NORTH SYDNEY BOYS HIGH SCHOOL

2022 YEAR 11 ASSESSMENT TASK 3

Mathematics Extension 1

General Instructions

- Working time – **70 minutes** + 5 mins reading time
- Write in this booklet
- Write using blue or black **pen**
- Board approved calculators may be used
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

Class Teacher: Please shade the circle.

- O Mr Berry
- O Ms Fu
- O Mr Ireland
- O Ms Lee
- O Mr Lin
- O Ms Moss
- O Mr Umakanthan

STUDENT NAME: ....

(To be used by the exam markers only.)

Section	A	B	C	D	E	F	Tot	Tot
Mark	/12	/12	/9	/10	/12	/8	/63	/100

Section A (12 marks)

1. Let α, β, and γ be the roots of $x^3 - 4x + 1 = 0$. **1**

Circle the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$:

(A) -4 **(B)** -1 **(C)** 1 **(D)** 4

2. The polynomial $P(x) = x^3 - x^2 - 5x - 3$ has a double root at $x = \alpha$.

What is the value of α ? **1**

(A) $\alpha = -\frac{5}{3}$ **(B)** $\alpha = -1$ **(C)** $\alpha = 1$ **(D)** $\alpha = \frac{5}{3}$

3. The polynomial $P(x)$ is given by $P(x) = 2x^3 + 7x^2 - 46x + 21$.

(i) Show that $P(x)$ is divisible by $(x - 3)$. **1**

(ii) Hence, fully factorise $P(x)$. **3**

4. The polynomial $P(x) = x^3 + 2x^2 + ax + b$ has a factor of $x + 2$ and when divided by $x - 2$ there is a remainder of 12. Find the values of a and b. **3**

5. The polynomial equation $4x^3 - 12x^2 + 5x + 6 = 0$ has roots α, β, and γ. It is known that one of the roots is the sum of the other two. Find α, β, and γ. **3**

Section B (12 marks)

1. What is the value of $cos^{-1}\left(-\frac{1}{2}\right)$? **1**

(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

2. Find the **exact** values of the following:

(i) $tan^{-1}\left(-\sqrt{3}\right)$ **1**

(ii) $sin^{-1}\left(\sin\frac{7\pi}{5}\right)$ **1**

(iii) $sin\left(cos^{-1}\frac{-2}{3}\right)$ **2**

(iv) $tan\left(2\,sin^{-1}\frac{3}{4}\right)$ **2**

3.

(i) State the domain and range of the function $f(x) = 3\, sin^{-1}\left(\frac{x}{2}\right)$ **2**

(ii) Sketch the graph $y = f(x)$ **2**

4.

Which diagram represents the domain of the function $f(x) = cos^{-1}\left(\frac{3}{x}\right)$? **1**

(A)

(B)

(C)

(D)

Section C (9 marks)

1 Find the coefficient of x^2 in the expansion of $(x^2 + \frac{2}{x})^7$ **2**

2. Consider the expansion of $(1 + 2x)^n$

(i) Write down an expression for the coefficient of the term in x^4 **2**

(ii) The ratio of the coefficient of the term in x^4 to that of the term in x^6 is $5 : 8$

Find the value of n. 3

3. Consider the binomial expansion $(1 + x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + ... + \binom{n}{n} x^n$

Show that $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + ... + n\binom{n}{n} = n\, 2^{n-1}$ **2**

Section D (10 marks)

1. What is the value of $\sin 2\theta$ given that $\sin\theta = \frac{\sqrt{3}}{4}$ and θ is obtuse? **1**

(A) $\frac{-\sqrt{39}}{8}$ (B) $\frac{-\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{39}}{8}$

2. Using the t-substitutions, or otherwise, prove the identity **3**

$$\frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$$

3. Using an appropriate products-to-sums relationship, or otherwise, solve the equation

$\sin 5x + \sin x = 0$ in the domain $0 \leq x \leq \frac{\pi}{2}$. **2**

4. The quadratic equation $ax^2 + bx + c = 0$ has roots $x = \tan\alpha$ and $x = \tan\beta$.

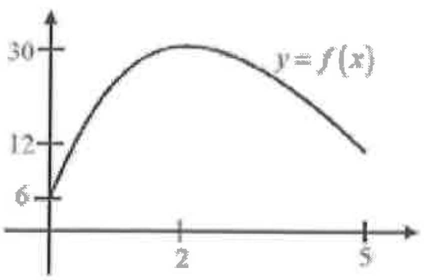
(i) Show that $\tan\,(\alpha + \beta) = \frac{b}{c-a}$ **2**

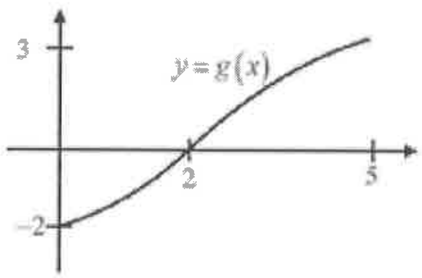
(ii) Show that $\tan^2\,(\alpha - \beta) = \frac{b^2-4ac}{(a+c)^2}$ **2**

Section E (12 marks)

1. Solve the inequality $\dfrac{2x}{1-x} \geq x$ **3**

2. Consider the following sketches of the functions $y = f(x)$ and $y = g(x)$. **1**

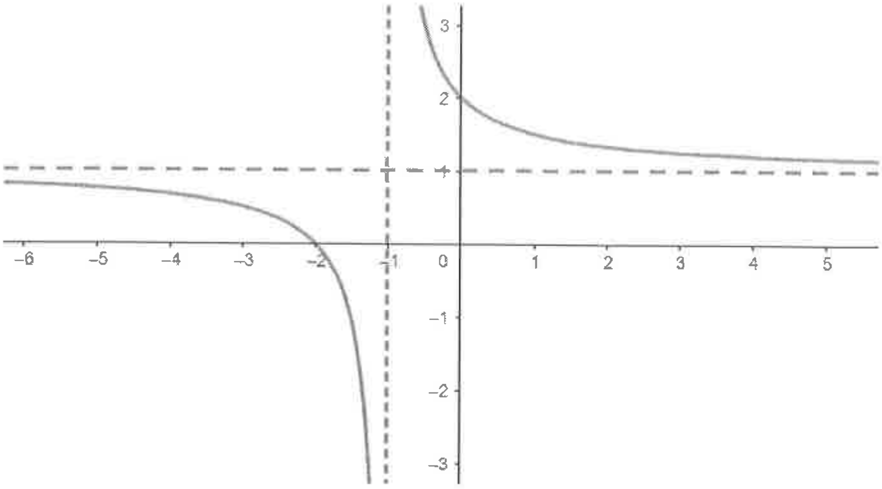




What is the range of the function $y = \dfrac{f(x)}{g(x)}$? [Circle the answer]

(A) $(-\infty, -3] \cup [4, \infty)$ (B) $[-3, 4]$ (C) $[-15, 2]$ (D) all real y

3. The graph of $y = f(x)$ is shown below:

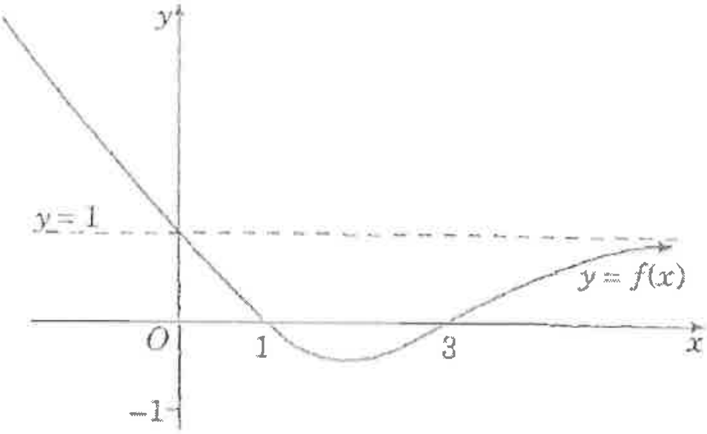


Draw separate one-third page sketches of the following curves, clearly indicating any important features such as asymptotes, intercepts and turning points.

(i) $y = |f(x)|$ **2**

(ii) $y = f(|x|)$ **2**

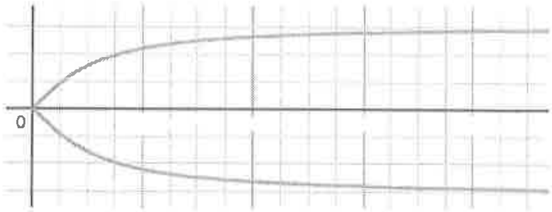
4. The graph shows the graph of the function $y = f(x)$. It has a horizontal asymptote $y = 1$.



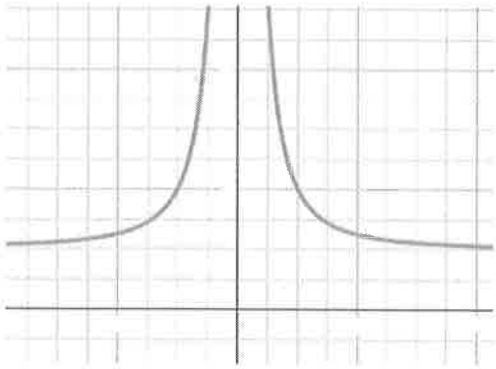
Draw a one-third page graph of $y = \sqrt{f(x)}$ showing any asymptotes and stating its domain and range. **3**

5. Which of the following graphs best represents $|y| = tan^{-1}(x)$? **1**

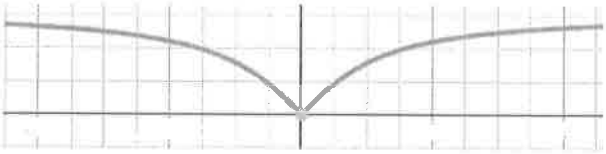
(A)



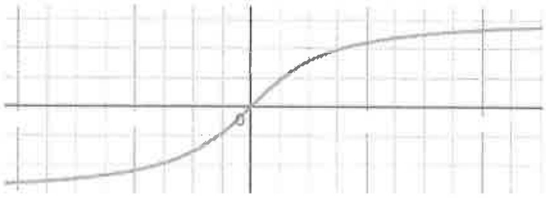
(B)



(C)



(D)



Section F (8 marks)

1. The surface area of a sphere of radius r metres is decreasing at a rate of $0 \cdot 8\, m^2$ per second at the instant when $r = 2 \cdot 3\, m$.

Calculate the rate of decrease of the radius of the sphere at this instant. **2**

2. A metal statuette is taken from a freezer at $-8°C$ into a room where the air temperature is $22°C$.

The rate at which the statuette warms follows Newton's law, that is $\frac{dT}{dt} = -k(T - 22)$ where k is a positive constant, time t is measured in minutes, and temperature T is measured in degrees Celsius.

(i) Show that $T = 22 - Ae^{-kt}$ is a solution of the equation $\frac{dT}{dt} = -k(T - 22)$, and find the value of A. **2**

(ii) The temperature of the statuette reaches $4°C$ in 90 minutes. Find the exact value of k. **2**

(iii) Find the temperature of the statuette after another 90 minutes. **1**

3. When added to water, 5 grams of a substance dissolves at a rate equal to 10% of the amount of undissolved chemical per hour. If x is the number of grams of *undissolved* chemical after t hours, which equation describes the rate of change of x? **1**

(A) $\frac{dx}{dt} = -\frac{1}{10}x$ (B) $\frac{dx}{dt} = -\frac{1}{5}x$ (C) $\frac{dx}{dt} = \frac{1}{5}(10 - x)$ (D) $\frac{dx}{dt} = \frac{1}{10}(5 - x)$

END OF EXAM

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Mark	/12	/12	/9	/10	/12	/8	/63	/100

Section A (12 marks)

1. Let α, β, and γ be the roots of $x^3 - 4x + 1 = 0$. **1**

Circle the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$:

(A) -4 **(B)** -1 **(C)** 1 **(D)** 4

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$$

$$= \frac{-4}{-1} = 4 \quad \therefore \text{(D)}$$

2. The polynomial $P(x) = x^3 - x^2 - 5x - 3$ has a double root at $x = \alpha$.

What is the value of α? **1**

(A) $\alpha = -\frac{5}{3}$ **(B)** $\alpha = -1$ **(C)** $\alpha = 1$ **(D)** $\alpha = \frac{5}{3}$

$$P'(x) = 3x^2 - 2x - 5$$

$$= (3x - 5)(x + 1) = 0$$

$\therefore x = -1$ or $x = \frac{5}{3}$

Since $P(-1) = -1 - 1 + 5 - 3 = 0$,

and there can only be 1 double root tops,

$\therefore$ double root is -1 $\therefore$ (B)

3. The polynomial $P(x)$ is given by $P(x) = 2x^3 + 7x^2 - 46x + 21$.

(i) Show that $P(x)$ is divisible by $(x - 3)$. **1**

$$P(3) = 2(3^3) + 7(3^2) - 46(3) + 21 = 54 - 63 - 138 + 21$$
$$= 0$$

(ii) Hence, fully factorise $P(x)$. $\therefore$ $x-3$ is factor. **3**

$$
\begin{array}{r|l}
 & 2x^2 + 13x - 7 \\
x-3 & 2x^3 + 7x^2 - 46x + 21 \\
 & 2x^3 - 6x^2 \\
 & 13x^2 - 46x \\
 & 13x^2 - 39x \\
 & -7x + 21 \\
 & -7x + 21 \\
 & 0
\end{array}
$$

$$\therefore\ P(x) = (x-3)(2x^2 + 13x - 7)$$

$$P(x) = (x-3)(2x-1)(x+7).$$

4. The polynomial $P(x) = x^3 + 2x^2 + ax + b$ has a factor of $x + 2$ and when divided by $x - 2$ there is a remainder of 12. Find the values of a and b. **3**

$P(-2) = 0 \quad \therefore \quad (-2)^3 + 2(-2)^2 + a(-2) + b = 0$

$-8 + 8 - 2a + b = 0$

$\therefore \quad -2a + b = 0 \quad \text{———} (1)$

$P(2) = 12$

$\therefore \quad 2^3 + 2(2^2) + 2a + b = 12$

$\therefore \quad 2a + b = -4 \quad \text{———} (2)$

$(1) + (2) \rightarrow \quad 2b = -4$

$\therefore \quad b = -2$

Thus $\quad 2a = -4 - b$

$= -4 + 2$

$= -2$

$a = -1$

$\therefore \quad a = -1$
$\quad b = -2$

5. The polynomial equation $4x^3 - 12x^2 + 5x + 6 = 0$ has roots α, β, and γ. It is known that one of the roots is the sum of the other two. Find α, β, and γ. **3**

$$4x^3 - 12x^2 + 5x + 6 = 0$$

$$\begin{cases} \alpha + \beta + \gamma = 3 & —(1) \\ \alpha\beta + \alpha\gamma + \beta\gamma = \frac{5}{4} & —(2) \\ \alpha\beta\gamma = -\frac{3}{2} & —(3) \end{cases}$$

✓

Let $\alpha = \beta + \gamma$

Sub into ① $\rightarrow$ $2\alpha = 3$

$\therefore \alpha = \frac{3}{2}$

sub this into ③ $\rightarrow$ $\frac{3}{2} \cdot \beta\gamma = \frac{-3}{2}$

$\therefore \beta\gamma = -1$ — ④

Also, $\beta + \gamma = \frac{3}{2}$ — ⑤

From ⑤, $\gamma = \frac{3}{2} - \beta$.

sub into ④ $\rightarrow$ $\beta\left(\frac{3}{2} - \beta\right) = -1$

$\therefore -\beta^2 + \frac{3}{2}\beta = -1$

$\therefore \beta^2 - \frac{3}{2}\beta - 1 = 0$

$2\beta^2 - 3\beta - 2 = 0$

$(2\beta + 1)(\beta - 2) = 0$

$\therefore \beta = -\frac{1}{2}$ or 2

(ie $\gamma = 2$ or $-\frac{1}{2}$)

So roots are $\frac{3}{2}, 2, -\frac{1}{2}$.

✓✓

Section B (12 marks)

1. What is the value of $\cos^{-1}\left(-\frac{1}{2}\right)$? $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3}$. **1**

(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$ $\therefore$ (C) ✓

2. Find the **exact** values of the following:

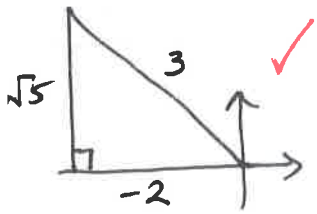
(i) $\tan^{-1}(-\sqrt{3})$ $= -\tan^{-1}(\sqrt{3}) = -\frac{\pi}{3}$ ✓ **1**

(ii) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$ $= \sin^{-1}\left[\sin\left(\pi - \frac{7\pi}{5}\right)\right]$ **1**

$= \sin^{-1}\left[\sin\left(-\frac{2\pi}{5}\right)\right]$

$= -\frac{2\pi}{5}$ ✓

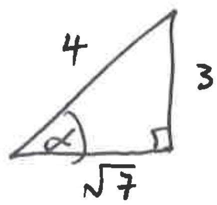
(iii) $\sin\left(\cos^{-1}\frac{-2}{3}\right)$ **2**



✓ $\sin\left[\cos^{-1}\frac{-2}{3}\right] = \frac{\sqrt{5}}{3}$ ✓

(iv) $\tan\left(2\sin^{-1}\frac{3}{4}\right)$ **2**

$\tan\left[2\sin^{-1}\frac{3}{4}\right]$



$= \frac{2\tan\alpha}{1-\tan^2\alpha}$

$= \frac{2\left(\frac{3}{\sqrt{7}}\right)}{1-\frac{9}{7}} = \frac{\frac{6}{\sqrt{7}}}{\frac{-2}{7}} = -3\sqrt{7}$ ✓✓

3.

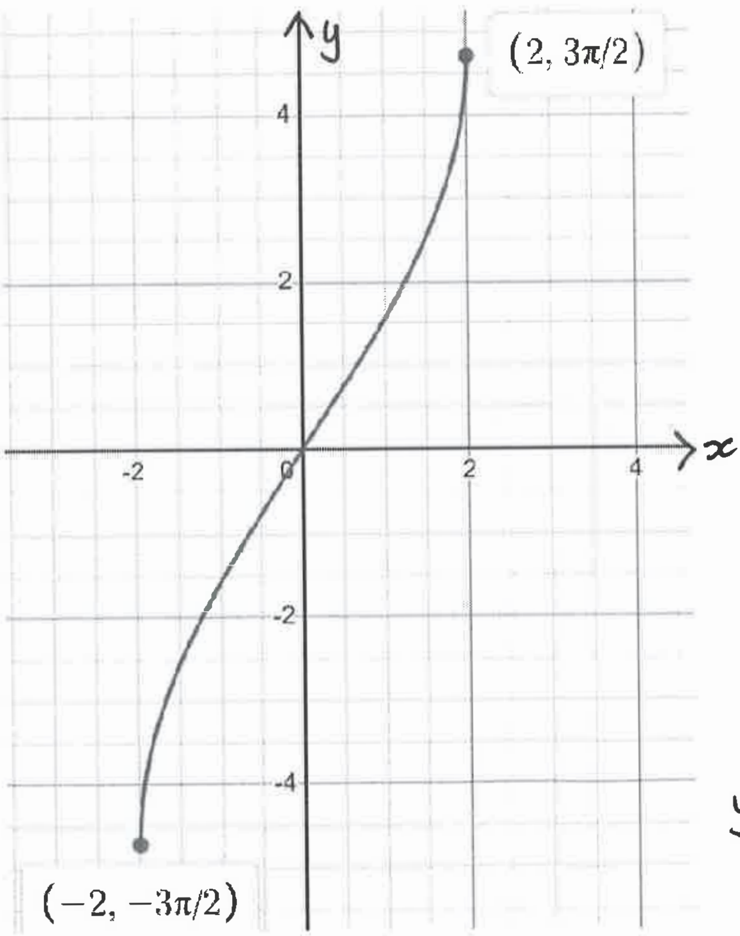
(i) State the domain and range of the function $f(x) = 3\,sin^{-1}\left(\frac{x}{2}\right)$ **2**

$-1 \leq \frac{x}{2} \leq 1 \quad \therefore \; -2 \leq x \leq 2$

So $\boxed{D: -2 \leq x \leq 2}$ ✓

& $\boxed{R: -\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}}$ ✓

(ii) Sketch the graph $y = f(x)$ **2**



* label axes

* vertical at end points

* oblique thru (0,0).

✓ ✓

$y = 3\sin^{-1}\left(\frac{x}{2}\right)$

4.

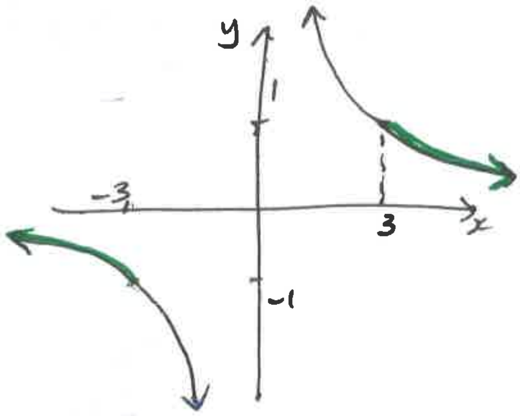
Which diagram represents the domain of the function $f(x) = cos^{-1}\left(\frac{3}{x}\right)$? **1**

(A)

we need

$-1 \le \frac{3}{x} \le 1$

(B)



(C)

(D)

So $x \le -3$ or $x \geq 3$.

$\therefore$ (A) ✓

Section C (9 marks)

1 Find the coefficient of x^2 in the expansion of $(x^2 + \frac{2}{x})^7$ 2

The general term is $\binom{7}{k}(x^2)^{7-k}\left(\frac{2}{x}\right)^k$

$= \binom{7}{k} x^{14-3k} 2^k$

$\therefore$ we need $14 - 3k = 2$

$\therefore k = 4$

Thus coefficient of $x^2 = \binom{7}{4} \cdot 2^4$

$= 560$

2. Consider the expansion of $(1+2x)^n$

(i) Write down an expression for the coefficient of the term in x^4 **2**

$$(1+2x)^n = \sum_{k=0}^{n} \binom{n}{k} 2^k x^k$$

$\therefore$ coefficient of x^4 term is $\binom{n}{4} \cdot 2^4$

(-1 if gives $\binom{n}{4} \cdot 2^4 x^4$)

(ii) The ratio of the coefficient of the term in x^4 to that of the term in x^6 is 5 : 8

Find the value of n. **3**

Likewise, coefficient of x^6 term

is $\binom{n}{6} 2^6$

Thus ratio of x^4 term to x^6 term is

$$\frac{\binom{n}{4} \cdot 2^4}{\binom{n}{6} \cdot 2^6} = \frac{\frac{n!}{4!(n-4)(n-5)(n-6)!}}{\frac{n!}{(6)(5)4!\ (n-6)!} \cdot 2^2}$$

$$= \frac{30}{4(n-4)(n-5)} = \frac{15}{2(n-4)(n-5)}$$

Thus $\frac{15}{2(n-4)(n-5)} = \frac{5}{8}$

$\therefore (n-4)(n-5) = 12$

$n^2 - 9n + 8 = 0$

$\therefore (n-8)(n-1) = 0$

$\therefore n = 8$ (as $n=1$ produces no x^4 or x^6 terms)

3. Consider the binomial expansion $(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots + \binom{n}{n}x^n$

Show that $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots\ldots + n\binom{n}{n} = n\,2^{n-1}$ **2**

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$$

Differentiate both sides with respect to x :-

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + \cdots + n\binom{n}{n}x^{n-1} \quad ✓$$

Substitute $x=1$:-

$$n(1+1)^{n-1} = \binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n}$$

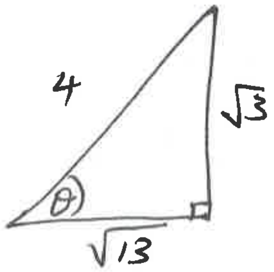
i.e. $$n \cdot 2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} \quad ✓$$

as required #

Section D (10 marks)

1. What is the value of $\sin 2\theta$ given that $\sin\theta = \frac{\sqrt{3}}{4}$ and θ is obtuse? **1**

(A) $\frac{-\sqrt{39}}{8}$ (B) $\frac{-\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{39}}{8}$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{\sqrt{3}}{4} \cdot -\frac{\sqrt{13}}{4}$

$= \frac{-\sqrt{39}}{8}$ $\therefore$ (A)

2. Using the t-substitutions, or otherwise, prove the identity **3**

$$\frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$$

Let $t = \tan\theta$

$\therefore LHS = \frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta}$

$= \frac{\frac{2t}{1-t^2} - t}{\frac{2t}{1-t^2} + \frac{1}{t}}$

$= \frac{\frac{2t - t + t^2}{1-t^2}}{\frac{2t^2 + 1 - t^2}{t(1-t^2)}}$

$= \frac{t^2(1+t^2)}{t^2+1}$ $= t^2$

$= \tan^2\theta$

$= RHS.$

3. Using an appropriate products-to-sums relationship, or otherwise, solve the equation

$\sin 5x + \sin x = 0$ in the domain $0 \leq x \leq \frac{\pi}{2}$. **2**

$$\sin 5x + \sin x = 0$$

$$\therefore \sin(3x+2x) + \sin(3x-2x) = 0$$

$$\therefore 2 \sin 3x \cos 2x = 0$$

$\therefore \sin 3x = 0$ or $\cos 2x = 0$

$3x = 0, \pi, 2\pi, 3\pi, \ldots$

$x = 0, \frac{\pi}{3}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$x = \frac{\pi}{4}$

$$\therefore x = 0, \frac{\pi}{3} \text{ or } \frac{\pi}{4}$$

4. The quadratic equation $ax^2 + bx + c = 0$ has roots $x = \tan\alpha$ and $x = \tan\beta$.

(i) Show that $\tan(\alpha + \beta) = \frac{b}{c-a}$ **2**

Sum of roots $= \tan\alpha + \tan\beta = -\frac{b}{a}$

Product of roots $= \tan\alpha \tan\beta = \frac{c}{a}$

So $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = \frac{-b}{a-c} = \frac{b}{c-a}$$

as required.

(ii) Show that $\tan^2(\alpha - \beta) = \frac{b^2-4ac}{(a+c)^2}$ **2**

$$\tan^2(\alpha-\beta) = \left(\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right)^2$$

$$= \frac{\tan^2\alpha - 2\tan\alpha\tan\beta + \tan^2\beta}{(1+\tan\alpha\tan\beta)^2}$$

$$= \frac{(\tan\alpha + \tan\beta)^2 - 4\tan\alpha\tan\beta}{(1+\tan\alpha\tan\beta)^2}$$

$$= \frac{\left(\frac{b}{a}\right)^2 - \frac{4c}{a}}{\left(1 + \frac{c}{a}\right)^2}$$

$$= \frac{b^2 - 4ac}{(a+c)^2}$$

Section E (12 marks)

1. Solve the inequality $\dfrac{2x}{1-x} \geq x$ $(x \neq 1)$ **3**

$$\frac{2x}{1-x} \geqslant x$$

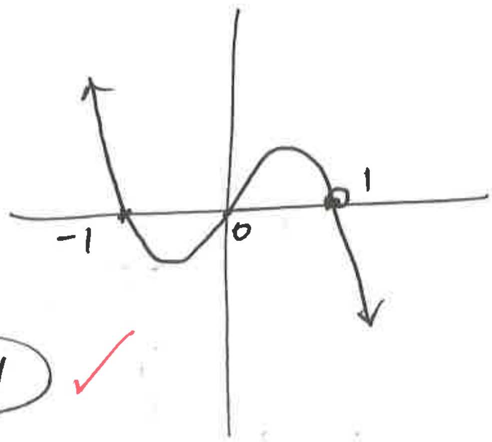
$$\therefore \quad 2x(1-x) \geqslant x(1-x)^2$$

$$\therefore \quad 2x(1-x) - x(1-x)^2 \geqslant 0$$

$$x(1-x)\left[2-(1-x)\right] \geqslant 0$$

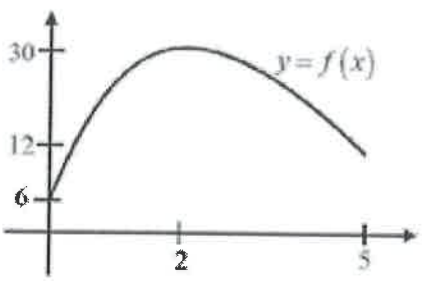
$$x(1-x)(1+x) \geqslant 0$$

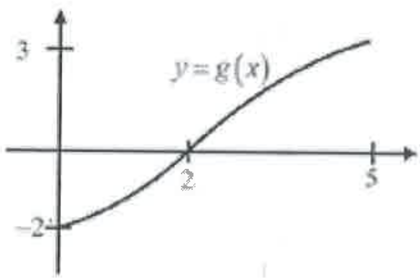
$$-x(x-1)(x+1) \geqslant 0$$



$$\therefore \quad x \leq -1 \text{ or } 0 \leq x < 1$$

2. Consider the following sketches of the functions $y = f(x)$ and $y = g(x)$. **1**



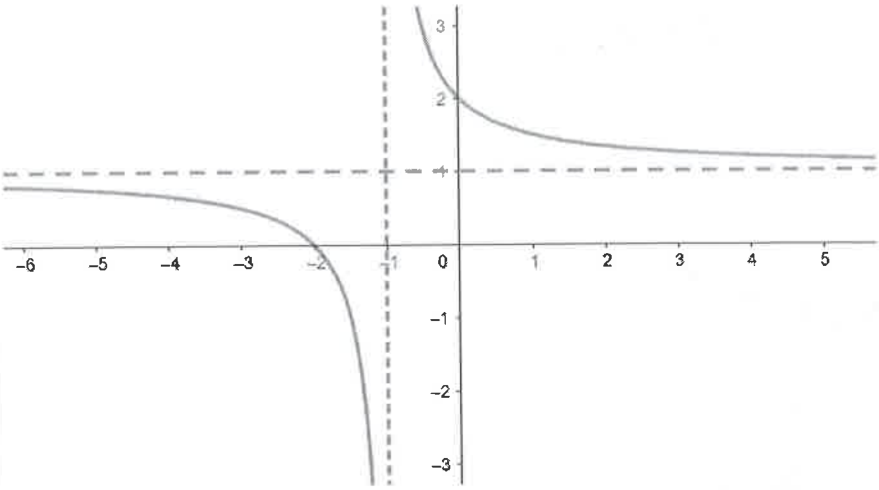


What is the range of the function $y = \dfrac{f(x)}{g(x)}$? [Circle the answer]

(A) $(-\infty, -3] \cup [4, \infty)$ (B) $[-3, 4]$ (C) $[-15, 2]$ (D) all real y

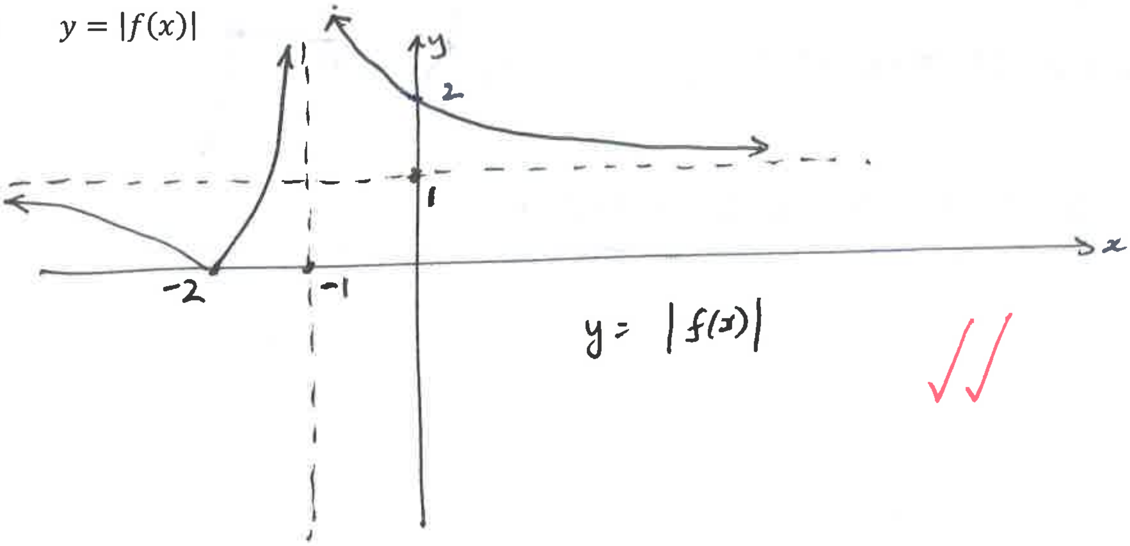
answer: A

3. The graph of $y = f(x)$ is shown below:



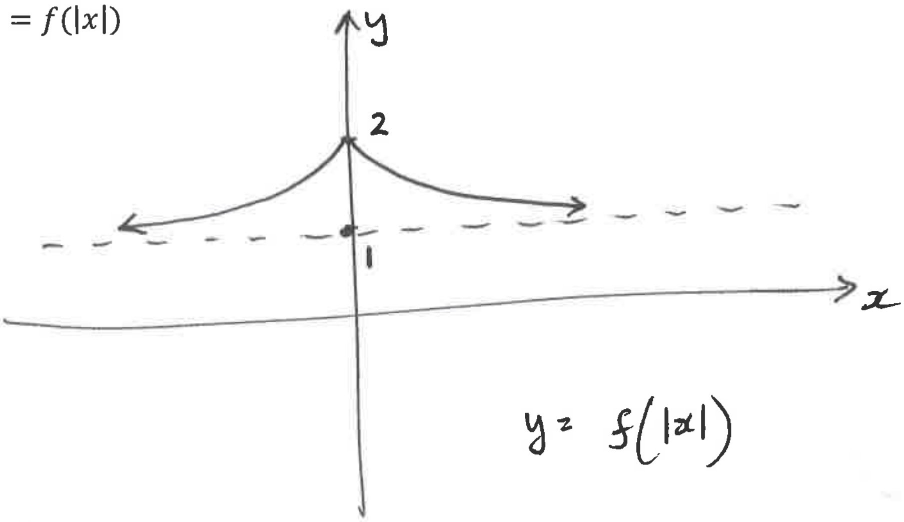
Draw separate one-third page sketches of the following curves, clearly indicating any important features such as asymptotes, intercepts and turning points.

(i) $y = |f(x)|$ **2**



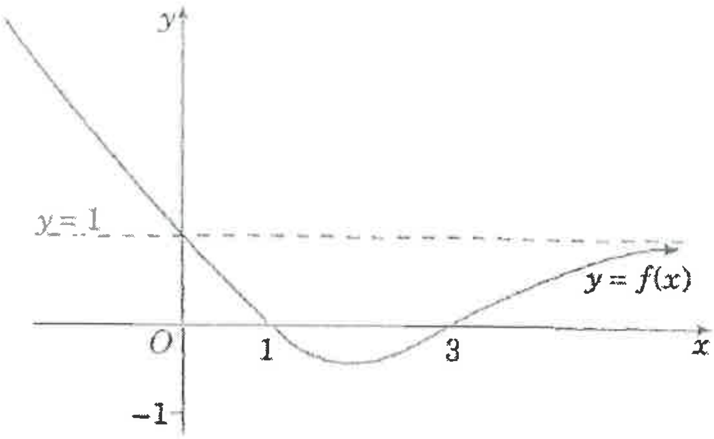
✓✓

(ii) $y = f(|x|)$ **2**

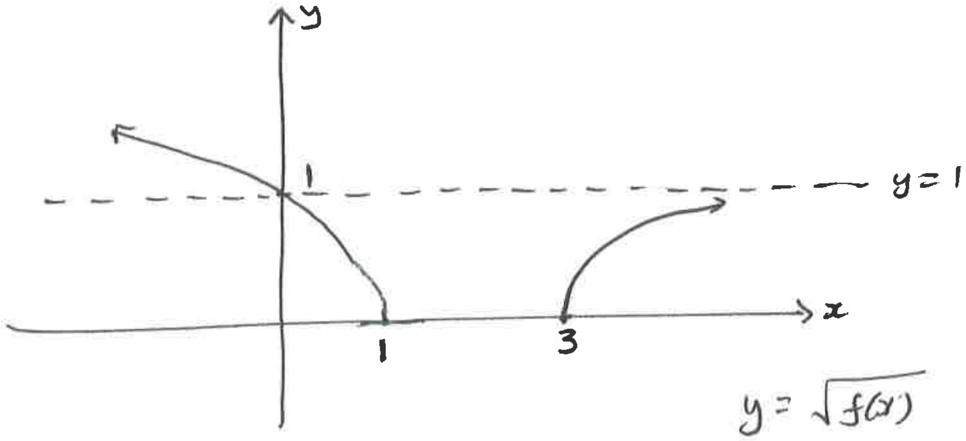


✓✓

4. The graph shows the graph of the function $y = f(x)$. It has a horizontal asymptote $y = 1$.



Draw a one-third page graph of $y = \sqrt{f(x)}$ showing any asymptotes and stating its domain and range. **3**

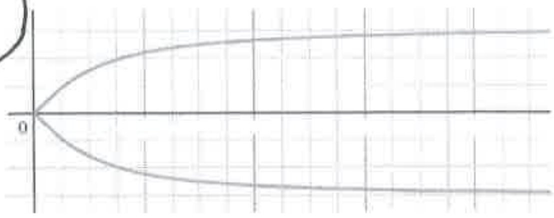


D: $x \leq 1$ or $x \geq 3$

R: $y \geq 0$.

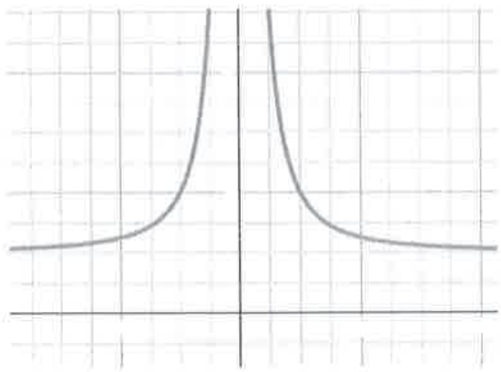
5. Which of the following graphs best represents $|y| = tan^{-1}(x)$? **1**

(A)

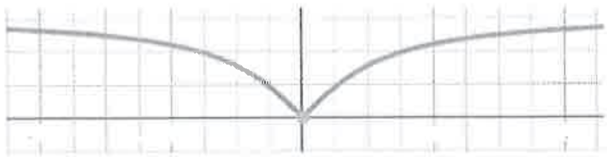


ANSWER: A

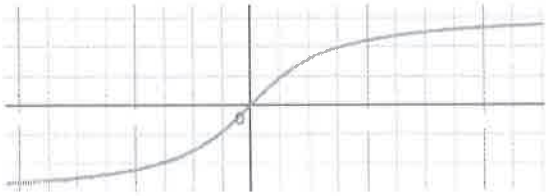
(B)



(C)



(D)



Section F (8 marks)

1. The surface area of a sphere of radius r metres is decreasing at a rate of $0 \cdot 8\, m^2$ per second at the instant when $r = 2 \cdot 3\, m$.

Calculate the rate of decrease of the radius of the sphere at this instant. **2**

$S = 4\pi r^2 \quad \therefore \quad \frac{dS}{dr} = 8\pi r \quad . \quad \frac{dS}{dt} = 0.8 \ m^2/s.$

$$\frac{dr}{dt} = \frac{dr}{dS} \cdot \frac{dS}{dt}$$

$$= \frac{1}{8\pi r} \cdot (0.8)$$

$\therefore$ when $r = 2.3$,

$$\frac{dr}{dt} = \frac{0.8}{8\pi(2.3)} = \frac{1}{23\pi} \ m/s$$

$$\doteqdot 0.0138396 \ m/s$$

2. A metal statuette is taken from a freezer at $-8°C$ into a room where the air temperature is $22°C$.

The rate at which the statuette warms follows Newton's law, that is $\frac{dT}{dt} = -k(T-22)$ where k is a positive constant, time t is measured in minutes, and temperature T is measured in degrees Celsius.

(i) Show that $T = 22 - Ae^{-kt}$ is a solution of the equation $\frac{dT}{dt} = -k(T-22)$, and find the value of A. **2**

$$\frac{dT}{dt} = (-k)(-Ae^{-kt})$$
$$= kAe^{-kt}$$
$$= k(22-T)$$
$$= -k(T-22)$$

$\therefore$ is solution.

when $t=0$, $T=-8$

$\therefore -8 = 22 - A$

$\therefore A = 30$

(ii) The temperature of the statuette reaches $4°C$ in 90 minutes. Find the exact value of k. **2**

when $t = 90$, $T = 4$

$$\therefore 4 = 22 - 30e^{-90k}$$
$$30e^{-90k} = 18$$
$$e^{-90k} = \frac{18}{30} = \frac{3}{5}$$
$$\therefore -90k = \ln\frac{3}{5}$$
$$\therefore k = -\frac{1}{90}\ln\left(\frac{3}{5}\right)$$
$$\left(k = \frac{\ln 5 - \ln 3}{90}\right)$$

(iii) Find the temperature of the statuette after another 90 minutes. **1**

when $t = 180$,

$$T = 22 - 30e^{-180k}$$
$$= 22 - 30e^{-180\left[-\frac{1}{90}\ln\frac{3}{5}\right]}$$
$$= 22 - 30e^{2\ln\frac{3}{5}} = 22 - 30 \times \frac{9}{25} = 22 - \frac{54}{5}$$
$$T = 11.2°C$$

3. When added to water, 5 grams of a substance dissolves at a rate equal to 10% of the amount of undissolved chemical per hour. If x is the number of grams of *undissolved* chemical after t hours, which equation describes the rate of change of x? **1**

(A) $\frac{dx}{dt} = -\frac{1}{10}x$ (B) $\frac{dx}{dt} = -\frac{1}{5}x$ (C) $\frac{dx}{dt} = \frac{1}{5}(10 - x)$ (D) $\frac{dx}{dt} = \frac{1}{10}(5 - x)$

answer : A

END OF EXAM