Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2011 YEAR 11 YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Working time – 2.5 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Attempt all questions
- Marks may be deducted for incomplete or illegible working

Class Teacher:
(Please tick or highlight)

- O Mr Weiss
- O Mr Ireland
- O Mr Berry
- O Mrs Collins
- O Mr Lam
- O Mr Fletcher/Ms Everingham

Student Name:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	Total	Total
Mark	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	/100

Question 1 (12 Marks) – START A NEW PAGE **Marks**

(a) Evaluate $\frac{\sqrt{2^5-\pi}}{\sqrt[3]{7}}$ to 3 significant figures. **2**

(b) Solve $2^{3x+1} = \frac{1}{2}$ **2**

(c) Fully simplify $\frac{\sin 30°+\tan 60°}{\cos^2 45°}$, giving the answer as an exact value. **3**

(d) Find the coordinates of the point P that divides the points $(-2,3)$ and $(6,1)$ externally in the ratio 1:5. **3**

(e) Solve $|7x+2| = 5$ **2**

Question 2 (12 Marks) – START A NEW PAGE

(a) Given that $\frac{2}{\sqrt{3}-2} = a\sqrt{3} - b$ find the values of a and b **2**

(b) Find the acute angle between the lines $2x - 5y + 1 = 0$ and $x + 2y - 3 = 0$. Answer to the nearest degree. **3**

(c) Solve for x: $\frac{x-1}{x+2} < 5$ **3**

(d) Factorise fully $x^3 - 8$ **2**

(e) Sketch $y = \sqrt{4-x}$ showing all relevant features. **2**

Question 3 (12 Marks) – START A NEW PAGE **Marks**

(a) Differentiate $y = \dfrac{1}{x}$ from first principles. **3**

(b) Differentiate each of the following with respect to x:

i) $\sqrt{x^5}$ **2**

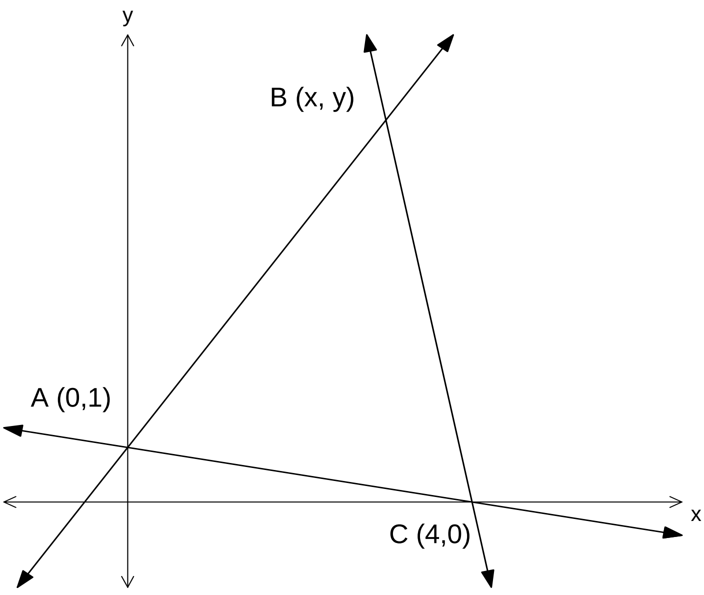
ii) $2x^2(x^2+5)^5$ **3**

(c) Find the equation of the normal to $y = x^3 - 2x^2$ when $x = 1$. **4**

Question 4 (12 Marks) – START A NEW PAGE

(a) Find the equation of the line that passes through the point $(-1, 2)$ and the point of intersection of the lines $x + 2y - 5 = 0$ and $2x - 5y - 1 = 0$. **3**

(b)



The lines AB and CB have equations $y = 2x + 1$ and $y = -7x + 28$ respectively.

i) State the equation of the line AC **1**

ii) Calculate the exact distance of AC **2**

iii) Find the coordinates of B **2**

iv) Calculate the area of triangle ABC **4**

Question 5 (12 Marks) – START A NEW PAGE **Marks**

For the function: $(x) = \frac{2}{x^2-4}$:

(a) Find whether it is odd, even or neither (justify your answer with working) **1**

(b) Find the *y* intercept. **1**

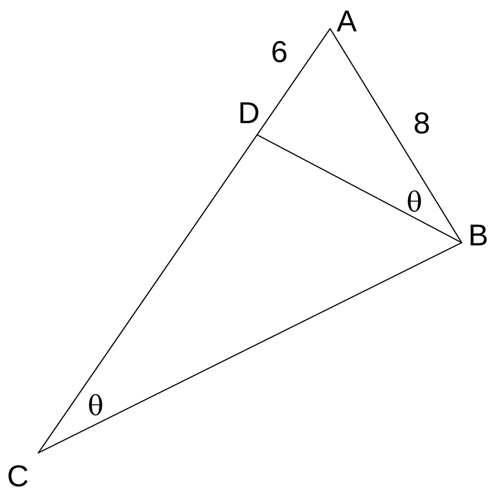
(c) State the equations of any vertical asymptotes **1**

(d) Find any stationary points and determine their nature **3**

(e) Show that there are no points of inflexion **3**

(f) Draw a neat sketch showing all relevant features **3**

Question 6 (12 Marks) – START A NEW PAGE



Not to scale

(a) In the above figure *AB* = 8 cm and *AD* = 6 cm. Copy the diagram into your answer booklet.

i) Show that $\Delta ABC \ ||| \ \Delta ABD$ **2**

ii) Find the length of *AC* **2**

iii) If $\angle ADB = 90^{\circ}$ find the area of *ABC* in terms of $\cos\theta$ **2**

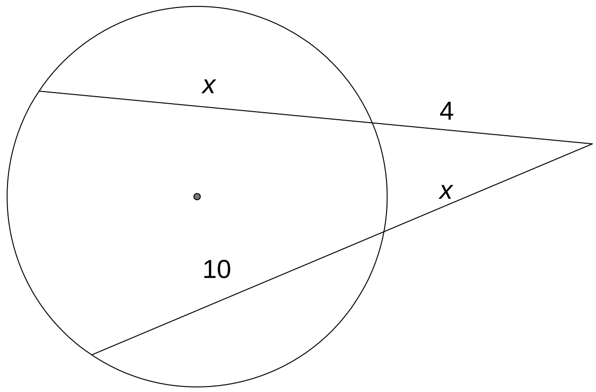
(b) A rhombus with diagonals *x* cm and (*x*+2) cm has an area of 24 cm^2

Find the value of *x* **3**

(c) Solve $2\log_3 5 - \log_3 x = 2$ **3**

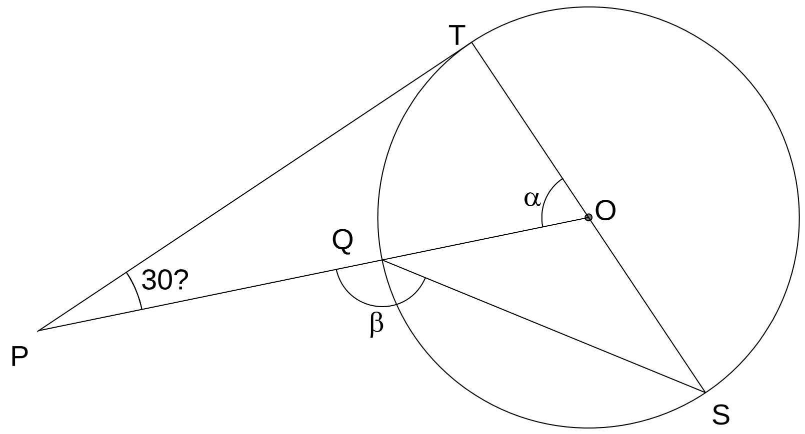
Question 7 (12 Marks) – START A NEW PAGE **Marks**

(a)



Find the value of x. No reasons necessary. **2**

(b)

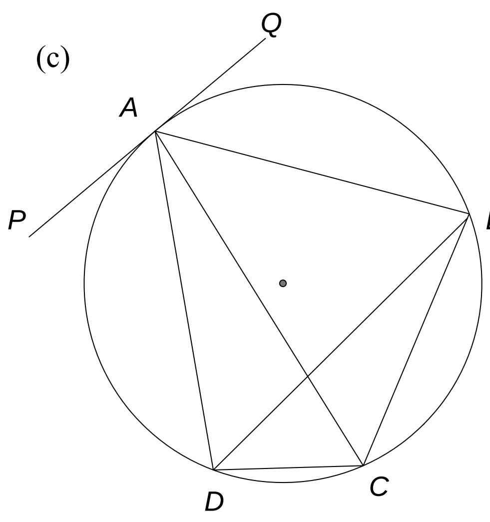


O is the centre of the circle and PT is a tangent touching the circle at T.

PO cuts the circle at Q. TS is a diameter. Find the values of α and β.

Show all necessary reasoning. **3**

(c)



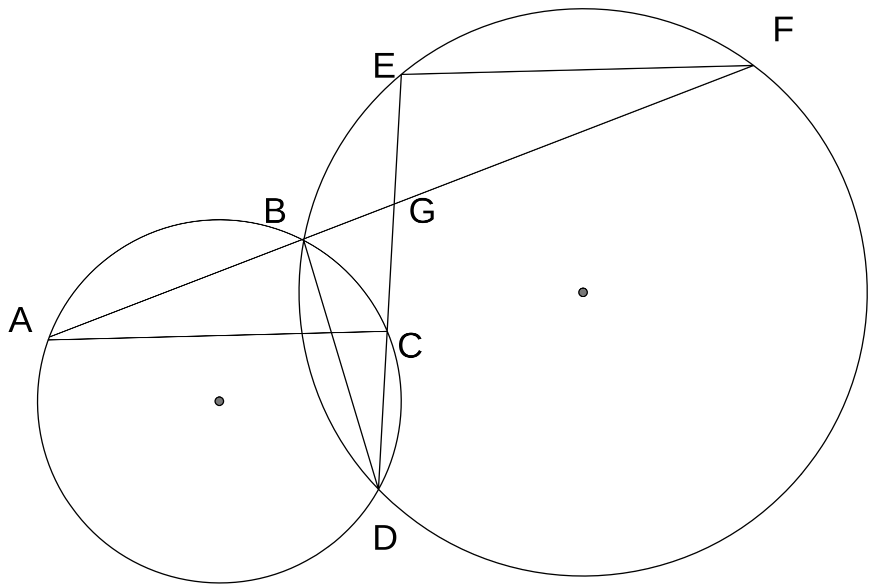
ABCD is a cyclic quadrilateral such that the tangent to the circle at *A* is parallel to *BD*

Copy this diagram in your answer booklet and show that:

i) *AC* bisects $\angle BCD$ **3**

ii) ΔABD is isosceles **1**

(d) **Marks**



In the diagram above *ABF* and *DCE* are straight lines

Copy the diagram into your answer booklet and prove *AC* is parallel to *EF*. **3**

Question 8 (12 Marks) – START A NEW PAGE

(a) A series is defined by $18 + 24k + 32k^2 + \ldots$

i) Prove that the series is geometric and find the common ratio **1**

ii) For what values of *k* will the series have a limiting sum **2**

iii) Find the limiting sum if $k = \frac{1}{6}$ **2**

(b) Find an expression for the *n*th term of a series if the sum of the first *n* terms is given by $S_n = n^2 + 5n$ **2**

(c) The sum of the first 4 terms of an arithmetic series is 42 and the sum of the 3rd and 7th term is 46. Find the sum of the first 20 terms. **3**

(d) Evaluate $\sum_{k=4}^{20} 2^{k-4}$ **2**

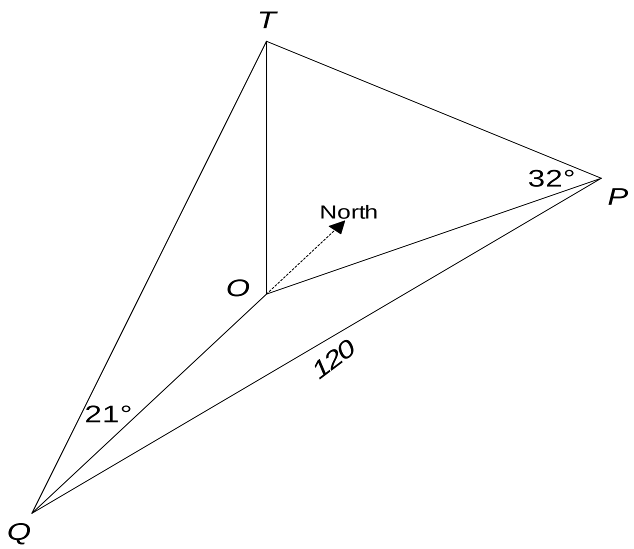
Question 9 (12 Marks) – START A NEW PAGE **Marks**

(a) Neatly sketch the graph of $y = 3\sin\frac{x}{2}$ for $0° \le x \le 360°$ **2**

(b) Solve $2\cos\theta + \sqrt{3} = 0$ for $0° \le \theta \le 360°$ **3**

(c) Prove that $\frac{2}{\tan A + \cot A} = \sin 2A$ **2**

(d) Point *P* is north east of a tower. The angle of elevation to the top of the tower from *P* is 32°. Point *Q* is due South of the tower. The angle of elevation to the top of the tower from *Q* is 21°. P and Q are 120 metres apart.



i) Show that OP and OQ are equal to $h\cot 32°$ and $h\cot 21°$ respectively, where *h* is the height of the tower. **1**

ii) Hence, or otherwise, find the height of the tower correct to one decimal place. **4**

Question 10 (12 Marks) – START A NEW PAGE

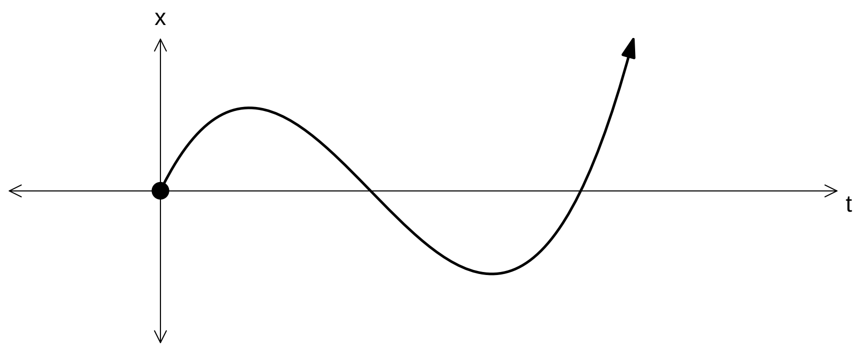
a) The surface area, in cm^2, of a balloon is given below by the formula, where *t* is the time in seconds.

$$SA = 2t^2 + 5t + 7$$

At what rate is the surface area increasing after 5 seconds? **2**

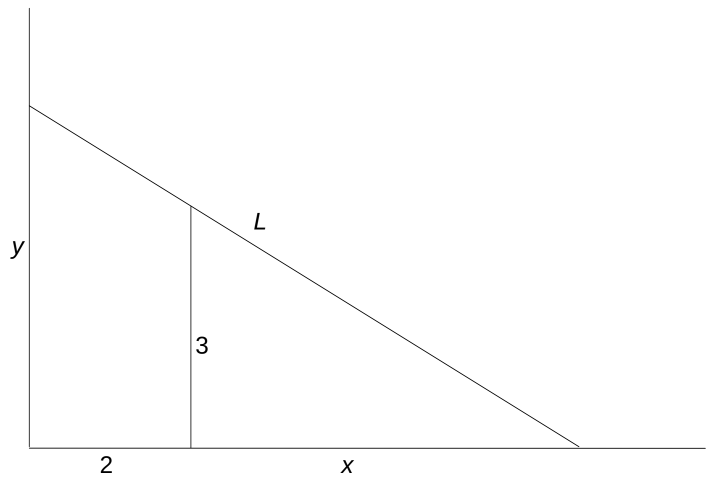
Question 10 continues on the next page.

b) The displacement of a particle is shown in the graph below. Copy the diagram into your answer booklet.



On two different sets of axes, vertically below this diagram, draw the graphs for the velocity and acceleration showing all relevant features (i.e. x-intercepts and stationary points) **4**

(c) A 3 metre vertical fence stands 2 metres from a high vertical wall. A ladder is placed from the horizontal ground to the wall , resting on the fence. The base of the ladder is x metres from the fence.



i) If L represents the length of the ladder, show that:

$$L^2 = (x+2)^2\left(1+\frac{9}{x^2}\right)$$ **2**

ii) By first finding $\frac{dL^2}{dx}$, or otherwise, calculate the shortest ladder that can reach from the ground outside the fence to the wall, correct to 2 decimal places. Show all working. **4**

EXTENSION YEAR 11 SOLUTIONS SEPT 2011

Question 1

(a) $\dfrac{\sqrt{2^5-\pi}}{\sqrt[3]{7}} = 2.80825\ldots$ $= 2.81$ (3 s.f.)	✓ answer ✓ rounding [2]
(b) $2^{3x+1} = \frac{1}{2}$ $= 2^{-1}$ $\therefore 3x+1 = -1$ $x = -\frac{2}{3}$	[2] ✓ equating indices ✓ correct answer
(c) $\dfrac{\sin 30^\circ + \tan 60^\circ}{\cos^2 45^\circ} = \dfrac{\frac{1}{2}+\sqrt{3}}{\left(\frac{1}{\sqrt{2}}\right)^2}$ $= \dfrac{\frac{1}{2}+\sqrt{3}}{\frac{1}{2}}$ $= 2\left(\frac{1}{2}+\sqrt{3}\right)$ $= 1+2\sqrt{3}$	✓ correct numerator [3] ✓ correct denominator ✓ fully simplified
(d) $(-2,3)\quad(6,1)$, $1:-5$ $P=\left(\dfrac{-2\times-5+1\times6}{1-5}, \dfrac{3\times-5+1\times1}{1-5}\right)$ $=\left(\dfrac{16}{-4}, \dfrac{-14}{-4}\right)$ $=\left(-4, 3\frac{1}{2}\right)$ $(-2,3)\quad(6,1)$, $-1:5$ $P=\left(\dfrac{-2\times5+-1\times6}{-1+5}, \dfrac{3\times5+-1\times1}{-1+5}\right)$ $=\left(\dfrac{-16}{4}, \dfrac{14}{4}\right)$ $=\left(-4, 3\frac{1}{2}\right)$	[3] ✓ correct working for x coord ✓ correct working for y coord. ✓ correct answer
e) $\lvert 7x+2\rvert = 5$ $7x+2=5$ $\quad$ $7x+2=-5$ $7x=3$ $\quad$ $7x=-7$ $x=\frac{3}{7}$ and $x=-1$	[2] ✓ two correct equations ✓ both solutions

Question 2

(a) $\frac{2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2} = \frac{2\sqrt{3}+4}{-1}$

$= -2\sqrt{3}-4$

compare $a\sqrt{3}-b$

$\therefore a=-2,\ b=4$

[2]

✓ simplified surd

✓ both correct values

(b) $2x-5y+1=0 \Rightarrow m_1 = \frac{2}{5}$

$x+2y-3=0 \Rightarrow m_2 = -\frac{1}{2}$

using $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$ gives:

$= \left|\frac{\frac{2}{5}+\frac{1}{2}}{1-\frac{2}{5}\cdot\frac{1}{2}}\right|$

$= \frac{9}{8}$

$\theta = \tan^{-1}\left(\frac{9}{8}\right)$

$= 48.3664\ldots$

$= 48°$ (nearest degree)

[3]

✓ correct gradients

wrong formula can only get 1 max.

✓ correct substitution

(no penalty for not rounding)

✓ correct answer

(c) $\frac{x-1}{x+2} < 5$ $\qquad \frac{(x-1)}{(x+2)} \times (x+2)^2 < 5(x+2)^2$

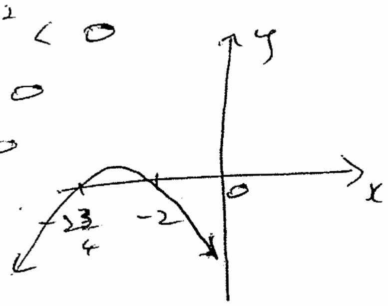
$(x-1)(x+2) - 5(x+2)^2 < 0$

$(x+2)[x-1-5(x+2)] < 0$

$(x+2)(-4x-11) < 0$

$-(x+2)(4x+11) < 0$

$\therefore x < -2\frac{3}{4}$, and $x > -2$



[3]

✓ working

✓ correctly tests region

✓ both correct answers

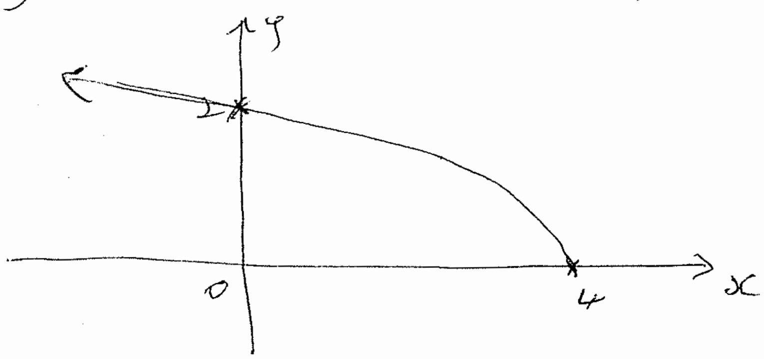
(d) $x^3-8 = x^3-2^3$

$= (x-2)(x^2+2x+4)$

[2]

✓✓ each factor

(e) $y=\sqrt{4-x}$



[2]

✓ correct intercepts

✓ correct shape.

semicircle [0]

Question 3

(a) $\lim_{h \to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ ✓

$= \lim_{h \to 0} \dfrac{\left(\frac{x - (x+h)}{x(x+h)}\right)}{h}$ [3]

$= \lim_{h \to 0} \dfrac{\left(\frac{-h}{x(x+h)}\right)}{\cancel{h}}$ ✓

$= \lim_{h \to 0} \dfrac{-1}{x(x+h)}$

$= \dfrac{-1}{x(x+0)}$

$= \dfrac{-1}{x^2}$ ✓

(b) i) $\frac{d}{dx} x^{5/2}$ ✓ [2]

$= 5/2\, x^{3/2}$ ✓

ii) $2x^2(x^2+5)^5$

$u = 2x^2$ $\quad$ $v = (x^2+5)^5$ ✓

$u' = 4x$ $\quad$ $v' = 10x(x^2+5)^4$

$u'v + v'u$ ✓ [3]

$= 4x(x^2+5)^5 + 10x(x^2+5)^4 \times 2x^2$ ✓

$= 4x(x^2+5)^4 [x^2+5 + 5x^2]$

$= 4x(x^2+5)^4(6x^2+5)$

(c) $y = x^3 - 2x^2$

when $x = 1$

$y = 1^3 - 2(1)^2$

$= -1$

$\therefore$ point $(1, -1)$ ✓

$y' = 3x^2 - 4x$ ✓

$m_{tgt} = 3(1)^2 - 4(1)$

$= -1$

$\therefore m_{norm} = 1$ ✓

[4]

$y - y_1 = m(x - x_1)$

$y + 1 = 1(x - 1)$ ✓

$\therefore x - y - 2 = 0$ or $y = x - 2$

Question 4

(a) $x + 2y - 5 + k(2x - 5y - 1) = 0$

$(-1) + 2(2) - 5 + k(2(-1) - 5(-2) - 1) = 0$

$\therefore -2 - 13k = 0$

$13k = -2$

$k = -2/13$

$\therefore x + 2y - 5 - 2/13(2x - 5y - 1) = 0$

$13x + 26y - 65 - 4x + 10y + 2 = 0$

$9x + 36y - 63 = 0$

$\therefore x + 4y - 7 = 0$

[3]

(b) i) $y = -1/4\, x + 1$

or

$x + 4y - 4 = 0$

[1]

ii) $AC = \sqrt{(4-0)^2 + (0-1)^2}$

$= \sqrt{17}$

[2].

iii) $y = 2x + 1$ ... (1)

$y = -7x + 28$ ... (2)

(1) = (2)

$2x + 1 = -7x + 28$

$9x = 27$

$x = 3$ sub into (1)

[2]

$y = 2(3) + 1$

$= 7$

B is $(3, 7)$

iv) let d be the perpendicular distance

$$d = \frac{|(1)(3) + (4)(7) - 4|}{\sqrt{1^2 + 4^2}}$$ ✓

$$= \frac{27}{\sqrt{17}}$$ ✓

$$A = \frac{1}{2} \times d \times AC$$ ✓ [4]

$$= \frac{1}{2} \cdot \frac{27}{\sqrt{17}} \times \sqrt{17}$$

$$= 13\tfrac{1}{2} \text{ units}^2$$ ✓

$\boxed{Q5}$ $f(x) = \dfrac{2}{x^2-4}$

(a) Let a be a number in the Domain of $f(x)$.

Then $f(a) = \dfrac{2}{a^2-4}$

and $f(-a) = \dfrac{2}{(-a)^2-4}$

$= \dfrac{2}{a^2-4}$

$= f(a)$. Hence $f(x)$ is even. ✓

(b) when $x=0$, $f(0) = \dfrac{2}{0-4} = -\dfrac{1}{2}$

$\therefore$ y-int. is $-\frac{1}{2}$. ✓

(c) $f(x) = \dfrac{2}{(x-2)(x+2)}$, hence there are

vertical asymptotes $x=2$, $x=-2$. ✓ must give equation

(d) $y' = \dfrac{-4x}{(x^2-4)^2}$, $y'' = \dfrac{12x^2+16}{(x^2-4)^3}$ ✓ for y'

For stationary points, $y'=0$

$\therefore$ $x=0$, $y=-\frac{1}{2}$ is a stationary point. ✓

At $(0,-\frac{1}{2})$, $y'' = \dfrac{0+16}{-64} < 0$

$\therefore$ $(0,-\frac{1}{2})$ is a maximum turning point. ✓ testing

ALTERNATIVELY :-

x	-1	0	1
y'	$+$	0	$-$
	/	—	\

$\therefore$ max. turning point at $(0,-\frac{1}{2})$

$\boxed{Q5}$ – continued

(e) $y' = \dfrac{-4x}{(x^2-4)^2}$

$$\therefore\ y'' = \frac{-4\cdot(x^2-4)^2 - (-4x)\cdot 2\cdot 2x\cdot(x^2-4)}{(x^2-4)^4}$$

$$= \frac{-4(x^2-4)^2 + 16x^2(x^2-4)}{(x^2-4)^4}$$

$$= \frac{-4(x^2-4) + 16x^2}{(x^2-4)^3}$$

$$\therefore\ y'' = \frac{12x^2+16}{(x^2-4)^3}$$

✓ Correct second derivative.

For inflexions, y'' must $= 0$, $\therefore 12x^2+16 = 0$

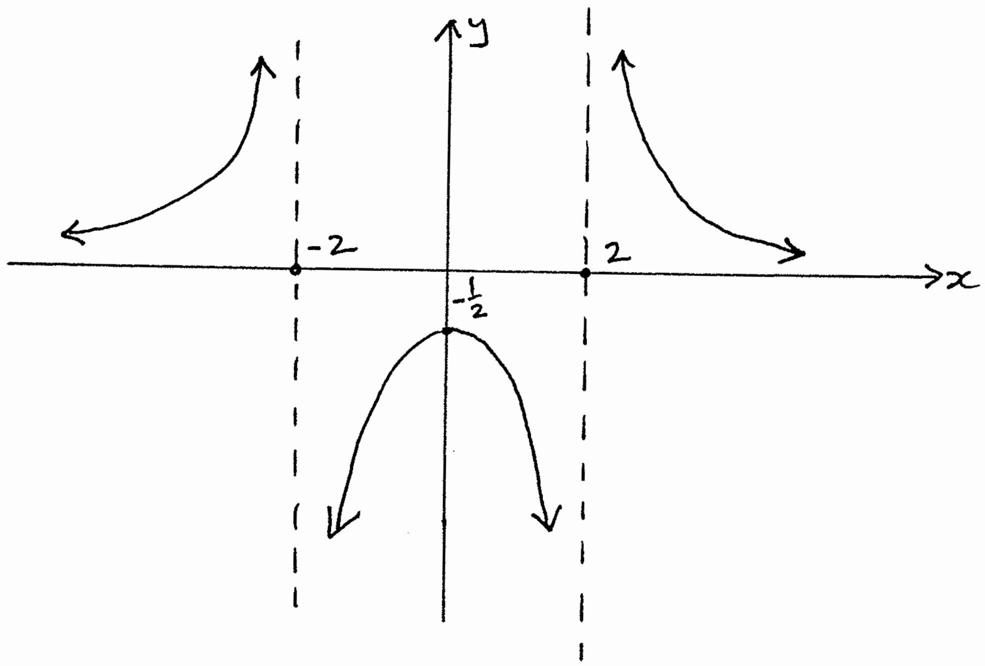
But $12x^2+16 > 0$ for all x,

thus y'' is never zero

$\therefore$ there can be no inflexion points.

✓✓ Correctly simplified, solved, & conclusion.

(f)
$\begin{cases} \text{as } x \to \infty, & y \to 0^+ \\ \text{as } x \to -\infty, & y \to 0^+ \end{cases}$



✓✓✓

Question 6

(a) i) In Δs ABC & ABD

$\angle ABD = \angle ACB$ (θ - given) ✓

$\angle BAD = \angle CAB$ (common) [2]

$\therefore \Delta ABC ||| \Delta ABD$ (equiangular) ✓

ii) $\frac{AD}{AB} = \frac{AB}{AC}$ (corresponding sides in similar triangles) ✓

$\frac{6}{8} = \frac{8}{AC}$ [2]

$\therefore 6AC = 64$

$AC = 64/6$ cm. or $10\frac{2}{3}$ cm. ✓

iii) $\cos\theta = \frac{DB}{8}$

$DB = 8\cos\theta$ ✓

$A = \frac{1}{2} \times AC \times DB$

$= \frac{1}{2} \times 10\frac{2}{3} \times 8\cos\theta$ [2]

$= \frac{128\cos\theta}{3}$ cm^2

(b) $A = \frac{1}{2} \cdot$ diagonals

$\therefore 24 = \frac{1}{2}(x)(x+2)$ ✓

$\therefore x^2 + 2x = 48$

$x^2 + 2x - 48 = 0$ ✓

$(x+8)(x-6) = 0$ [3]

$\therefore x = 6$ cm since length > 0 ✓

(c) $2\log_3 5 - \log_3 x = 2$ ✓

$\log_3 25 - \log_3 x = 2$ ✓

$\log_3\left(\frac{25}{x}\right) = 2$

$\frac{25}{x} = 3^2$ [3]

$x = \frac{25}{9}$ ✓

Q7

(a) $(x+4)(4) = (x+10)(x)$ ✓

$\therefore\ 4x + 16 = x^2 + 10x$

$x^2 + 6x - 16 = 0$

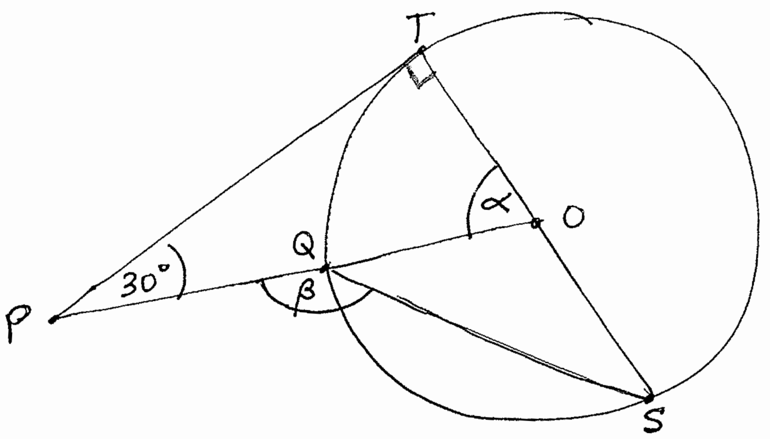
$(x+8)(x-2) = 0$

$\therefore\ x = -8,\ 2$

but $x > 0$

$\therefore\ x = 2.$ ✓

(b)



(Find α, β values)

$\angle PTO = 90°$ (tangent ⊥ radius to point of contact)

$\therefore\ \alpha = 60°$ ($\angle$ sum of ΔPTO)

$\angle OQS = \angle OSQ$ (base $\angle$'s of isosceles ΔOQS, which has 2 equal radii as sides).

But $\alpha = \angle OQS + \angle OSQ$ (exterior $\angle$ of ΔOQS)

$\therefore\ \angle OQS = \frac{1}{2}\alpha = 30°$

$\therefore\ \beta = 180 - 30$ (supplementary $\angle$'s on straight line PO).

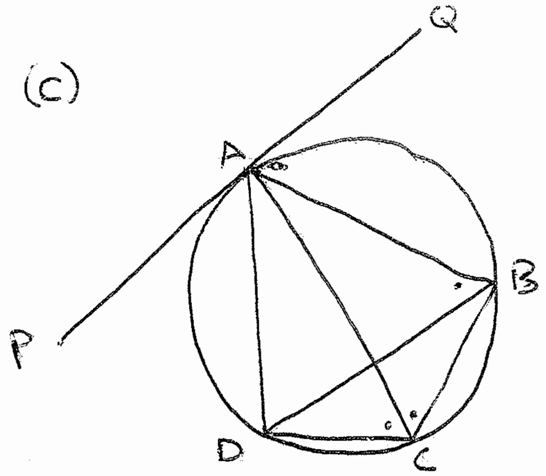
$\therefore\ \beta = 150°$

$\alpha = 60$ ✓

$\beta = 150$ ✓

Reasons ✓

(c)



(i) $\angle QAB = \angle ACB$ (alternate segment theorem) ✓

$\angle ABD = \angle QAB$ (alternate angles, $PQ \parallel BD$) ✓

$\angle ACD = \angle ABD$ (angles standing on same arc)

$\therefore \angle ACD = \angle ACB$ ✓

$\therefore$ AC bisects $\angle BCD$

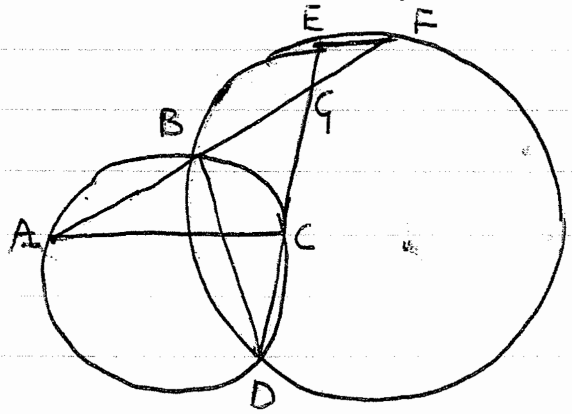
(ii) $\angle QAB = \angle ABD$ (alternate $\angle$s, $PQ \parallel BD$)

$\angle ADB = \angle QAB$ (alternate segment theorem) ✓

$\therefore \angle ADB = \angle ABD$

$\therefore \Delta ABD$ is isosceles (base $\angle$s ADB & ABD are equal)

(d)



$\angle BDE = \angle EFB$ ($\angle$s standing on same arc) ✓

$\angle BAC = \angle BDE$ ($\angle$s standing on same arc) ✓

$\angle BAC = \angle EFB$

$\therefore AC \parallel EF$ (alternate $\angle$s are equal) ✓

Question 8

(a) $\frac{T_3}{T_2} = \frac{32k^2}{24k}$

$= \frac{4k}{3}$

[1]

$\frac{T_2}{T_1} = \frac{24k}{18}$

$= \frac{4k}{3}$ ✓

$\therefore$ this is a GP where $r = 4k/3$

(b) $-1 < r < 1$

$-1 < 4k/3 < 1$

$-3 < 4k < 3$ [2]

$-3/4 < k < 3/4$

✓ ✓

(iii) $S_n = \frac{a}{1-r}$ ✓

$= \frac{18}{1-(4/3 \times 1/6)}$ [2]

$= \frac{18}{(14/18)}$

$= 23\ 1/7$ or $162/7$ ✓

(b) $T_n = S_n - S_{n-1}$

$= n^2 + 5n - [(n-1)^2 + 5(n-1)]$ ✓

$= n^2 + 5n - [n^2 - 2n + 1 + 5n - 5]$ [2]

$= \cancel{n^2} + \cancel{5n} - \cancel{n^2} + 2n - 1 - \cancel{5n} + 5$

$= 2n + 4$ ✓

(c) $42 = \frac{4}{2}(2a + (4-1)d)$

$21 = 2a + 3d$

[3]

$46 = a + (3-1)d + a + (7-1)d$

$46 = 2a + 8d$

$46 = 2a + 8d \quad \ldots (1)$ ✓ equations

$21 = 2a + 3d \quad \ldots (2)$

(1) − (2)

$25 = 5d$

$d = 5$

substitute into (2)

$21 = 2a + 3(5)$ ✓ a & d

$2a = 6$

$a = 3$

$S_{20} = \frac{20}{2}(2(3) + (20-1)5)$

$= 1010$ ✓ answer

(d) $\sum_{k=4}^{20} 2^{k-4}$

this form a - GP ✓ any 3 of these 4 points correct

- $a = 1$
- $r = 2$
- $n = 17$

[2]

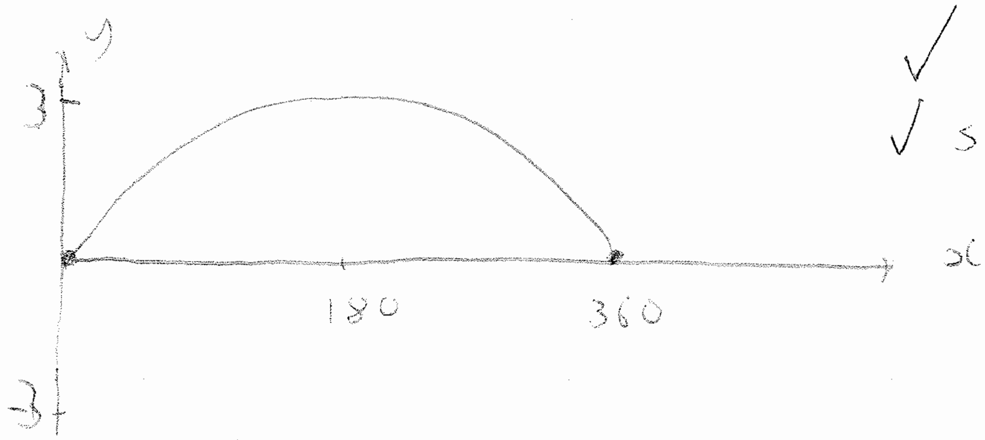
$\therefore S_n = \frac{a(r^n - 1)}{r-1}$

$= 2^{17} - 1$

$= 131\ 071$ ✓

Question 9

(a)



✓ period [2]
✓ shape & amplitude

(b) $2\cos\theta + \sqrt{3} = 0$

$2\cos\theta = -\sqrt{3}$

$\cos\theta = -\sqrt{3}/2$ ✓ [3]

$\theta = 150°$ ✓ or $210°$ ✓

(c) $LHS = \dfrac{2}{\tan A + \cot A}$

$= \dfrac{2}{\sin A/\cos A + \cos A/\sin A}$ ✓

$= \dfrac{2}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$ [2]

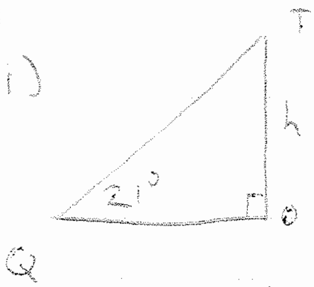
$= \dfrac{2\sin A\cos A}{1}$ ✓

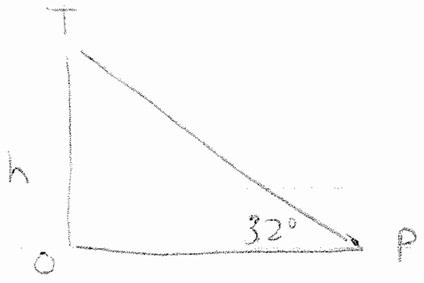
$= \sin 2A$

$= LHS$, as req'd.

(d)

i)





[✓]

(1)

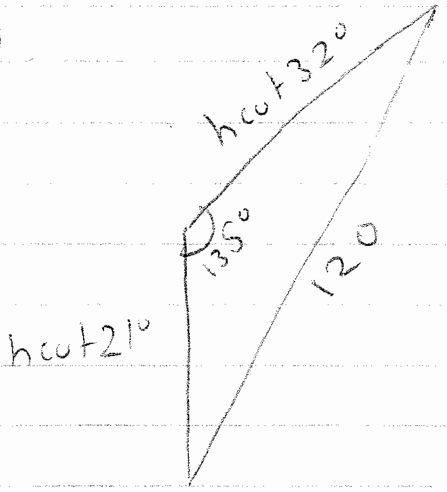
$\cot 21° = \dfrac{OQ}{h}$

$\therefore OQ = h\cot 21°$

$\cot 32° = \dfrac{OP}{h}$

$OP = h\cot 32°$

ii)



✓ cos rule

✓ 135°

✓ making h the subject

✓ answer

[4]

$$h^2\cot^2 21° + h^2\cot^2 32 - 2h\cot(21°)\,h\cot 32°\cos 135 = 120^2$$

$$\therefore h^2\left(\cot^2 21° + \cot^2 32° - 2\cot(21°)\cot(32°)\cos 135°\right) = 120$$

$$\therefore h = \sqrt{\frac{120^2}{\tan^2 69 + \tan^2 58 - 2\tan(69°)\tan 58°\cos(135}}$$

$= 30{\cdot}7\text{m}$

Suggested Solutions

Question 10 (Lam)

(a) (2 marks)

✓ [1] for correct differentiation.

✓ [1] for substitution and final correct answer.

$$A = 2t^2 + 5t + 7$$
$$\frac{dA}{dt} = 4t + 5$$

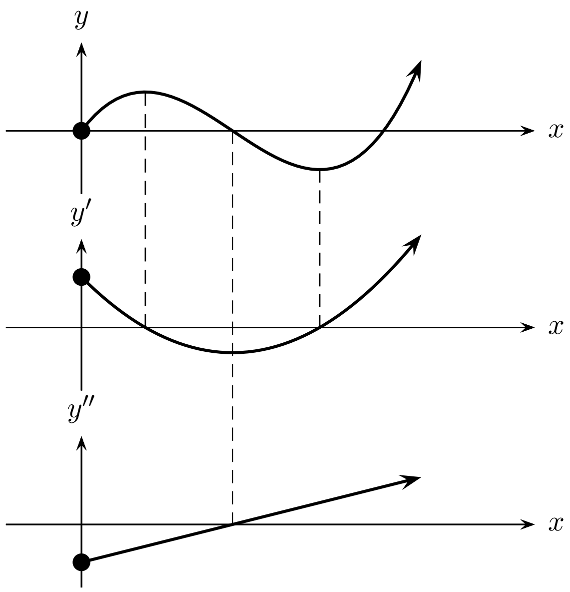
When $t = 5$,

$$\frac{dA}{dt} = 4t + 5\Big|_{t=5} = 25\,\mathrm{cm}^2\mathrm{s}^{-1}$$

(b) (4 marks)

✓ [2] for correct shape/intercepts of first derivative.

✓ [2] for correct shape/intercepts of second derivative.



(c) i. (2 marks)

✓ [1] for Pythagoras' Theorem relationship.

✓ [1] for final correct answer.

By Pythagoras' Theorem,

$$L^2 = y^2 + (2+x)^2$$

By similarity,

$$\frac{y}{3} = \frac{2+x}{x}$$
$$\therefore y = \frac{3(x+2)}{x}$$

Substitute into L^2,

$$\begin{aligned} L^2 &= \left(\frac{3(x+2)}{x}\right)^2 + (x+2)^2 \\ &= (x+2)^2\left(\frac{9}{x^2} + 1\right) \end{aligned}$$

ii. (4 marks)

✓ [1] for using product rule.

✓ [1] for correct derivative.

✓ [1] for checking for local maximum.

✓ [1] for final answer.

$$L^2 = (x+2)^2\left(\frac{9}{x^2}+1\right)$$

Applying the product rule,

$$\begin{aligned} u &= (x+2)^2 & v &= 9x^{-2} + 1 \\ u' &= 2(x+2) & v' &= -2(9)x^{-3} \\ & & &= -18x^{-3} \end{aligned}$$

$$\begin{aligned} \frac{dL^2}{dx} &= uv' + vu' \\ &= -18x^{-3}(x+2)^2 + 2(x+2)\left(\frac{9}{x^2}+1\right) \\ &= 2(x+2)\left(\frac{-9(x+2)}{x^3} + \frac{9}{x^2} + 1\right) \\ &= 2(x+2)\left(\frac{-9x - 18 + 9x + x^3}{x^3}\right) \\ &= 2(x+2)\left(\frac{x^3 - 18}{x^3}\right) \end{aligned}$$

Stationary pts when $\frac{dL^2}{dx} = 0$. Since $x > 0$ as it is a length,

$$x^3 - 18 = 0$$
$$x = \sqrt[3]{18} \approx 2.62\cdots$$

x		$\sqrt[3]{18} \approx 2.62$	
$\frac{dL^2}{dx}$	$-$	0	$+$
L^2	$\searrow$	min	$\nearrow$

Hence $x = \sqrt[3]{18}$ minimises L.

$$
\begin{aligned}
L &= \left(18^{\frac{1}{3}} + 2\right)\sqrt{\left(\frac{9}{18^{2/3}} + 1\right)} \\
&\approx 7.02\,\mathrm{m}
\end{aligned}
$$