NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2016 Preliminary Course Yearly Examination

31st of August, 2016

General instructions

- Working time – 2 hours 30 minutes (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.

SECTION I

- **Questions 1 - 8**
- Mark your answers in the section provided in your answer booklet

SECTION II

- **Questions 9 - 16**
- Commence each new question on a **new page**. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED:** .....

Class (please ✔)

○ Mr Berry ○ Miss Lee

○ Mr Hwang ○ Mr Lin

○ Mr Ireland ○ Ms Ziaziaris

○ Dr Jomaa

Marker's use only.

QUESTION	1–8	9	10	11	12	13	14	15	16	Total
MARKS	$\overline{8}$	$\overline{8}$	$\overline{11}$	$\overline{9}$	$\overline{18}$	$\overline{14}$	$\overline{17}$	$\overline{15}$	$\overline{15}$	$\overline{115}$

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Section I: Objective response

Mark your answers on the multiple choice section in the answer booklet. **Marks**

1. What is the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures? **1**

(A) 0.40

(B) 0.041

(C) 0.0409

(D) 0.0410

2. What is the centre and radius of the circle with equation $x^2 + y^2 + 6x - 8y - 11 = 0$? **1**

(A) Centre $(-3, -4)$ and radius 36

(B) Centre $(-3, 4)$ and radius 36

(C) Centre $(-3, -4)$ and radius 6

(D) Centre $(-3, 4)$ and radius 6

3. What are the coordinates of the focus of the parabola $x^2 = 2(y - 1)$? **1**

(A) $(0, \frac{1}{2})$

(B) $(0, \frac{3}{2})$

(C) $(\frac{1}{2}, 0)$

(D) $(\frac{3}{2}, 0)$

4. What is the value of $\dfrac{a}{b}$ if the lines $ax + 2y = 6$ and $4y = bx - 9$ are parallel? **1**

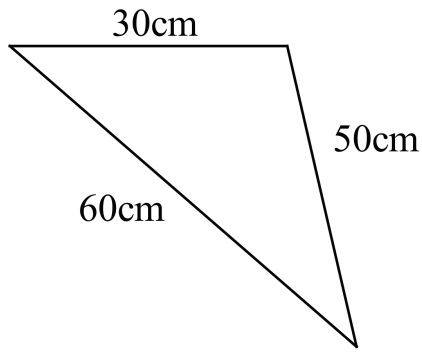
(A) $\dfrac{1}{2}$

(B) $-\dfrac{1}{2}$

(C) -2

(D) 2

5. The following triangle has sides 30cm, 50cm and 60 cm. **1**



Angle C is the largest angle. Which of the following expressions is correct for angle C?

(A) $\cos C = \dfrac{30^2 + 60^2 - 50^2}{2 \times 30 \times 60}$

(B) $\cos C = \dfrac{50^2 + 30^2 - 60^2}{2 \times 50 \times 30}$

(C) $\cos C = \dfrac{50^2 + 60^2 - 30^2}{2 \times 50 \times 60}$

(D) $\cos C = \dfrac{50^2 + 30^2 - 60^2}{2 \times 50 \times 60}$

6. What is the solution to the equation $\cos(\dfrac{\theta}{2} + 20^\circ) = \sin\theta$ for $0^\circ \le \theta \le 90^\circ$? **1**

(A) $\theta = 20^\circ$

(B) $\theta = 40^\circ$

(C) $\theta = \dfrac{160^\circ}{3}$

(D) $\theta = \dfrac{140^\circ}{3}$

7. What is the minimum value of $\cos\theta + 2\cos(\theta + 240^\circ)$? **1**

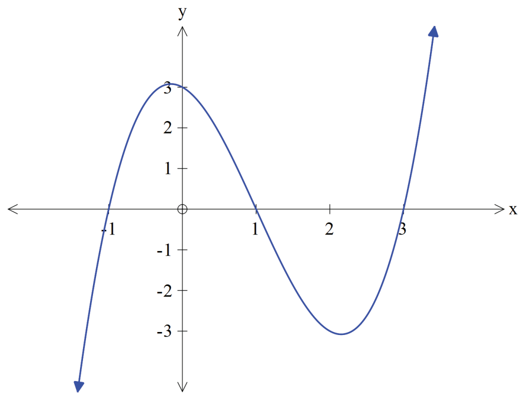
(A) $-2\sqrt{3}$

(B) $-\sqrt{3}$

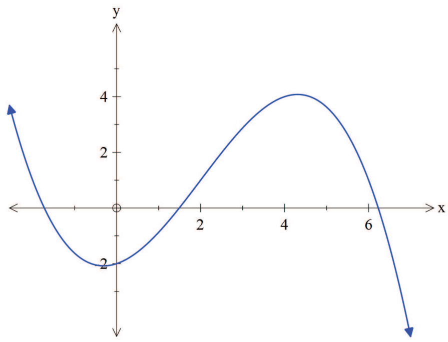
(C) $-\dfrac{\sqrt{3}}{2}$

(D) $-\dfrac{\sqrt{3}}{4}$

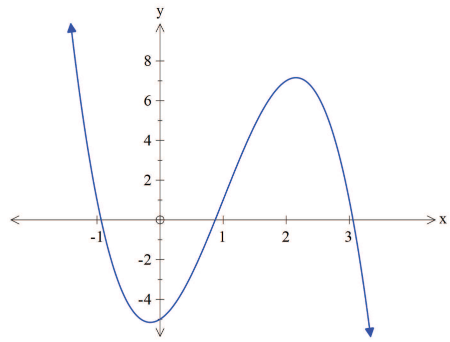
8. The graph of a function f, with domain R, is as shown. **1**



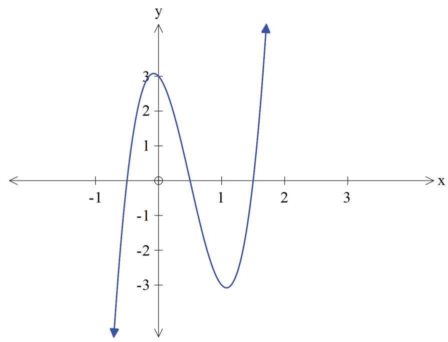
The graph which best represents $1 - f(2x)$ is



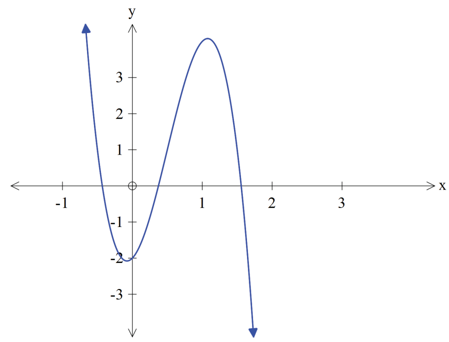
(A)



(C)



(B)



(D)

End of Section I.
Examination continues overleaf.

Section II

Write your answers in the writing booklet supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (8 Marks) Commence a NEW page. **Marks**

(a) Show that $\dfrac{4}{2+\sqrt{5}} - \dfrac{1}{9-4\sqrt{5}}$ is rational **3**

(b) Solve $|2x-1| = 3x$ **3**

(c) Solve $|5-3x| \geq 4$ **2**

Question 10 (11 Marks) Commence a NEW page. **Marks**

(a) If $\sin\theta = \dfrac{3}{4}$ and $90^\circ \leq \theta \leq 180^\circ$ find the exact value of $\sec\theta$ **2**

(b) i. Show $\dfrac{\cos x + \sin x}{\cos x - \sin x} + \dfrac{\cos x - \sin x}{\cos x + \sin x} = \dfrac{2}{2\cos^2 x - 1}$ **3**

ii. Use the above result to solve **3**

$$\frac{\cos x + \sin x}{\cos x - \sin x} + \frac{\cos x - \sin x}{\cos x + \sin x} = 4 \text{ for } 0^\circ \leq \theta \leq 360^\circ$$

(c) Sketch the region defined by $(x-2)^2 + y^2 < 4$ *and* $y \geq (x-2)^2$ **3**
(i.e. Find the intersection of these two graphs)

Question 11 (9 Marks) Commence a NEW page. **Marks**

(a) Sketch the following showing essential features

i. $y = \dfrac{3}{x+1}$ **2**

ii. $y = |x-2| + 4$ **2**

(b) i. Find the domain of $y = \sqrt{1-x^2}$ **1**

ii. Find the domain of $y = \sqrt{x^2-1}$ **1**

iii. Find the range of $y = \sqrt{x^2-1}$ **1**

(c) State whether $y = \dfrac{x^3}{1+x}$ is an odd function, an even function or neither, justifying your answer with working out. **2**

Question 12 (18 Marks) Commence a NEW page. **Marks**

(a) Differentiate

i. $y = (2 - 3x)^5$ **1**

ii. $y = \dfrac{3x}{\sqrt{x}}$ **2**

iii. $y = \dfrac{x^3 - 2x}{x^2}$ **2**

iv. $y = x(x - 3)^7$ **2**

(b) Find the equation of the normal to the curve $y = 2x^2 - 4$ at the point where $x = 1$. **3**

(c) Given the function $y = x^3 - 3x^2 - 12$

i. Find the stationary points and determine their nature. **4**

ii. Find any points of inflexion. **2**

iii. Hence, sketch the curve. **2**

Question 13 (14 Marks) Commence a NEW page. **Marks**

(a) Solve the inequality $\dfrac{x}{2x - 1} \leq 5$ **3**

(b) Find the acute angle between the lines $2x - 3y + 1 = 0$ and $y = 2x - 1$ to the nearest degree **2**

(c) The point $P(5, 7)$ divides the interval AB joining the points $A(-1, 1)$ and $B(3, 5)$ in the ratio $m : 1$. Find the value of m **2**

(d) Without using calculus, sketch on a number plane the graph of $y = \dfrac{x - 1}{x^2 - 9}$. Label any asymptotes and the x and y intercepts. **3**

(e) Find the domain of the function $f(x) = \sqrt{4 - x^2} + \dfrac{1}{x + 2}$ **2**

(f) If $\sin 2A = \dfrac{1}{2}$, find the exact value of $\dfrac{1}{\sin A \cos A}$ **2**

Question 14 (17 Marks) Commence a NEW page. **Marks**

(a) Find the equation of the straight line passing through $A(3, -2)$ inclined at an angle of $120°$ with the positive direction of the x-axis. (Leave values in exact form) **2**

(b) Solve for x the equation $4^x - 3(2^x) + 2 = 0$ **3**

(c) Find the values of k for which $kx^2 + (k+3)x + 4$ is positive definite **3**

(d) One of the roots of the equation $2x^2 - 15x + c = 0$ is four times the other. Find the roots of the equation and hence the value of c **3**

(e) i. Prove that $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ **2**

ii. If the roots of the quadratic $x^2 + 4x + 2 = 0$ are α and β, using the result in part (i) or otherwise, find a value for **2**

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3}$$

(f) Find the Cartesian equation of a curve whose parametric equations are $x = 2\cos\theta$ and $y = \sqrt{3}\sin\theta$. (You do not need to simplify). **2**

Question 15 (15 Marks) Commence a NEW page. **Marks**

(a) The point $A(2at, at^2)$ lies on the parabola with equation $x^2 = 4ay$

i. Find the equation of the tangent to the parabola at A **2**

ii. Find the coordinates of B and C, the x and y-intercepts of this tangent. **2**

iii. Show that B is the midpoint of AC **2**

(b) Two circles meet in A and B. PQ is a common tangent and $PR||QA$. RA is produced to meet the other circle in S.

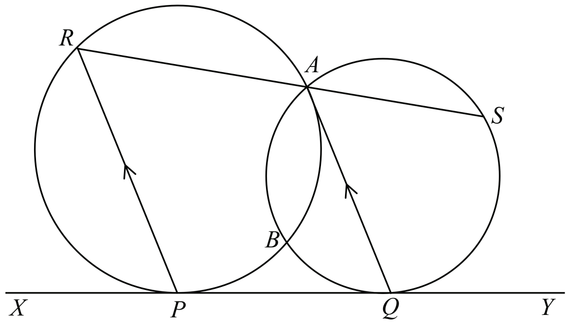


DIAGRAM NOT TO SCALE

Copy the diagram into your writing booklet.

i. Prove that $PRSQ$ is cyclic **2**

ii. Prove that $PA||QS$ **2**

(c) $ABCDE$ is a pentagon inscribed in a circle, where $AB = AE$, BE meets AC and AD at P and Q respectively.

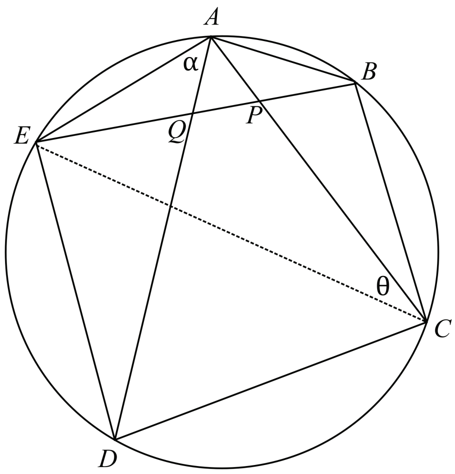


DIAGRAM NOT TO SCALE

Given $\angle ACE = \theta$ and $\angle DAE = \alpha$,

i. Copy the diagram onto your answer booklet and show that $\angle AEB = \theta$, giving reasons **2**

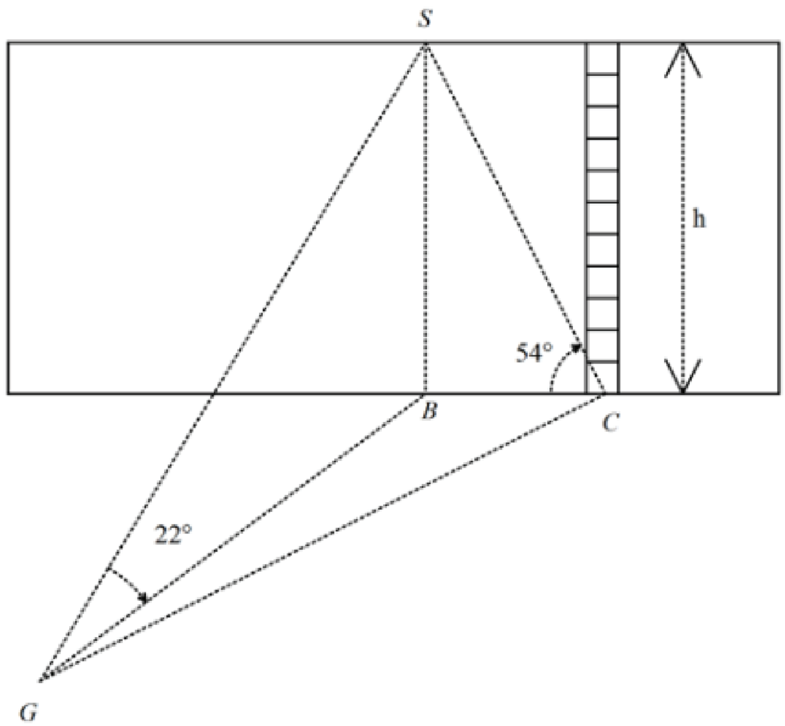
ii. Hence, or otherwise, show that $CPQD$ is a cyclic quadrilateral **3**

Question 16 (15 Marks) Commence a NEW page. **Marks**

(a) i. Expand $\tan(A+B)$ **1**

ii. The roots of $x^2 - 2x - 1 = 0$ are $\tan A$ and $\tan B$. If $A+B$ is acute, find the size of $A+B$ **3**

(b) A child climbs down a ladder and into a gorilla enclosure at Cincinnati zoo.
A gorilla (point G on the diagram) notices the child (point C on the diagram) standing at the bottom of the ladder.
A zookeeper (point S on the diagram) is standing on top of the enclosure wall.
The zookeeper aims a tranquiliser at the gorilla with an angle of depression of $22°$ and will shoot the gorilla if he approaches the child.
The angle of elevation from the child to the zookeeper is $54°$.
The distance of the gorilla from the child is 3 times the height of the wall.
The wall runs East-West.

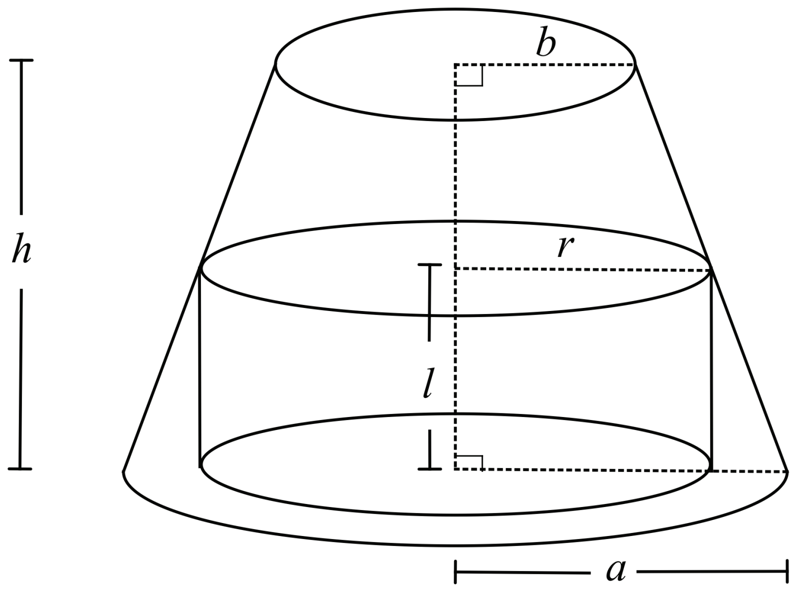


i. Show that $GB = h\cot 22°$ **2**

ii. Similarly, write an expression for the length of BC. **1**

iii. Hence, or otherwise, find the bearing of the child from the gorilla **3**

(c) A solid block of wood is in the shape of a frustrum of a right circular cone, the radii of the end faces being a and b $(a > b)$.The frustrum has fixed height h. A right circular cylinder of height l is to be cut from the frustrum, with its axis along the axis of the cone. **5**



Show that the volume of the cylinder is a maximum when the height of the cylinder is

$$\frac{ah}{3(a-b)}$$

End of paper.

2016 Ext 1 Prelim Yearly - Suggested Solutions

MCQ

1. D
2. D
3. B
4. B
5. B
6. D
7. B
8. D

Question 9

3 a) $\dfrac{4(2-\sqrt{5})}{4-5} - \dfrac{(9+4\sqrt{5})}{81-16\times 5}$

$= \dfrac{8-4\sqrt{5}}{-1} - \dfrac{(9+4\sqrt{5})}{1}$

$= 4\sqrt{5} - 8 - 9 - 4\sqrt{5}$

$= -17.$

3 b) $|2x-1| = 3x$

$2x-1 = 3x$ OR $2x-1 = -3x$

$-1 = x$ OR $5x = 1$

$x = \frac{1}{5}$

Check $x = -1$

$|2(-1)-1| = -3$

not poss.

Check $x = \frac{1}{5}$

$\left|2\times\frac{1}{5} - 1\right| = 3\left(\frac{1}{5}\right)$

Only soln $x = 1/5$.

2 c) $|5-3x| \geq 4$

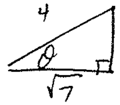
$5-3x \geq 4$ OR $5-3x \leq -4$

$-3x \geq -1$ $\quad$ $-3x \leq -9$

$x \leq \frac{1}{3}$ OR $x \geq 3$.

Question 10

2 a) $\sin\theta = \frac{3}{4}$



$\sec\theta = -\frac{4}{\sqrt{7}}$

3 b) (i) LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x} + \dfrac{\cos x - \sin x}{\cos x + \sin x}$

$= \dfrac{(\cos x + \sin x)^2 + (\cos x - \sin x)^2}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + 2\sin x\cos x + \sin^2 x + \cos^2 x - 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \dfrac{2}{\cos^2 x - \sin^2 x}$

$= \dfrac{2}{\cos^2 x - (1 - \cos^2 x)}$

$= \dfrac{2}{2\cos^2 x - 1}$

= RHS.

3 (ii) $\therefore \dfrac{2}{2\cos^2 x - 1} = 4$

$2 = 8\cos^2 x - 4$

$8\cos^2 x = 6$

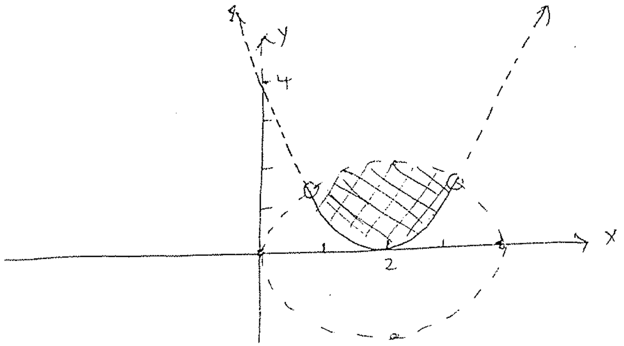
$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\therefore$ x lies in all quads

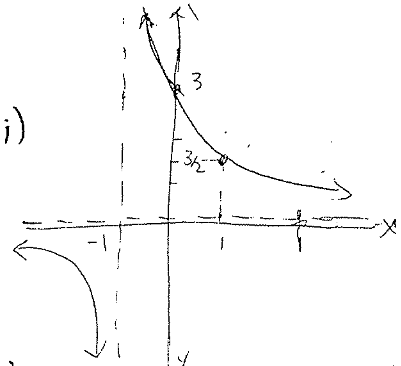
$x = 30°, 150°, 210°, 330°$.

3 c)

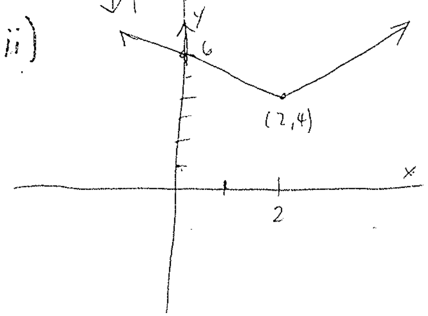


Question 11

2 a) i)



2 ii)



Question 11 continued

1 b) i) $y = \sqrt{1-x^2}$

$D: -1 \leq x \leq 1$

1 ii) $y = \sqrt{x^2-1}$

$x^2 - 1 \geq 0$.

$(x-1)(x+1) \geq 0$

$x \leq -1 \, , \, x \geq 1$

1 iii) $y \geq 0$.

2 c) $y = \dfrac{x^3}{1+x}$

$f(a) = \dfrac{a^3}{1+a}$

$f(-a) = \dfrac{(-a)^3}{1+(-a)}$

$= \dfrac{-a^3}{1-a}$

$-f(a) = \dfrac{-a^3}{1+a}$

Neither.

Question 12

1 a) (i) $5(2-3x)^4 \cdot -3$

$= -15(2-3x)^4$

2 (ii) $\dfrac{3x}{\sqrt{x}}$

$= 3x^{\frac{1}{2}}$

$y' = \frac{1}{2} \times 3x^{-1/2}$

$= \frac{3}{2} x^{-1/2}$

$= \dfrac{3}{2\sqrt{x}}$

2 (iii) $\dfrac{x^3}{x^2} - \dfrac{2x}{x^2}$

$x - \dfrac{2}{x}$

$x - 2x^{-1}$

$= 1 + 2x^{-2}$

$= 1 + \dfrac{2}{x^2}$

2 (iv) $(x-3)^7 \cdot 1 + x \cdot 7(x-3)^6$

$= (x-3)^7 + 7x(x-3)^6$

$(x-3)^6(8x-3)$

3 b) $y = 2x^2 - 4$

$\dfrac{dy}{dx} = 4x$

At $x = 1, y = -2$.

$\dfrac{dy}{dx} = 4$

$\therefore$ m of normal $= -\frac{1}{4}$

Eqn of normal

$y + 2 = -\frac{1}{4}(x-1)$

$4y + 8 = -x + 1$

4 c) i) $y = x^3 - 3x^2 - 12$

$\frac{dy}{dx} = 3x^2 - 6x$

$\frac{d^2y}{dx^2} = 6x - 6$

Stat Pts: $\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, \quad x = 2$

$y = -12 \quad y = -16$

Test $(0,-12)$

$\frac{d^2y}{dx^2} = -6 < 0 \therefore \boxed{\max (0,-12)}$

Test $(2,-16)$

$\frac{d^2y}{dx^2} = 6 > 0 \therefore \boxed{\min (2,-16)}$

2 ii) Inflex. Pts: $\frac{d^2y}{dx^2} = 0$ and change in sign.

$6x - 6 = 0$

$x = 1$

$y = -14$

Test $(1,-14)$

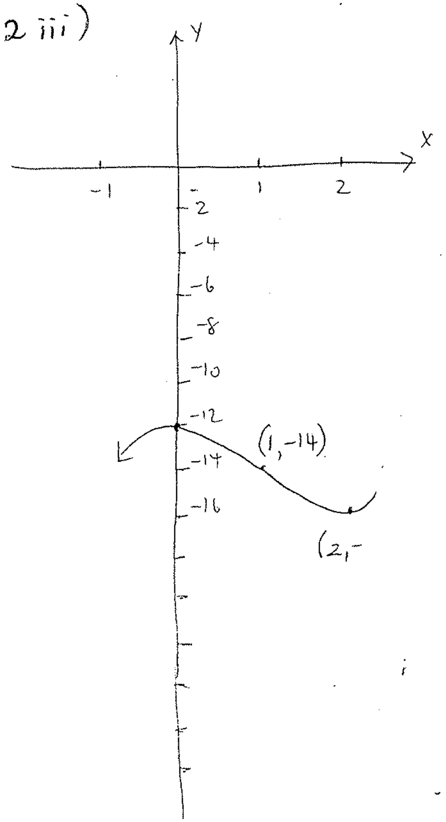
x	0	1	2
$\frac{d^2y}{dx^2}$	-6	0	6

$\therefore$ Inflex Pt at $(1,-14)$

Yint: $x = 0$

$y = -12$.

2 iii)



Question 13

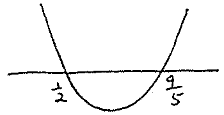
3 a) $\frac{x}{2x-1} \leq 5 \qquad x \neq \frac{1}{2}$

$x(2x-1) \leq 5(2x-1)^2$

$0 \leq (2x-1)[5(2x-1) - x]$

$0 \leq (2x-1)(10x - 5 - x)$

$0 \leq (2x-1)(9x-5)$



$\therefore \quad x < \frac{1}{2}$ or $x \geq \frac{5}{9}$

2 b) $m_2 = \frac{2}{3}, \quad m_1 = 2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2 - \frac{2}{3}}{1 + 2(\frac{2}{3})}\right|$

$= \frac{4}{7}$

$\therefore \theta = 30°$

c) $A(-1,1) \qquad B(3,5)$

$m : 1$

$\frac{-1+3m}{m+1} = 5$ ①

$\frac{1+5m}{m+1} = 7$ ②

$-1 + 3m = 5m + 5$

$-6 = 2m$

$\therefore m = -3$

$1 + 5m = 7m + 7$

$2m$

3 d) $y = \dfrac{x-1}{x^2-9}$

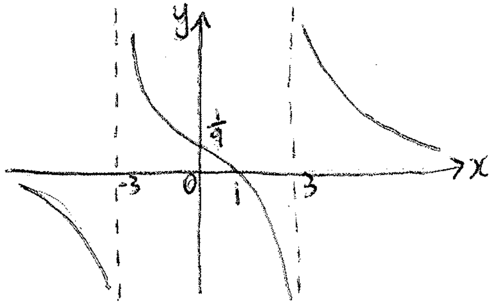
$= \dfrac{x-1}{(x+3)(x-3)}$

Vertical asymptotes : $x = \pm 3$

Horizontal asymptotes : $y = 0$

x-intercept : $(1, 0)$

y-intercept : $(0, \frac{1}{9})$



3 e) Domain of $\sqrt{4-x^2}$ is $-2 \leq x \leq 2$

Domain of $\dfrac{1}{x+2}$ is $x \neq -2$

$\therefore$ domain of $f(x)$ is $-2 < x \leq 2$

2 f) $\dfrac{1}{\sin A \cos A} = \dfrac{2}{\sin 2A}$

$= 2 \div \frac{1}{2}$

$= 4$

Question 14 continued.

e)

2 i) $RHS = (a+b)^3 - 3ab(a+b)$

$= (a+b)\left[(a+b)^2 - 3ab\right]$

$= (a+b)\left[a^2+2ab+b^2-3ab\right]$

$= (a+b)(a^2 - ab + b^2)$

$= a^3 + b^3$

$= LHS$

2 ii) $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\beta^3+\alpha^3}{\alpha^3\beta^3}$

$= \dfrac{(\alpha+b)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$

$= \dfrac{(-4)^3 - 3(2)(-4)}{2^3}$

$= -5$

2 f) $x = 2\cos\theta \rightarrow \cos\theta = \frac{x}{2}$

$y = \sqrt{3}\sin\theta \rightarrow \sin\theta = \frac{y}{\sqrt{3}}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{3}}\right)^2 = 1$

Question 14

2 a) $m = \tan\theta$
$= \tan 120$
$= -\sqrt{3}$

$y + 2 = -\sqrt{3}(x - 3)$

3 b) $4^x - 3(2^x) + 2 = 0$

Let $2^x = u$

$u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u = 1 \qquad u = 2$

$2^x = 1 \qquad 2^x = 2$

$\therefore x = 0, 1$

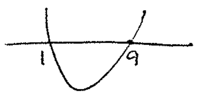
2 c) Positive definite: coeff. of $x^2 > 0$ and $\Delta < 0$

i.e. $k < 0$ and $(k+3)^2 - 4(k)(4) < 0$

$k^2 + 6k + 9 - 16k < 0$

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0$



$\therefore \quad 1 < k < 9$

3 d) Let the roots be $\alpha, 4\alpha$.

$\alpha + 4\alpha = \frac{15}{2}$

$10\alpha = 15$

$\alpha = \frac{3}{2}$

$4\alpha = 6$

$4\alpha^2 = \frac{c}{2}$

$9 = \frac{c}{2}$

$c = 18$

Question 15

a)

2 i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$

When $x = 2at$, $m = \frac{2at}{2a}$

$= t$

Equation of tangent:

$y - y_1 = m(x - x_1)$

$y - at^2 = t(x - 2at)$

$= tx - 2at^2$

$\therefore \boxed{y = tx - at^2}$

2 ii) $B(at, 0)$

$C(0, -at^2)$

Midpoint AC:

$x = \frac{2at + 0}{2}$

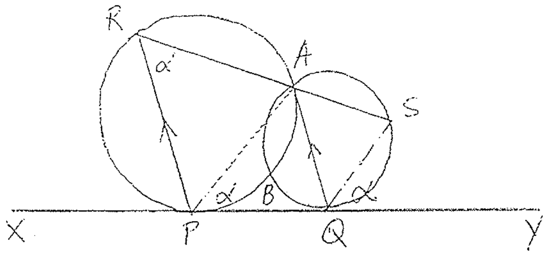
$= at$

$y = \frac{at^2 - at^2}{2}$

$= 0$

$\therefore B$ is the midpoint of AC

Question 15(b)



(i) Join SQ

Let $\angle SQY = \alpha$

then $\angle QAS = \alpha$ (angle in alternate segment)

$\angle ARP = \alpha$ (corresponding angles in parallel lines $AQ \parallel RP$)

$\therefore \angle SQY = \angle ARP$

$\therefore$ PQSR is a cyclic quad (exterior angle of cyclic quad equals opposite interior angle.

(ii) Join PA

$\angle APQ = \angle ARP$ (angle in alternate segment)

$\therefore \angle APQ = \angle SQY$ (both α from(i)).

$\therefore AP \parallel SQ$ (corresponding angles are equal)

uestion 15 continued.

c)

2 i) $A\hat{C}E = A\hat{B}E = \theta$ (Angles in same segment) ✓

$A\hat{B}E = A\hat{E}B$ (Equal $\angle$'s opp. equal sides, $AE = AB$ given)

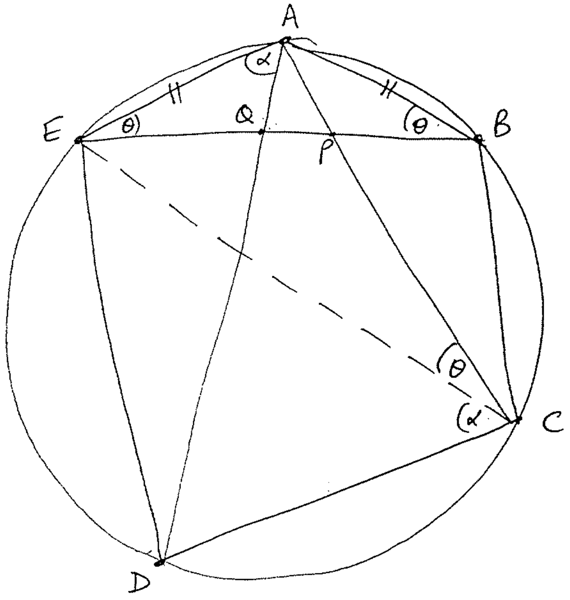
$\therefore A\hat{E}B = \theta$

3 ii) $D\hat{A}E = D\hat{C}E = \alpha$ (Angles in same segment)

$A\hat{Q}B = D\hat{A}E + A\hat{E}B$ (Exterior $\angle$ of Δ AQE equal to sum of two opp. interior $\angle$'s).

$= \alpha + \theta$

As $P\hat{C}D = A\hat{Q}B = \alpha + \theta$, PCDQ is a cyclic quad. as the exterior angle is equal to the interior opposite angle.



Question 16

1 a) i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

3 ii) $\tan A + \tan B = 2$

$\tan A \tan B = -1$

$\tan(A+B) = \dfrac{2}{1-(-1)}$

$= 1$

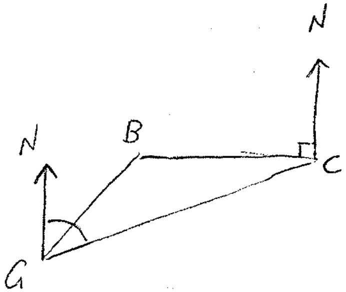
$\therefore A+B = 45^\circ$

b) i) $\tan 22 = \dfrac{h}{GB}$

$GB = \dfrac{h}{\tan 22}$

$= h \cot 22$

ii) $BC = h \cot 54$



iii) $\cos B\hat{C}G = \dfrac{BC^2 + GC^2 - GB^2}{2(BC)(GC)}$

$= \dfrac{(h\cot 54)^2 + (3h)^2 - (h\cot 22)^2}{2(h\cot 54)(3h)}$

$= \dfrac{\cot^2 54 + 9 - \cot^2 22}{6\cot 54}$

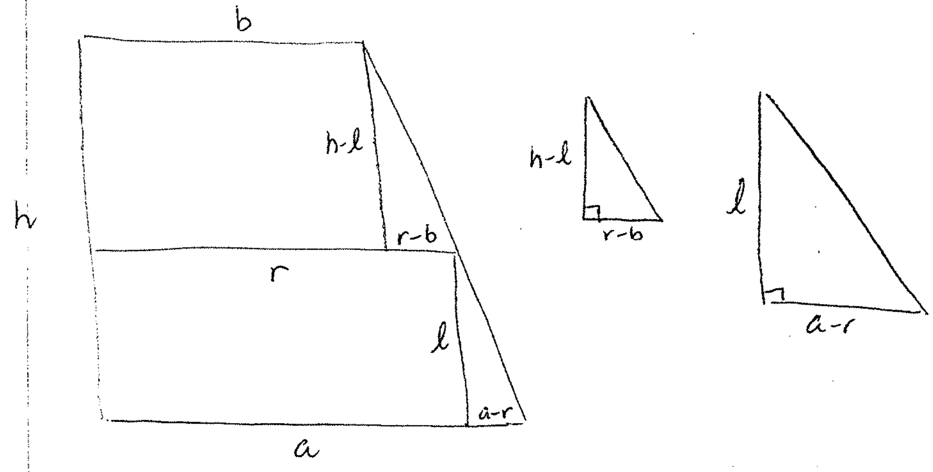
$B\hat{C}C = \cos^{-1}\left(\dfrac{\cot^2 54 + 9 - \cot^2 22}{6\cot 54}\right)$

$= 38^\circ 42'$ (to newest min.)

$\therefore$ bearing is $180^\circ - 90^\circ - 38^\circ 42' = 51^\circ 18'$ (to newest min).

Method 1

5 c) $a, h, r, (a-b) > 0$



By similar triangles,

$\dfrac{h-l}{l} = \dfrac{r-b}{a-r}$

$(h-l)(a-r) = l(r-b)$

$ah - hr - al + \cancel{rl} = \cancel{rl} - bl$

$ah - hr = al - bl$

$\therefore l = \dfrac{h(a-r)}{(a-b)}$

The volume of the cylinder:

$V = \pi r^2 l$

$= \pi r^2 \dfrac{h(a-r)}{a-b}$

$= \dfrac{\pi h}{a-b}\left[r^2(a-r)\right]$

$= \dfrac{\pi h}{a-b}\left[ar^2 - r^3\right]$

$\dfrac{dV}{dr} = \dfrac{\pi h}{a-b}\left[2ar - 3r^2\right]$

$\dfrac{d^2V}{dr^2} = \dfrac{\pi h}{}\left[2a - 6r\right]$

$$\frac{dV}{dr} = 0$$

$$2ar - 3r^2 = 0$$

$$r(2a - 3r) = 0$$

$$r \neq 0 \quad , \quad r = \frac{2a}{3}$$

When $r = \frac{2a}{3}$, $\quad V'' = \frac{\pi h}{a-b}\left[2a - 6\left(\frac{2a}{3}\right)\right]$

$$= \frac{\pi h}{a-b}\left[-4a\right]$$

$$< 0 \quad \text{since } a, h, r, (a-b) > 0$$

$\therefore$ volume is maximum when $r = \frac{2a}{3}$.

and $\quad l = \frac{h}{a-b}\left(a - \frac{2a}{3}\right)$

$$= \frac{h}{a-b}\left(\frac{a}{3}\right)$$

$$= \frac{ah}{3(a-b)}$$

Method 2

$$r = \frac{ah - al + bl}{h}$$

$$V = \pi l \left(\frac{ah - al + bl}{h}\right)^2$$

$$\frac{dv}{dl} = \pi\left(\frac{ah - al + bl}{h}\right)^2 + 2\pi l \frac{(b-a)}{h}\left(\frac{ah - al + bl}{h}\right)$$

$$= \pi\left(\frac{ah - al + bl}{h}\right)\left[\frac{ah - al + bl + 2l(b-a)}{h}\right]$$

$$= \pi\left[\frac{ah + (b-a)l}{h}\right]\left[\frac{ah + 3l(b-a)}{h}\right]$$

$$\frac{dv}{dl} = 0 \therefore l = \frac{ah}{a-b} \text{ or } l = \frac{ah}{3(a-b)}$$

$$\frac{d^2v}{dl^2} = \frac{\pi(b-a)}{h^2}\left[4ah + 6l(b-a)\right]$$

at $l = \frac{ah}{3(a-b)}$, $\frac{d^2v}{dl^2} = \frac{\pi(b-a)}{h^2}\left[4ah - 2ah\right] < 0$ since $b - a < 0$

$l = \frac{ah}{a-b}$, $\frac{d^2v}{dl^2} = \frac{\pi(b-a)}{h^2}\left[4ah - 6ah\right] > 0$

$$\therefore l = \frac{ah}{3(a-b)}.$$

Method 3

Let r denote the radius of the right circular cylinder and l its height.
Then the volume is given by

$$V = \pi r^2 l$$

l lies on the line connecting the origin to the top of the frustrum. The equation of this line is $y = \frac{h}{a-b} x$ since it has slope $\frac{h}{a-b}$

(since the height of the frustrum is h and the horizontal distance between the lower edge & upper edge is $a-b$). Therefore

$$l = \frac{h(a-r)}{a-b}$$

Hence the volume of the cylinder is given by

$$V = \pi r^2 \left(\frac{h(a-r)}{a-b}\right)$$

$$= \frac{\pi r^2 (ha - hr)}{a-b}$$

$$= \frac{\pi r^2 ha - \pi h r^3}{a-b}$$

$$\frac{dV}{dr} = \frac{2\pi r h a - 3\pi h r^2}{a-b}$$

$$\frac{dV}{dr} = 0 : \quad 2\pi r h a - 3\pi h r^2 = 0$$

$$\pi r h (2a - 3r) = 0$$

$$r = 0 \text{ or } r = \frac{2a}{3}$$

$$\frac{d^2V}{dr^2} = \frac{2\pi h a - 6\pi h r}{a-b} < 0 \text{ when } r = \frac{2a}{3}$$