Maths

NORTH SYDNEY BOYS HIGH SCHOOL

2012 YEAR 11 YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Working time – 2.5 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Attempt all questions
- Marks may be deducted for incomplete or illegible working

Class Teacher:
(Please tick or highlight)

- O Mr Rezcallah/ Ms Juhn
- O Mr Fletcher
- O Ms Ziaziaris
- O Mr Lucas
- O Mr Lam
- O Mr Berry

Student Name:

(To be used by the exam markers only.)

Question No	1-10	11	12	13	14	15	16	17	18	Total	Total
Mark	/10	/14	/11	/11	/10	/14	/12	/13	/15	/110	/100

Question 1

Evaluate $\frac{(7.12)^3 - \sqrt{8}}{\sqrt[3]{2}}$ to 3 significant figures:

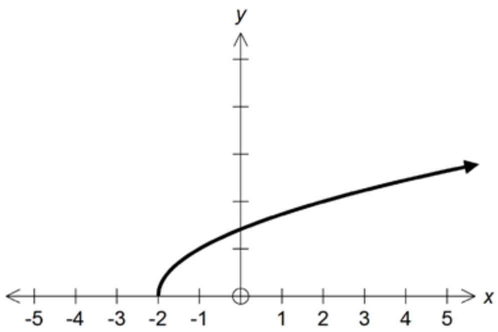
a) 1.11 b) 84.4 c) 253 d) 284

Question 2

Which of the following is a function?

a) $x = y^2$ b) $x = \frac{1}{y}$ c) $x^2 + y^2 = 1$ d) {1,2}, {2,5}, {1,-4}

Question 3



The function above can be best described as:

a) $y = \sqrt{x - 2}$ b) $y = \sqrt{x + 2}$ c) $y = \sqrt{x^2 - 2}$ d) $y = \sqrt{x^2 + 2}$

Question 4

If $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0$, which of the following is the value of $\tan\theta$?

a) $-\frac{\sqrt{21}}{2}$ b) $-\frac{2}{\sqrt{21}}$ c) $\frac{2}{\sqrt{21}}$ d) $\frac{\sqrt{21}}{2}$

Question 5

The perpendicular distance of (2, 3) from $2x - y + 3 = 0$ is given by:

a) $\frac{\sqrt{5}}{4}$ b) $\frac{\sqrt{13}}{4}$ c) $\frac{4}{\sqrt{13}}$ d) $\frac{4}{\sqrt{5}}$

Question 6

Which of the following is not a test for congruent triangles?

a) SAS b) SSA c) AAS d) SSS

Question 7

Solve for *x*: $7^x = 200$.

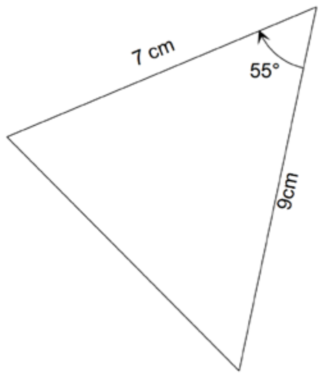
a) 0.328 b) 0.367 c) 2.72 d) 237

Question 8

The derivative of $\frac{2}{x^3}$ is:

a) $\frac{2x^2}{3}$ b) $-\frac{6}{x^4}$ c) $\frac{6}{x^2}$ d) $6x^4$

Question 9



The area of the above triangle is given by:

a) $\frac{63 \sin 55^o}{2}$ b) $63 \sin 55°$ c) $\frac{63 \cos 55^o}{2}$ d) $63 \cos 55°$

Question 10

The sum of the first *n* terms of an arithmetic series with a first term of 10 and a common difference of -2 is given by:

a) $3 - 2n$ b) $7 - 2n$ c) $5n - 2n^2$ d) $11n - n^2$

Question 11 *(14 Marks)*

(a) Express $\frac{5}{\sqrt{5}-\sqrt{3}}$ with a rational denominator. **2**

(b) Solve for *x*: $|3x+2|=5$ **2**

(c) Solve $\frac{4x-3}{x} \geq 5$ and sketch your solution on a number line. **3**

(d) Clearly sketch the region specified by: $(x-5)^2+(y+4)^2<4$ **3**

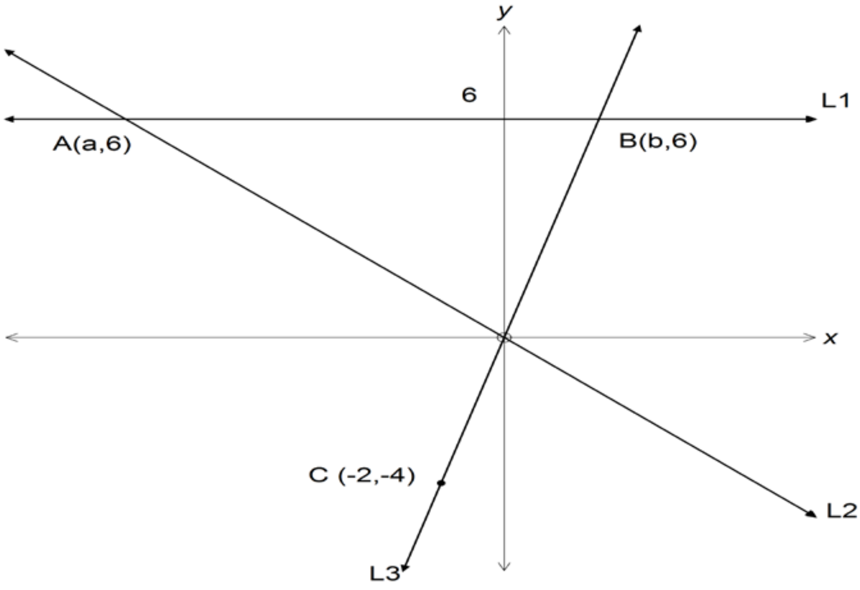
(e) i) Sketch $f(x)=x^2-3x-4$ showing all relevant features. **2**

ii) Hence, or otherwise, sketch $f(x)=\frac{1}{x^2-3x-4}$ on a separate number plane. **2**

(do not use calculus)

Question 12 *(11 Marks)*

(a) In the diagram below, the line *L1* is parallel with the *x*-axis and crosses the *y*-axis at 6. Lines *L2* and *L3* each pass through the origin, *O*, and intersect with *L1* at the points *A* (*a*,6) and *B* (*b*,6) respectively. The point *C* (-2, -4) lies on the line *L3*.



i) Show that the equation of the line *L3* is $2x-y=0$ **2**

ii) Show that the *x*-ordinate of point *B* is 3. **1**

iii) Find the coordinates of point A such that $\angle AOB$ is a right angle **3**

iv) Write down the distance between points *A* and *B*. Hence, or otherwise calculate the area of ΔAOB. **2**

(b) *P*(0, 0) divides *AB* externally in the ratio 2 : 1, where *A* is (*x*, *y*) and *B* is (4, 9). Find the values of *x* and *y*. **3**

Question 13 *(11 Marks)*

(a) Differentiate the following with respect to *x* and simplify your answer:

i) $y = x^2\sqrt{x^3}$ **2**

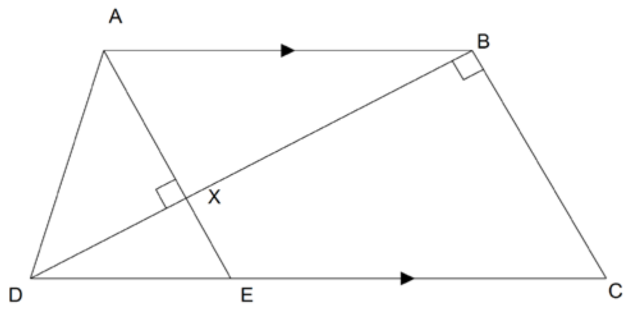
ii) $y = x^2(x-5)^3$ **3**

iii) $y = \frac{x^3+2}{5-x}$ **3**

(b) Find the equation of the tangent to the curve $y = \sqrt{x+2}$ at the point where $x = 5$. **3**

Question 14 *(10 Marks)*

(a) *ABCD* is a trapezium where *AB* // *DC*. Also *BD* is perpendicular to *BC* and *AE* is perpendicular to *BD* at *X*.



i) Copy the diagram into your writing booklet and find *AX* given that *AD* = 41 cm and *DX* = 9 cm. **1**

ii) What type of quadrilateral is *ABCE*. Give reasons. **2**

iii) Show that $\Delta DXE \;|||\; \Delta DBC$ **2**

iv) Hence show that *BX* . *XE* = 360. **3**

(b) Draw a neat sketch of $y = \log_2(x-1)$ clearly showing all relevant features. **2**

Question 15 *(14 Marks)*

(a) Consider the curve $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$

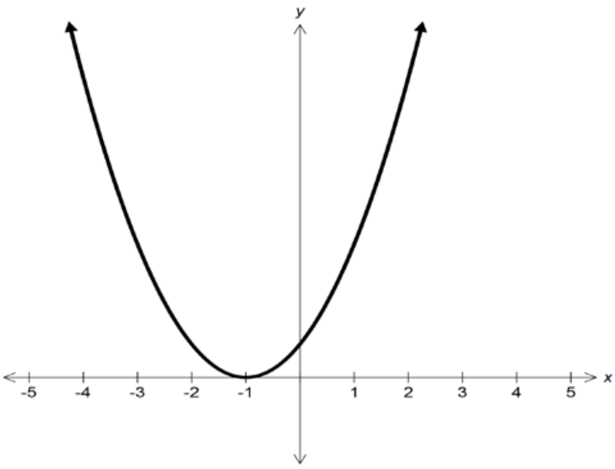
(i) Find the coordinates of any stationary points and determine their nature. **4**

(ii) Find any point(s) of inflexion. **2**

(iii) Sketch the curve in the domain $-6 \leq x \leq 3$. **2**
(You do not need to find the x intercepts)

(iv) What is the maximum value of $f(x)$ in the given domain? **1**

(b) Consider the function $y = f(x)$, whose ***gradient*** function is shown as a parabola in the sketch below.



(i) Comment on the sign of the gradient function for all $x \neq -1$ **1**

(ii) What can you conclude about $y = f(x)$ when $x = -1$ **1**

(iii) Sketch a graph for $y = f''(x)$ **1**

(iv) Sketch a possible graph for $y = f(x)$ **2**

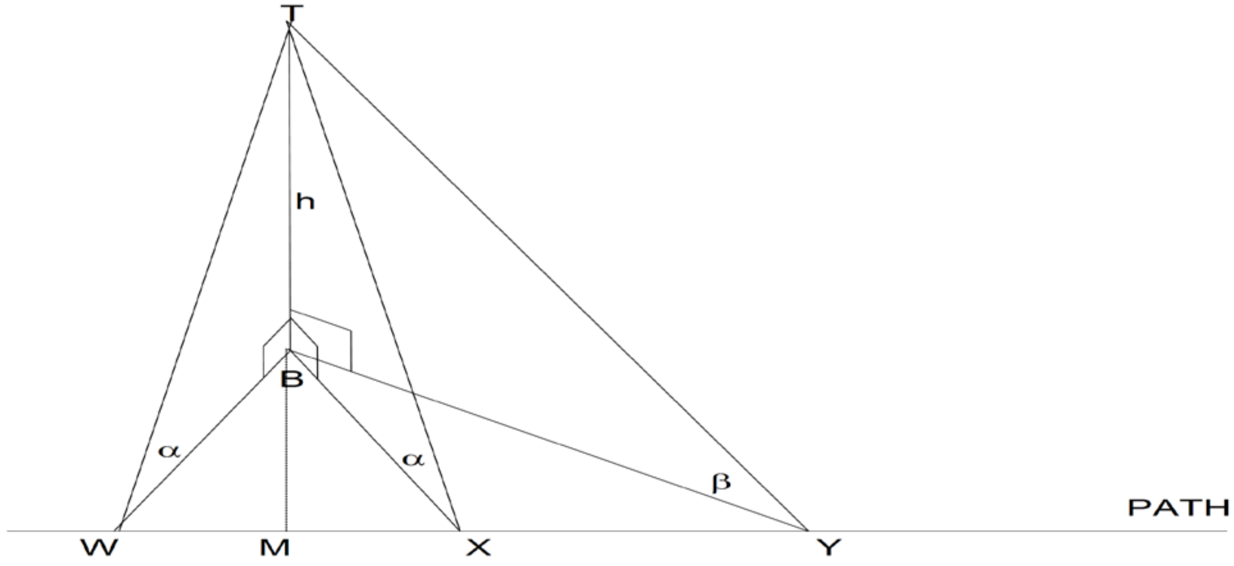
Question 16 *(12 Marks)*

(a) In ΔABC, AC = 8 cm, BC = 10 cm and $\angle ABC = 50^\circ$. Find the $\angle BAC$ to the nearest minute. **3**

(b) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$ **3**

Question 16 Continued

(c) Three points W, X and Y lie on a straight level path, where $WX = XY = 40$ m. The base, B, of a flagpole, TB, is level with the path. M is the midpoint of WX. The angles of elevation to the top of the flagpole from the points W, X and Y are α, α and β respectively. The height of the flagpole is h.



i) Prove that ΔBWX is isosceles. **2**

ii) Find the length of BM in terms of h and α. **1**

iii) Hence, or otherwise, show that the height of the flagpole is given by:

$$h = \frac{40\sqrt{2}}{\sqrt{\cot^2\beta - \cot^2\alpha}}$$ **3**

Question 17 *(13 Marks)*

(a) The first and last terms of an arithmetic series are 10 and 66, respectively, and the sum of the series is 570. Find:

i) The number of terms in the series. **2**

ii) The common difference **2**

(b) The general term of a geometric series is defined by $T_n = (x-2)^n$.
Find:

i) The value(s) of x for which the series has a limiting sum: **1**

ii) This limit in terms of x. **2**

(c) For the series $0 + \log_{10}x^2 + \log_{10}x^4 + \ldots$

i) Prove that this series is arithmetic and hence find the common difference. **2**

ii) Hence, or otherwise, find the sum of the first 10 terms. **1**

(d) Find the value of x if $\displaystyle\sum_{n=0}^{\infty} \frac{9}{x^{n+1}} = 18$ **3**

Question 18 *(15 Marks)*

(a) A particle moves in such a way that its distance, x metres, from the origin after t seconds is given by $x = 3 + 18t - t^3$ for $t \geq 0$.

i) Find an equation for its velocity after t seconds. **1**

ii) At what time does the particle stop? **1**

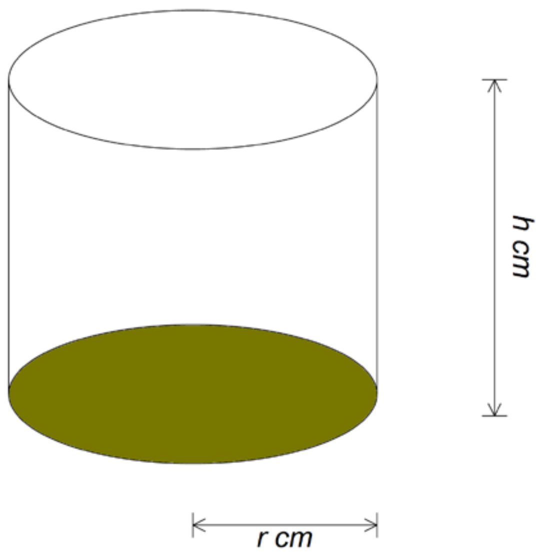
iii) Where is the particle initially? **1**

iv) Find the velocity after 1 second. **1**

v) How far has the particle travelled in the first second? **1**

vi) What is the particle's acceleration after 2 seconds? **2**

(b) A cylindrical can with an open top has a base radius of r cm and a height of h cm. It is to be made using 75π cm^2 of tin.



(i) Show that $h = \dfrac{75 - r^2}{2r}$ **2**

(ii) Find an expression for the volume of the can, V, in terms of r. **1**

(iii) Show that the maximum volume of this can in 125π cm^3. **4**

(iv) Find the height of the can from (iii). **1**

End of examination

$\boxed{\text{1 mark each}}$

1: D

2: B

3: B

4: B

5: D

6: B

7: C

8: B

9: A

10: D. #

(11)

(a) i) $\frac{5}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{5(\sqrt{5}+\sqrt{3})}{2}$

(b) $|3x+2| = 5$

$\therefore 3x+2 = 5$ or $3x+2 = -5$

$3x = 3 \qquad 3x = -7$

$x = 1 \qquad x = \frac{-7}{3}$

(c)

$\boxed{\text{method 1}}$

$\frac{4x-3}{x} \geq 5$

$x \neq 0$

$\frac{4x-3}{x} = 5$

$4x-3 = 5x$

$\therefore x = -3$

or

$\boxed{\text{method 2}}$

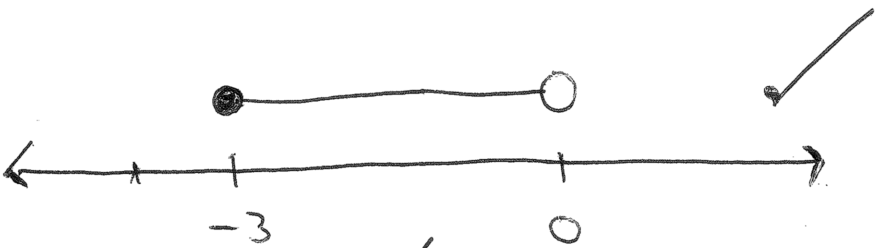
$\frac{4x-3}{x} \times x^2 \geq 5 \times x^2$

$4x^2 - 3x \geq 5x^2$

$\therefore x^2 + 3x \leq 0$

$x(x+3) \leq 0$

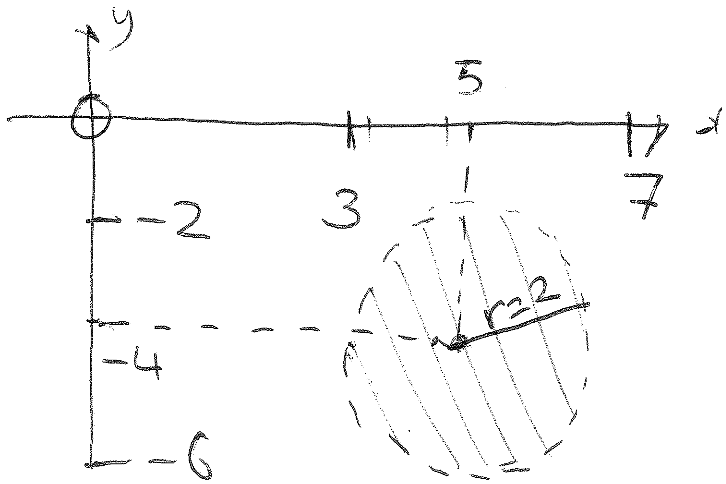
$\therefore -3 \leq x < 0$, as $x \neq 0$



x ✓ x ← testing a point in each region

$\therefore -3 \leq x < 0$

(d)

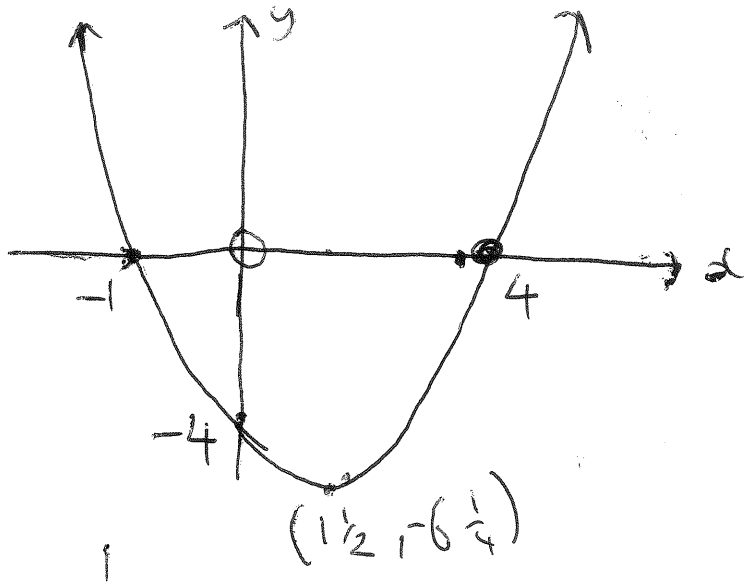


✓ dotted line

✓ shade region

✓ correct circle

(e) i) $y = (x-4)(x+1)$

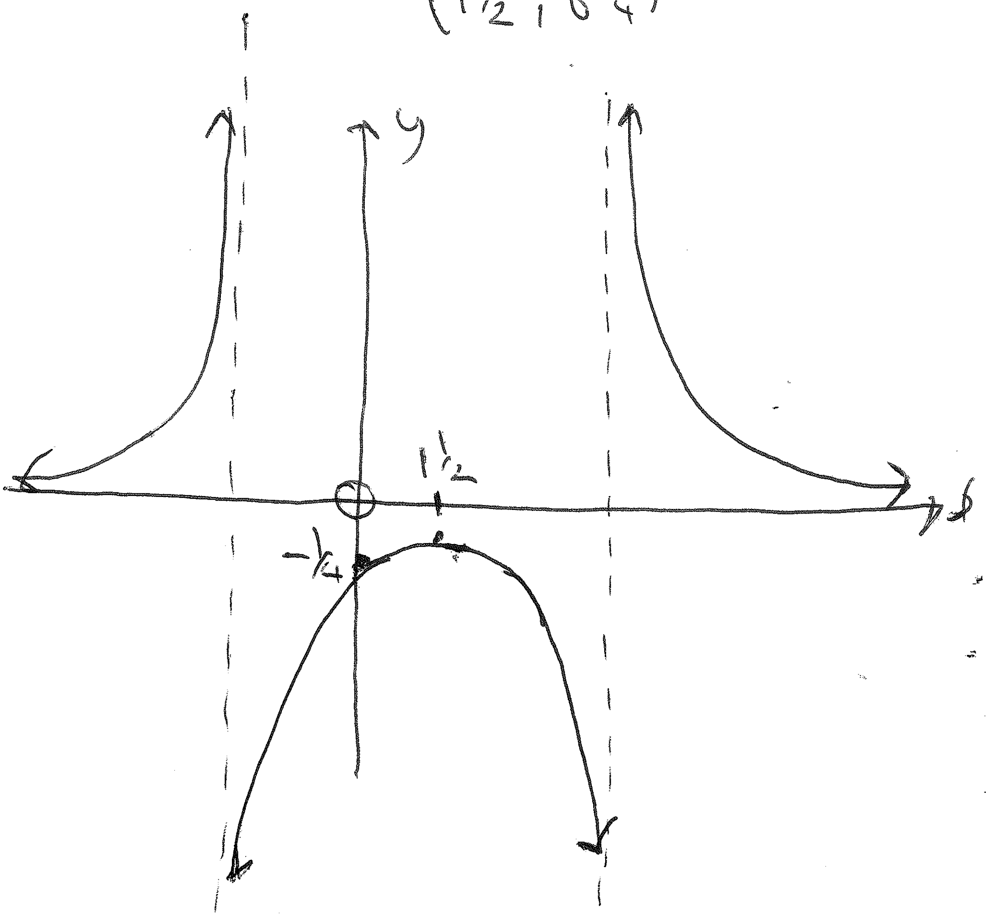


✓ intercepts

✓ vertex

(0 if wrong shape)

ii)



✓ shape

✓ asymptotes/point

(12)

(a) i) $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\therefore \frac{y-0}{x-0} = \frac{-4-0}{-2-0}$ ✓

$\therefore -2y = -4x$

$\therefore 4x-2y = 0$

$2x-y = 0$, as req'd ✓

ii) when $y=6$

$2x-6 = 0$ (eqn of L3)

$2x = 6$ ✓

$x = 3$

iii) m of $L3 = 2$. ✓

$\therefore m_{L3} \times m_{L2} = -1$

$\therefore m_{L2} = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x$ (L2 goes through origin) ✓

when $y = 6$

$6 = -\frac{1}{2} \times x$

$\therefore x = -12$ ✓

iv) $AB = 15$ units ✓

$AOB = \frac{1}{2} \times 15 \times 6$ ✓

$= 45$ units^2

(b) x: $0 = \dfrac{m x_2 + n x_1}{m + 2}$

$= \dfrac{(2)(4) + (-1)(x)}{2 + (-1)}$ ✓

$= 8 - x$

$\therefore x = 8$.

y: $0 = \dfrac{m y_2 + n y_1}{m + n}$

$= \dfrac{(2)(9) + (-1)(y)}{2 + (-1)}$ ✓

$= 18 - y$

$\therefore y = 18$

$\therefore A(x, y)$ is $(8, 18)$ ✓

(full marks if they do a diagram and solve by inspection)

(b)

$y = (x+2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ — 1

$m = \frac{1}{2\sqrt{7}}$ — 1

$x = 5 \quad y = \sqrt{7}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{7} = \frac{1}{2\sqrt{7}}(x-5)$

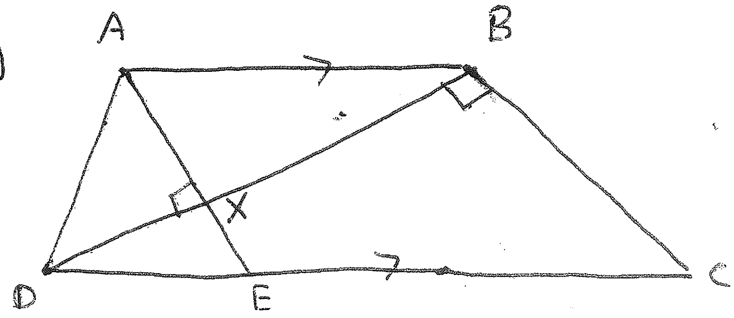
$2\sqrt{7}(y - \sqrt{7}) = x - 5$

$2\sqrt{7}y - 14 = x - 5$

$x - 2\sqrt{7}y + 9 = 0$ — 1

(14)

(a) i)



$(AX)^2 = (AD)^2 - (DX)^2$ (Pythagoras)

$\therefore AX = \sqrt{41^2 - 9^2}$

$= 40$ cm ✓

ii) $AB \parallel EC$ (given)

$AE \parallel BC$ (alt $\angle$s both $90°$) ✓

$\therefore$ parallelogram (opposite sides parallel) ✓

iii) In $\triangle$s DXE & DBC

$\angle DXE = \angle DBC$ (both $90°$) ✓

$\angle XDE = \angle BDC$ (common)

$\therefore \triangle DXE \mathbin{|||} \triangle DBC$ (equiangular) ✓

iv) $\frac{XE}{BC} = \frac{DX}{DB}$ (matching sides of similar $\triangle$s in ratio) ✓

(for BC=AE) ✓

$\therefore XE \cdot DB = BC \cdot DX$

$\therefore XE(BX+9) = 9(XE+40)$ (since $BC = AE$, opp sides of $\parallel$gram equal)

$\therefore BX \cdot XE + 9(\cancel{XE}) = 9(\cancel{XE}) + 360$

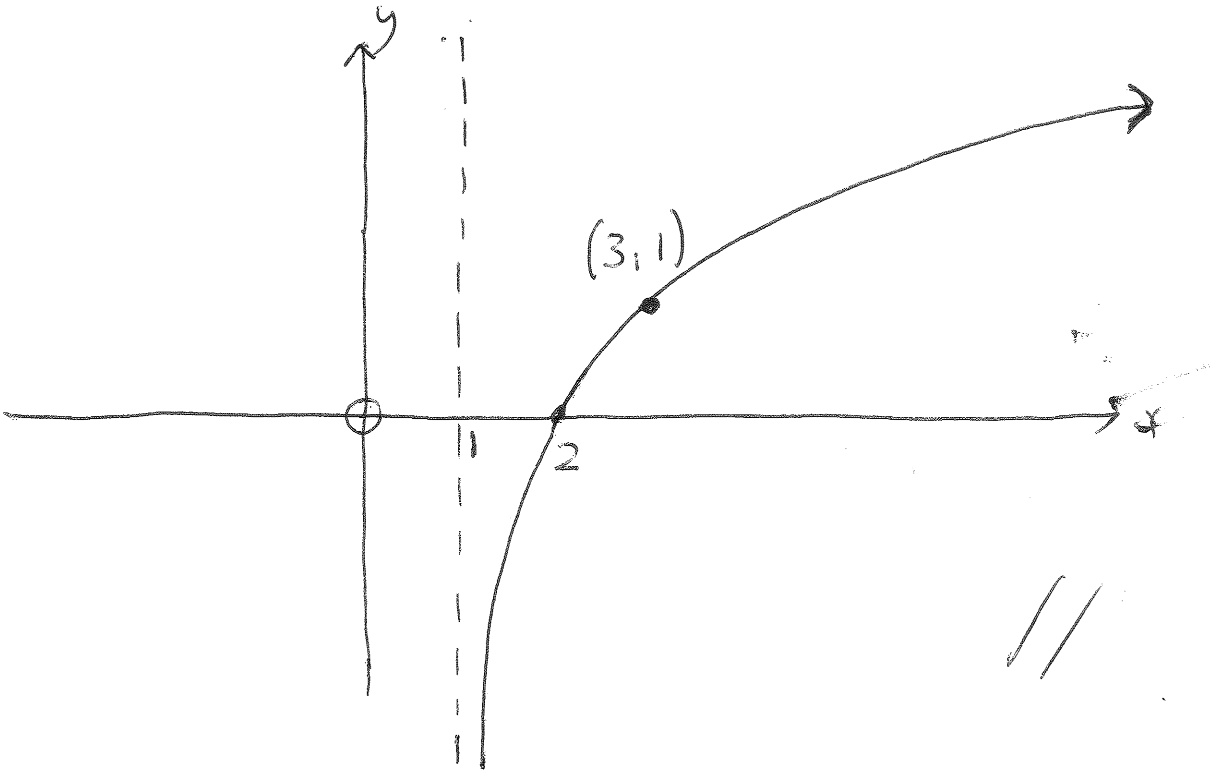
$\therefore BX \cdot XE = 360$, as req'd. ✓

alternately: Prove $\triangle AXB \mathbin{|||} \triangle EXD$

then $\frac{EX}{AX} = \frac{DX}{BX} \Rightarrow \frac{XE}{40} = \frac{9}{BX}$

$BX \cdot XE = 360$

(b)



* -1 for each mistake
* full marks even if $(3, 1)$ not labelled.

(15)

(a) i) $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$

$f'(x) = -x^2 - 2x + 3$

$= -(x^2 + 2x - 3)$

$= -(x+3)(x-1)$

$\therefore$ S Ps at -3 and 1

x	-4	-3	0	1	2
$f'(x)$	$-$	0	$+$	0	$-$
	\	—	/	—	\

$\therefore$ minima at $(-3, -8)$
maxima at $(1, 2\frac{2}{3})$

ii) $f''(x) = -2x - 2$

$f''(x) = 0$

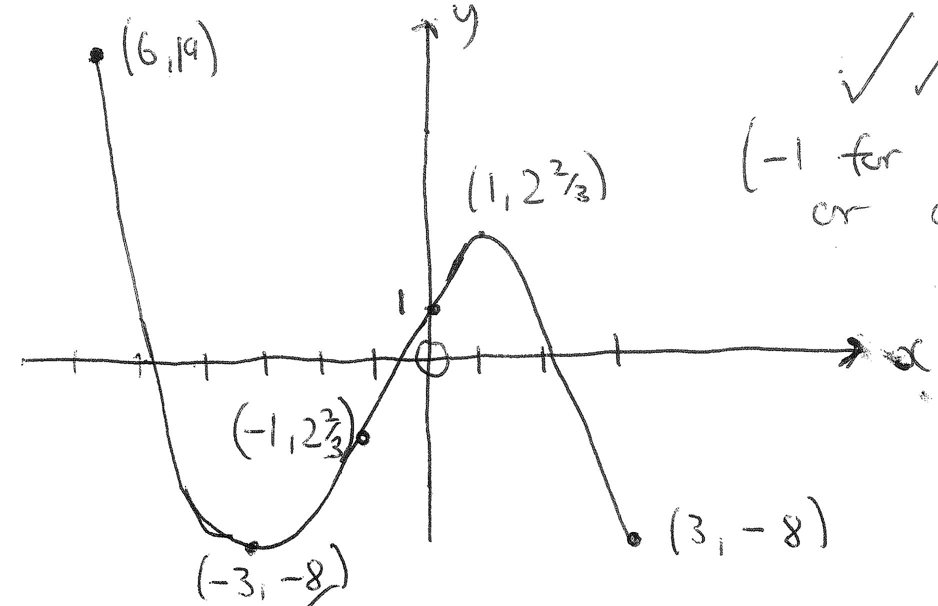
$\therefore 2x = -2$

$x = -1$

x	-2	-1	0
$f''(x)$	$+$	0	$-$
	$\cup$	.	$\cap$

concavity change means P.O.I.
$\therefore$ P.O.I at $(-1, -2\frac{2}{3})$

iii)



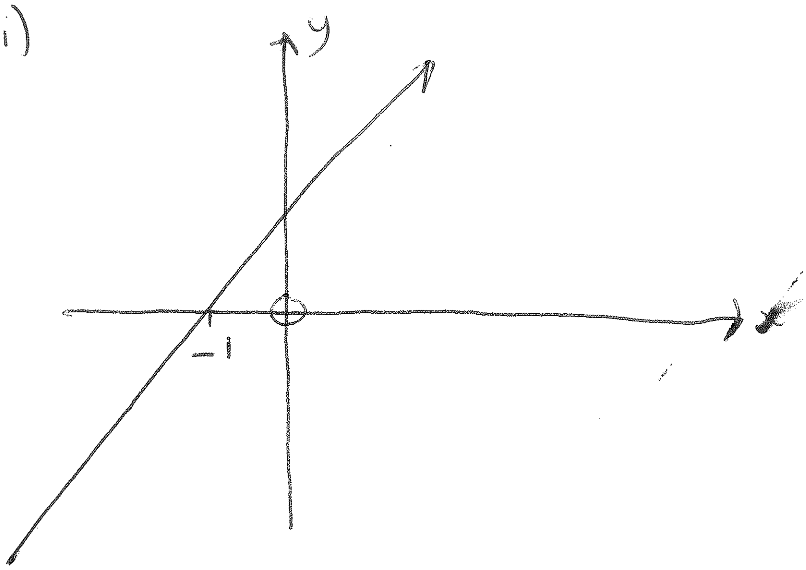
(−1 for each mistake or omission)

iv) $y=19$

(b) i) the gradient is positive ✓

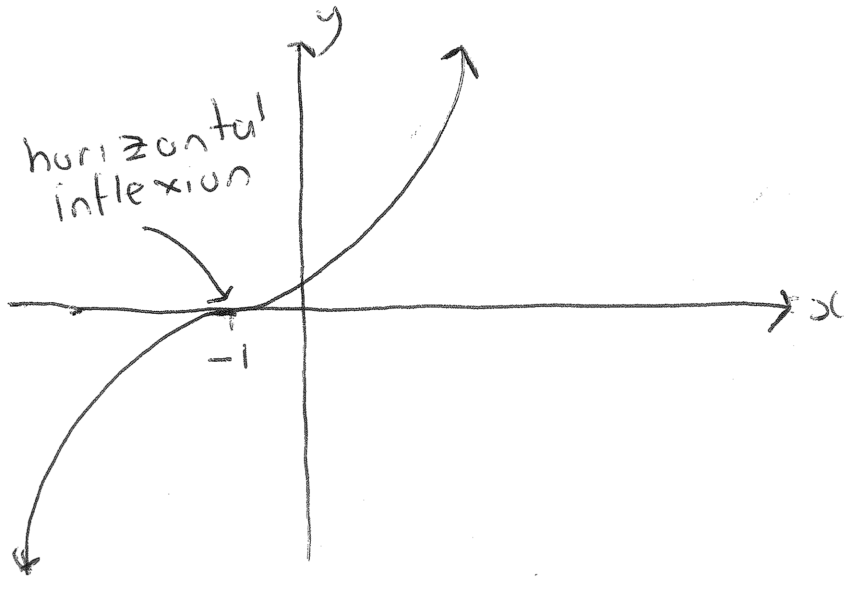
ii) there is a point of horizontal inflexion. ✓ must say 'horizontal' and 'inflexion'

iii)



✓

iv)

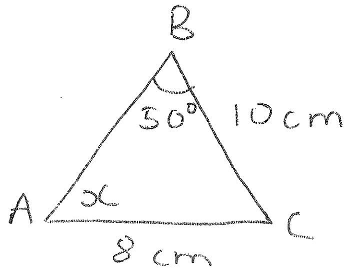


✓ S. P.

✓ inflexion

(16.)

(a)



$$\frac{\sin x}{10} = \frac{\sin 50^\circ}{8}$$

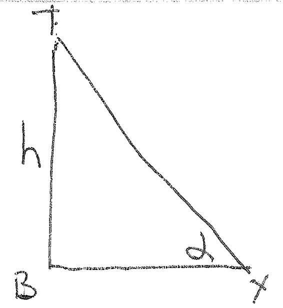
$$\therefore x = \sin^{-1}\left(\frac{10 \sin 50^\circ}{8}\right)$$

$$\therefore x = 73^\circ 15' \text{ or } 106^\circ 45'$$

(b)

$$\begin{aligned}
LHS &= (\cot\theta + \operatorname{cosec}\theta)^2 \\
&= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2 \\
&= \frac{(\cos\theta + 1)^2}{\sin^2\theta} \\
&= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta} \\
&= \frac{(\cos\theta + 1)(\cos\theta + 1)}{(1 + \cos\theta)(1 - \cos\theta)} \\
&= \frac{1 + \cos\theta}{1 - \cos\theta} \\
&= RHS, \text{ as required}
\end{aligned}$$

(c) i)



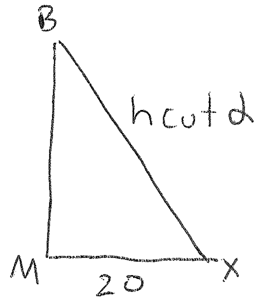
$\cot\alpha = \frac{BX}{h}$

$\therefore BX = h\cot\alpha$ ✓

similarly in ΔBTW, $BW = h\cot\alpha$ ✓

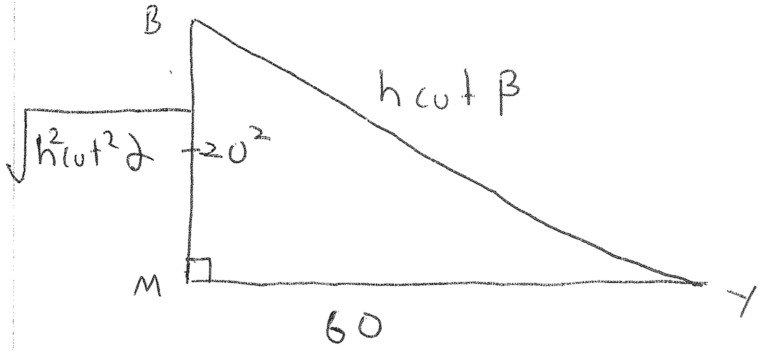
$\therefore \Delta BWX$ is isosceles as $BW = BX$

ii)



$\therefore BM = \sqrt{h^2\cot^2\alpha - 20^2}$ ✓

iii) $BY = h\cot\beta$ ✓



$60^2 + h^2\cot^2\alpha - 20^2 = h^2\cot^2\beta$ ✓

$\therefore h^2\cot^2\alpha + 3200 = h^2\cot^2\beta$

$h^2\cot^2\beta - h^2\cot^2\alpha = 3200$

$$h^2(\cot^2\beta - \cot^2\alpha) = 3200$$

$$\therefore h^2 = \frac{3200}{\cot^2\beta - \cot^2\alpha}$$

$$\therefore h = \sqrt{\frac{3200}{\cot^2\beta - \cot^2\alpha}}$$

$$\therefore h = \frac{40\sqrt{2}}{\sqrt{\cot^2\beta - \cot^2\alpha}}$$ ✓

(17)

(a) i) $S_n = \frac{n}{2}(a + l)$

$\therefore 570 = {}^{n}/_{2}(10 + 66)$

$\therefore 76n = 1140$

$\therefore n = 15$

ii) $T_n = a + (n-1)d$

$66 = 10 + (15-1)d$

$\therefore 14d = 56$

$\therefore d = 4$

(b) i) $r = \dfrac{(x-2)^{n+1}}{(x-2)^n}$

$r = x - 2$

$\therefore -1 < x - 2 < 1$

$\therefore 1 < x < 3$

ii) $S_\infty = \dfrac{a}{1 - r}$

$= \dfrac{x - 2}{1 - (x-2)}$

$= \dfrac{x-2}{3-x}$

(c) i) $T_3 - T_2 = \log_{10} x^4 - \log_{10} x^2$

$= 2\log_{10} x^2 - \log_{10} x^2$

$= \log_{10} x^2$

✓ proof

✓ $d = \log_{10} x^2$

$T_2 - T_1 = \log_{10} x^2 - 0$

$= \log_{10} x^2$

$\therefore T_3 - T_2 = T_2 - T_1$

$\therefore$ this is an AP where $d = \log_{10} x^2$

ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{10}{2}(2 \times 0 + (10-1)\log_{10} x^2)$

$= 5 \times 9 \log_{10} x^2$

$= 45 \log_{10} x^2$

$= 90 \log_{10} x$ or $\log_{10} x^{90}$ ✓

(d) this is a GP where:

$n = \infty$

$a = 9/x$

$r = 1/x$

$S_n = 18$

$S_n = \frac{a}{1-r}$ ✓

$\frac{9/x}{1 - 1/x} = 18$ ✓

$\therefore \frac{9}{x-1} = 18$

$\therefore 9 = 18x - 18$

$\therefore 18x = 36$

$\therefore x = 3/2$ ✓

Suggested Solutions

Question 18 (Lam)

(a) $x = 3 + 18t - t^3$

i. (1 mark)

$$\frac{dx}{dt} = 18 - 3t^2$$

ii. (1 mark)

$$\begin{aligned}\frac{dx}{dt} &= 18 - 3t^2 = 0\\ 3(6-t^2) &= 0\\ t^2 &= 6\\ \therefore t &= \sqrt{6}\end{aligned}$$

as $t > 0$.

iii. (1 mark)

$$x = 3$$

iv. (1 mark)

$$\frac{dx}{dt} = 18 - 3t^2\Big|_{t=1} = 15\,\text{ms}^{-1}$$

v. (1 mark)

$$\begin{aligned}x &= 3 + 18t - t^3\Big|_{t=1}\\ &= 3 + 18 - 1 = 20\,\text{m}\end{aligned}$$

Hence particle travels $20 - 3 = 17\,\text{m}$ in the first second (see part (iii))

vi. (2 marks)

✓ [1] for correct second derivative. Fatal error if this is not done properly.

✓ [1] for proper substitution and correct answer.

$$\frac{d^2x}{dt^2} = -6t\Big|_{t=2} = 12\,\text{ms}^{-2}$$

(b) i. (2 marks)

✓ [1] for progress towards answer.

✓ [2] for properly set out answer.

$$\begin{aligned}A = \pi r^2 + 2\pi rh &= 75\pi\\ \cancel{\pi}(r^2 + 2rh) &= 75\cancel{\pi}\\ r^2 + 2rh &= 75\\ 2rh &= 75 - r^2\\ \therefore h &= \frac{75 - r^2}{2r}\end{aligned}$$

ii. (1 mark)

$$\begin{aligned}V = \pi r^2 h &= \pi r^{\cancel{2}}\left(\frac{75 - r^2}{2\cancel{r}}\right)\\ &= \frac{\pi}{2}r\left(75 - r^2\right) = \frac{\pi}{2}\left(75r - r^3\right)\end{aligned}$$

iii. (4 marks)

✓ [1] for proper differentiation.

✓ [1] for $r = 5$ through showing working.

✓ [1] for testing local maximum.

✓ [1] for fully showing $V = 125\pi$.

$$\frac{dV}{dr} = \frac{\pi}{2}\left(75 - 3r^2\right)$$

Stationary points when $\dfrac{dV}{dr} = 0$:

$$\begin{aligned}\frac{\pi}{2}\left(75 - 3r^2\right) &= 0\\ \therefore 75 - 3r^2 &= 0\\ 3r^2 &= 75\\ r^2 &= 25\\ \therefore r &= 5\end{aligned}$$

as $r > 0$. Finding the second derivative,

$$\frac{d^2V}{dr^2} = \frac{\pi}{2} \times -6r = -3\pi r$$

When $r = 5$,

$$\frac{d^2V}{dr^2} < 0$$

$\therefore r = 5$ is a local maximum. Hence the maximum volume is

$$V = \frac{\pi}{2} \times 5\,(75 - 25) = 125\pi$$

iv. (1 mark)

$$h = \frac{75 - 5^2}{2 \times 5} = 5$$