta)$ **2**

$$\cos(\theta + \beta) = \cos\theta cos\beta - sin\theta sin\beta$$

Now θ and β are both acute angles **1**

Using triangles or otherwise, $\cos\theta = \frac{1}{2} \rightarrow sin\,\theta = \frac{\sqrt{3}}{2}$ and $\cos\beta = \frac{3}{5} \rightarrow sin\beta = \frac{4}{5}$

$$\cos(\theta + \beta) = \frac{1}{2} \times \frac{3}{5} - \frac{\sqrt{3}}{2} \times \frac{4}{5}$$

$$= \frac{3 - 4\sqrt{3}}{10}$$

(ii) If neither θ or β are acute angles, then the answer in part (i) is unchanged.

Briefly explain why this is so.

If neither θ or β are acute angles, then they are both angles in the 4th quadrant

Hence, $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\sin\beta = -\frac{4}{5}$ and the answer in part (i) is unchanged.

Question 7 (15 marks) **Start this on a new page**

(a) The roots of the cubic equation $x^3 - 3x^2 + 4x + 2 = 0$ are α, β, γ. **2**

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$\alpha\beta + \beta\gamma + \alpha\gamma = 4$ and $\alpha\beta\gamma = -2$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma}$$

$$= -2$$

(b) Consider the polynomial $P(x) = x^3 - 6x^2 + 10x - 3$.

1

(i) Show that $P(3) = 0$

$$P(3) = 27 - 54 + 30 - 3 = 0$$

(ii) Find all solutions to the polynomial equation $P(x) = 0$, leaving answers in exact form

3

$$P(x) = (x - 3)(x^2 - 3x + 1)$$

Solving $(x - 3)(x^2 - 3x + 1) = 0$

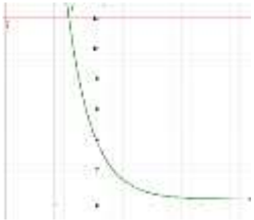
$$x = 3 \text{ or } x = \frac{3 \pm \sqrt{5}}{2}$$

(c) (i) Find the derivative of the function $y = 3e^{-2x}$ **1**

$$\frac{dy}{dx} = -6e^{-2x}$$

1

(ii) Sketch the curve $y = 3e^{-2x}$



(iii) The horizontal line $y = 12$ is drawn to intersect the curve $y = 3e^{-2x}$
Find the coordinates of the point of intersection, leaving your answer in exact form

$$12 = 3e^{-2x} \rightarrow e^{-2x} = 4$$

2

$$-2x = ln4$$

$$x = -ln2$$

Point of intersection is $(-ln2\,, 12\,)$

(iv) A particle is moving along the curve $y = 3e^{-2x}$ so that its x-coordinate is changing at a constant rate $\frac{dx}{dt} = 4$ units/second

Use an appropriate chain rule to find the rate of change $\frac{dy}{dt}$ of its y-coordinate when $x = 2$ (give your answer correct to 2 decimal places)

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -6e^{-2x} \times 4 = -24e^{-2x}$$

When $x = 2, \frac{dy}{dt} = -24e^{-4}$

3

$\frac{dy}{dt} = -0.44$ units/second

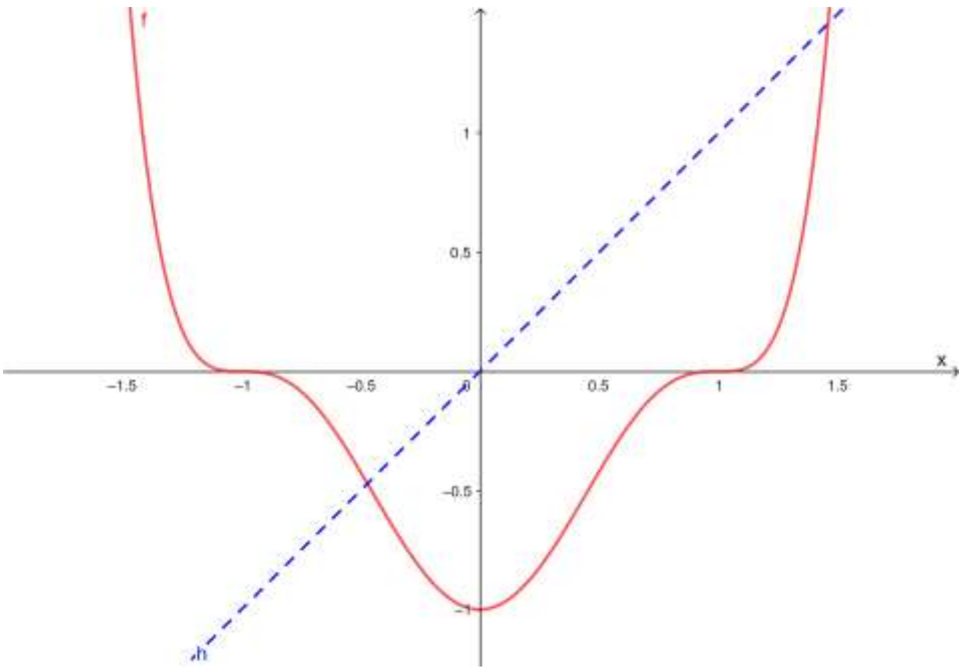
(v) Find the rate of change $\frac{dy}{dt}$ when $y = 12$ (give your answer in exact form) **2**

From part (iii), when $y = 12,\ e^{-2x} = 4.$

Hence, $\frac{dy}{dt} = -24 \times 4 = -96$ units/second

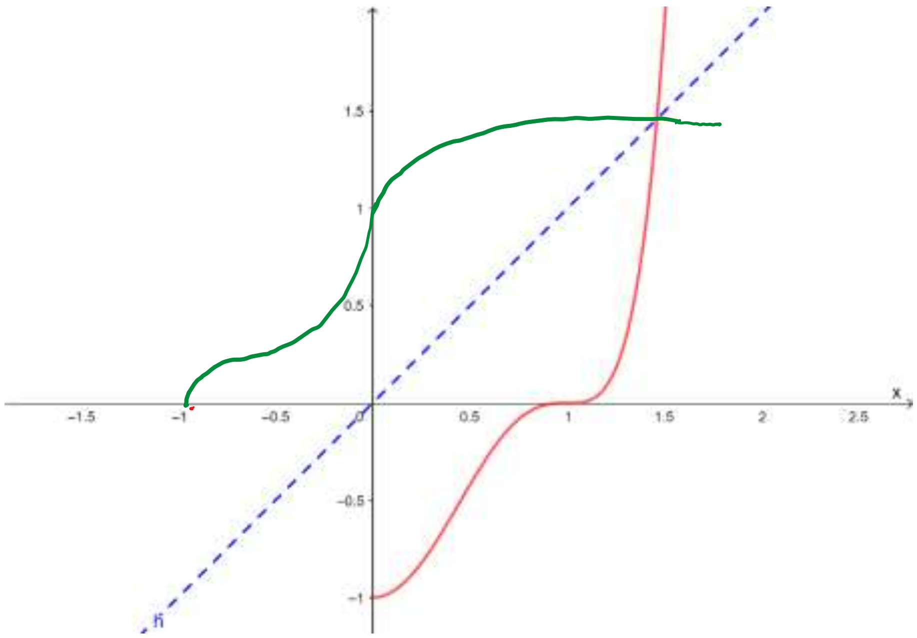
Question 8 (15 marks) **Start this on a new page**

(a) The graph shows the function $y = f(x) = (x^2 - 1)^3$, and the line $y = x$.



Question continues over the page

The domain of the function is restricted to $x \geq 0$ and the diagram below shows the restricted function.



(i) Carefully copy this diagram onto your working paper. **2**

Now carefully draw the inverse function $y = f^{-1}(x)$ on your number plane

(ii) State the domain and range of $y = f^{-1}(x)$ **1**

Domain is $[-1, \infty)$ and Range is $[0, \infty)$

(iii) Find the inverse function $f^{-1}(x)$ **2**

$$x = (y^2 - 1)^3$$

$$y^2 - 1 = \sqrt[3]{x}$$

$$y = \pm\sqrt{1 + \sqrt[3]{x}}$$

$f^{-1}(x) = \sqrt{1 + \sqrt[3]{x}}$ as Range of $f^{-1}(x)$ is $[0, \infty)$

(b) Gemma sketched the parabola $f(x) = x^2 - 4$

On the same axes Taube then sketched the curve $y = \sqrt{f(x)}$

(i) Find the x-coordinates of all points where Gemma's parabola meets Taube's curve. **3**

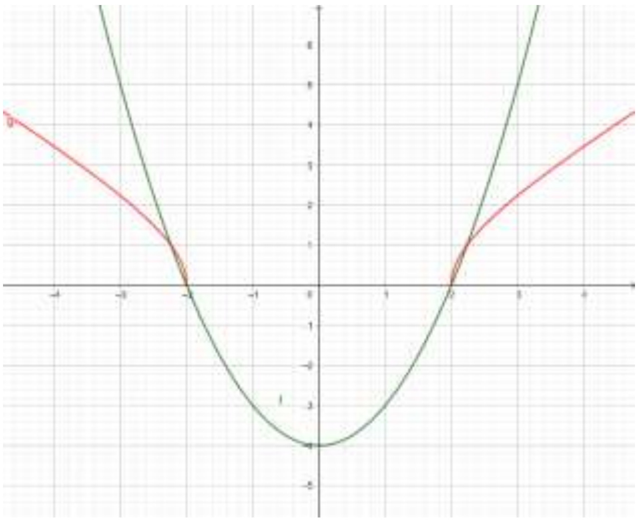
$f(x)$ meets $\sqrt{f(x)}$ when $f(x) = 1$ or $f(x) = 0$

$x^2 - 4 = 1 \rightarrow x = \pm\sqrt{5}$

2

$x^2 - 4 = 0 \rightarrow x = \pm 2$

(ii) Draw the two curves on the **same** half-page number plane



(c) Consider the function $y = x\sqrt{4-x^2}$

(i) Show that $\frac{dy}{dx} = \frac{4-2x^2}{\sqrt{4-x^2}}$ **3**

Let $u = x \rightarrow u' = 1$ and $v = \sqrt{4-x^2} \rightarrow v' = \frac{-x}{\sqrt{4-x^2}}$

$$\frac{dy}{dx} = \frac{-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$\frac{dy}{dx} = \frac{-x^2}{\sqrt{4-x^2}} + \frac{4-x^2}{\sqrt{4-x^2}}$$

$$\frac{dy}{dx} = \frac{4-2x^2}{\sqrt{4-x^2}}$$

(ii) The curve $y = x\sqrt{4-x^2}$ passes through the origin. **2**
Find the angle that the tangent at (0 , 0) makes with the x-axis, giving your answer in radians, correct to 2 decimal places.

When $x = 0$, $\frac{dy}{dx} = 2$

The angle θ is such that $\tan\theta = 2 \rightarrow \theta = 1.11\ radians\ (2dp)$

Question 9 (12 marks) **Start this on a new page**

(a) (i) Solve the equation $x + 2 = \frac{4}{x-1}$ **2**

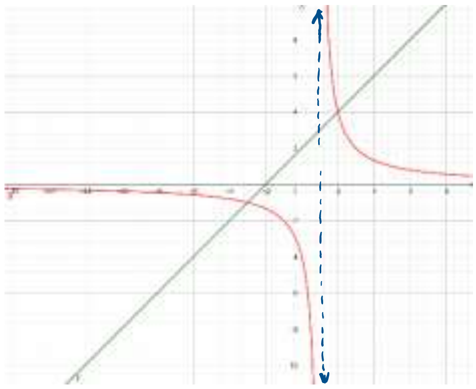
$$(x+2)(x-1) = 4$$
$$x^2 + x - 6 = 0$$
$$(x-2)(x+3) = 0$$
$$x = 2 \text{ or } x = -3$$

2

(ii) Hence, or otherwise, solve the inequality $x + 2 \leq \frac{4}{x-1}$

$x \neq 1$

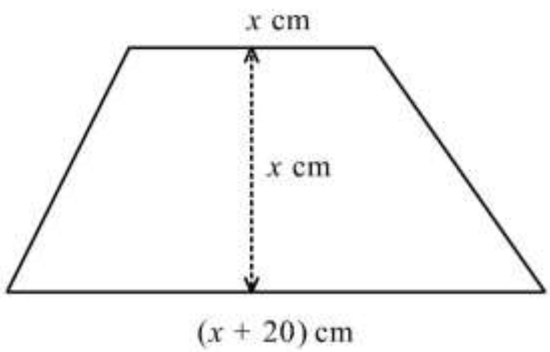
From the graphs,



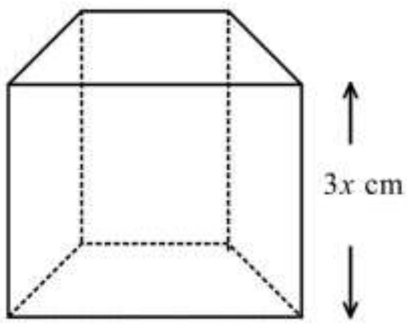
$x \leq -3 \ or \ 1 < x \leq 2$

(b) A prism has a trapezoidal base with parallel sides of length x cm and $(x + 20)$ cm.
The perpendicular distance between the parallel sides is also x cm.
The **height** of the **prism** is $3x$ cm, as shown in the diagram on the right.

BASE



PRISM



(i) Show that the volume of the prism is given by $V = 3x^3 + 30x^2$ **1**

$$\text{Area} = \frac{x(x+x+20)}{2}$$

$$\text{Area} = x^2 + 10x$$

$$\text{Volume} = (x^2 + 10x) \times 3x = 3x^3 + 30x^2$$

(ii) The height of the prism is increasing at 6 cm/minute. **2**

Find the rate at which the volume of the prism is increasing when the height of the prism is 3 centimetres.

$$h = 3x \rightarrow \frac{dh}{dx} = 3 \rightarrow \frac{dx}{dh} = \frac{1}{3}$$

$$V = 3x^3 + 30x^2 \rightarrow \frac{dV}{dx} = 9x^2 + 60x$$

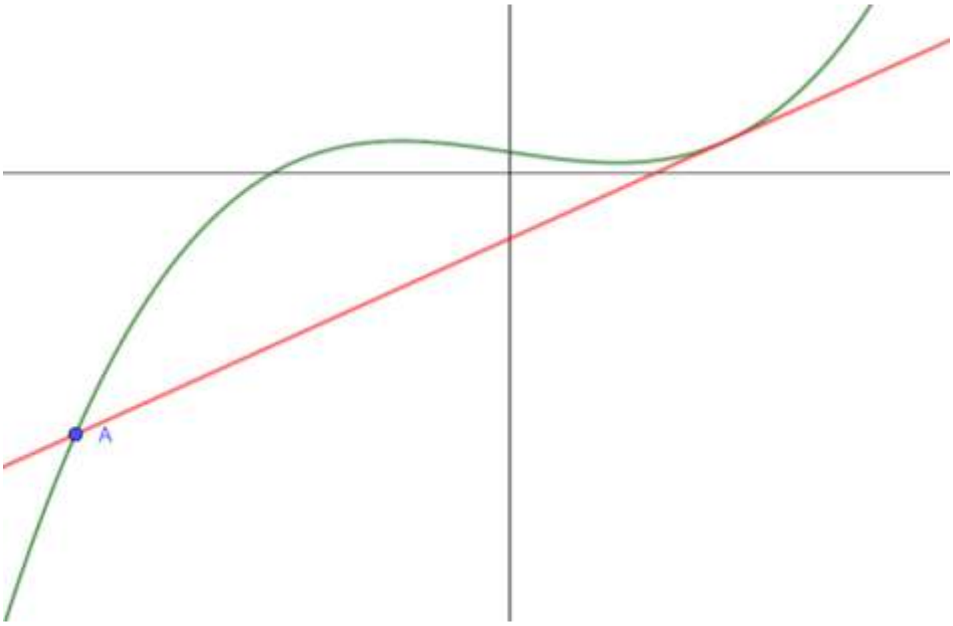
$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dh} \times \frac{dh}{dt}$$

$$\frac{dV}{dt} = (9x^2 + 60x) \times \frac{1}{3} \times 6$$

$$\frac{dV}{dt} = 18x^2 + 120x$$

Now when $h = 3, x = 1$ and $\frac{dV}{dt} = 138\ cm^3/min$

(c)



The curve shown has equation $y = x^3 - 3x + 4$

The point A is shown on the curve with coordinates $(-4, -48)$

A line is drawn to cut through the curve at point A.

This line is tangential to the curve at another point, as shown.

The aim of this question is to find the gradient of this tangent.

A small amount of scaffolding is given to you below but you have to do the rest of the work yourself

Scaffolding

Let m be the required gradient

(i) Show that the equation of the line through A with gradient m is given by: **1**

$$y = mx + 4m - 48$$

$$y + 48 = m(x + 4)$$

(ii) Find the value of m. **4**

Solving simultaneously, $y = mx + 4m - 48$............(1)

$$y = x^3 - 3x + 4\text{.................}(2)$$

We form the cubic equation in x,

$$x^3 - 3x + 4 = mx + 4m - 48$$

$$x^3 - x(m+3) + (52 - 4m) = 0\text{............}(3)$$

We know the line is a tangent. Therefore the cubic polynomial has a double root

We also know that $x = -4$ is the third root.

Hence, the roots can be written as $-4, \alpha$ and α.

Using the sum of the roots we see that $2\alpha - 4 = 0 \rightarrow \alpha = 2$

Hence, we can substitute $\alpha = 2$ into (3) to find the value of m.

$$8 - 2(m+3) + (52 - 4m) = 0$$

$$54 - 6m = 0$$

$$m = 9$$