2023

PRELIMINARY ASSESSMENT TASK 3

Extension 1 Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen only
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Calculators approved by NESA may be used
- DO NOT USE LIQUID PAPER OR CORRECTION TAPE

Total Marks – 52

Section I: 10 marks

- Attempt questions 1-10, using the answer sheet provided.
- Allow about 15 minutes for this section

Section II: 42 marks

- Attempt questions 11-14, answer the questions in the booklets provided.
- Allow about 75 minutes for this section

	M/C	11	12	13	14	TOTAL
Parametrics	/2 1, 10			b /6		/8
Rates of Change with Respect to Time	/1 2	c(i, iii, iv) /5	b /2			/8
Further Trigonometric Identities	/2 3, 4	a /2			c /4	/8
Inverse Trigonometric Functions	/2 5, 6		a /4		a /2	/8
Polynomials	/2 7, 8	c(ii) /3	c /5			/10
Exponential Growth & Decay	/ 1 9			a /4	b /5	/10
TOTAL						

STUDENT NUMBER: ______________________________ TEACHER:_______________

Section I

10 marks
Attempt Questions 1 to 10
Use the multiple-choice answer sheet for Questions 1 to 10

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1. Find the Cartesian equation for the curve with the parametric equations:

$$x = 4p - 6$$

$$y = 3p$$

(A) $y = \frac{1}{12}x + \frac{1}{2}$

(B) $y = \frac{3}{4}x + \frac{9}{2}$

(C) $y = \frac{1}{12}x - \frac{1}{2}$

(D) $y = \frac{3}{4}x + 6$

2. A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.

At a certain time $\dot{x} = -4$ m/s and $\ddot{x} = -5$ m/s^2.

Which statement describes the motion of the particle at that time?

(A) The particle is moving right with increasing speed.

(B) The particle is moving right with decreasing speed.

(C) The particle is moving left with increasing speed.

(D) The particle is moving left with decreasing speed.

3. Let $t = \tan\frac{\theta}{2}$ where $0 < \theta < 180°$. Which of the following gives the correct expression for $\sec\theta + \tan\theta$?

(A) $\dfrac{1-t}{1+t}$

(B) $\dfrac{1+2t+t^2}{1-t}$

(C) $\dfrac{1+t}{1-t}$

(D) $\dfrac{t^2-2t-1}{1+t^2}$

4. Which expression is equivalent to $\cos 3x \cos x + \sin 3x \sin x$?

(A) $2\cos 4x \sin 4x$

(B) $\cos 4x + \sin 4x$

(C) $\cos 4x$

(D) $\cos 2x$

5. Which of the following best describes $y = \arctan\theta$?

(A) Even function

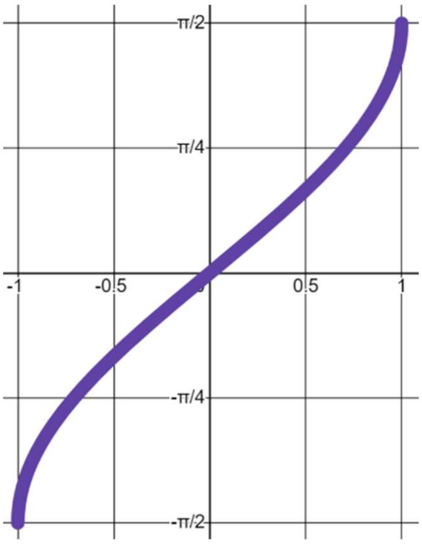
(B) Odd function

(C) Neither even nor odd function

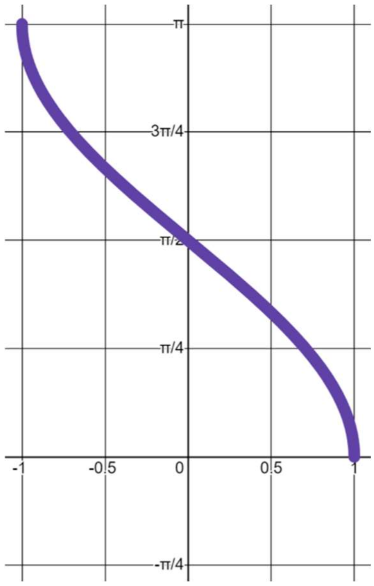
(D) Both even and odd function

6. Which graph best represents $y = \sin^{-1} x$?

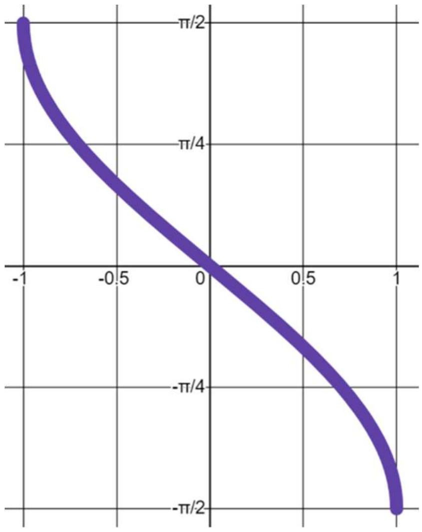
(A)



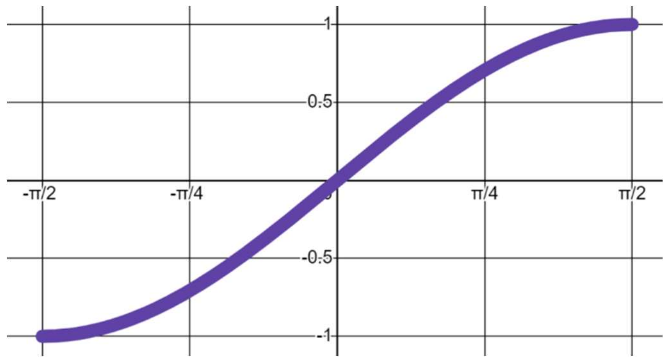
(B)



(C)



(D)



7. The roots of $4x^4 - 2x^3 + 9x - 10$ are α, β, γ and δ.

What is the value of $\alpha\beta\gamma\delta(\alpha + \beta + \gamma + \delta)$?

(A) $\dfrac{5}{4}$

(B) $-\dfrac{5}{4}$

(C) $\dfrac{9}{4}$

(D) $-\dfrac{9}{4}$

8. For a polynomial $P(x) = x^3 - 3x^2 + 1$, with roots α, β and γ it is known that:

$$\alpha^2 + \beta^2 + \gamma^2 = 9$$

What is the value of $\alpha^3 + \beta^3 + \gamma^3$?

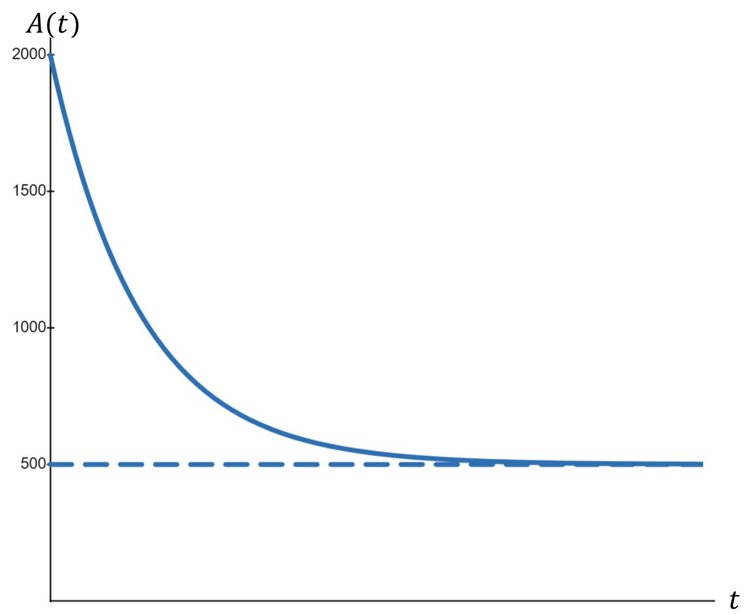
(A) 3

(B) 10

(C) 24

(D) 27

9. The population of ants $A(t)$, in a nest at time t is shown in the diagram below.



Which equation best represents this graph?

(A) $A(t) = 500 + 1500e^{-kt}$

(B) $A(t) = 500 - 2000e^{-kt}$

(C) $A(t) = 1500 + 500e^{-kt}$

(D) $A(t) = 2000 - 500e^{-kt}$

10. The Cartesian equation $y = 2x^2 - 5x + 3$, is being converted to parametric equations with $x = 5 - 3m$. Find the parametric equation for y in terms of m.

(A) $y = 18m^2 - 15m + 28$

(B) $y = 18m^2 - 45m + 28$

(C) $y = 18m^2 + 15m + 28$

(D) $y = 18m^2 + 45m + 28$

End of Section I

Section II

42 marks
Attempt Questions 11 to 14

Answer each question in a SEPARATE writing booklet.
Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (10 marks) Use a SEPARATE writing booklet.

(a) Prove that $\dfrac{\cos^2 x - \cos 2x}{\sin^2 x + \cos 2x} = \tan^2 x$ **2**

(b) Henry is enjoying a day at the zoo. He is standing outside the meerkat enclosure watching a particular meerkat walk along a straight fence. The displacement of the meerkat, x centimetres, in relation to Henry's position along the fence at time t seconds is given by $x = t^3 - 15t^2 + 54t - 40$.

(i) Find the initial position of the meerkat from Henry. **1**

(ii) The first time that the meerkat is directly in front of Henry is after 1 second. Find any other times when the meerkat is directly in front of Henry. **3**

(iii) Find any times when the meerkat stops moving, leaving your answer in exact form. **2**

(iv) Find the total distance covered by the meerkat over the first 12 seconds, correct to two decimal places. **2**

End of question 11

Question 12 (11 marks) Use a SEPARATE writing booklet.

(a) (i) State the domain and range of $y = 2\cos^{-1}(x+1)$ in interval notation. **2**

(ii) Sketch the graph of $y = 2\cos^{-1}(x+1)$, indicating clearly the coordinates of the endpoints of the graph. **2**

(b) Aimee notices a tap is turned on and water is flowing out of it. She quickly grabs an empty bucket with a capacity of 16 litres and puts it under the tap to collect the water. **2**

While the bucket is filling Aimee starts to turn the tap off. However, the tap is difficult to turn and the bucket fills up before she can finish turning the tap off.

The bucket is filling so that at time t seconds the volume of water V, in litres, is given by $V = -t^2 + 8t$.

By what percentage has Aimee reduced the flow rate of the tap in the first 3 seconds?

(c) Sketch the graph of $y = x^4 - 5x^3 - 3x^2 + 17x - 10$, given that it has a root at $x = 1$ which has multiplicity 2. **5**

Label all intercepts on your graph. Do NOT use calculus and do NOT calculate the coordinates of the vertices/turning points, just roughly sketch them appropriately, according to your scale and intercepts.

End of question 12

Question 13 (10 marks) Use a SEPARATE writing booklet.

(a) Rick is a mad scientist who is experimenting on humans to create zombies. After becoming self aware, 120 zombies escape from his basement. They immediately started attacking and infecting other people, turning them into zombies too.

The total number of zombies is given by $N(t) = 120e^{kt}$ where t is the number of days since the escape.

(i) After two days there was a total of 270 zombies. **1**

Show that $k = 0.405$, correct to 3 decimal places.

(ii) Find the number of zombies after 10 days. **1**

(iii) What is the rate of change of the number of zombies when $t = 20$? **2**

(b) Elena is standing in a football field playing with toy remote control car. She is controlling the car so that it is driving in a circle around herself. The centre of the football field is taken to be the origin $(0, 0)$. The position of a car in relation to the origin is given by the equation $x^2 + 4x + y^2 - 5y - 20 = 0$ where x is the horizontal displacement and y is the vertical displacement in metres from the origin.

(i) Show that Elena is standing at the coordinate $(-2,\ 2.5)$ on the football field. **1**

(ii) Show that the parametric equations that represent the horizontal and vertical displacements in terms of the angle, θ degrees, from the Elena are: **3**

$$x = \frac{11\cos\theta - 4}{2}$$

$$y = \frac{11\sin\theta + 5}{2}$$

(iii) Find the position on the football field of the toy car when it is on an angle of $135°$ from Elena. Give your answer as a coordinate in simplified exact form. **2**

End of question 13

Question 14 (11 marks) Use a SEPARATE writing booklet.

(a) Show that $\tan(\cos^{-1}(-\frac{2}{3})) = -\frac{\sqrt{5}}{2}$ **2**

(b) On Sunday, Gordon meal preps a delicious chicken curry. When he takes it off the stove it is at a temperature of 120°C. Gordon leaves the curry in the pot on the bench to cool down before dividing it into containers for his lunches for the following week. The room temperature where the curry is cooling down is 25°C.

After t minutes, the temperature of the curry in degree Celsius is T.

The temperature of the curry can be modelled using the differential equation $\frac{dT}{dt} = -k(T - 25)$, where k is the decay constant.

(i) Show that $T = 25 + 95e^{-kt}$ satisfies both this equation and the initial condition. **2**

(ii) Gordon's containers have a safety warning, stating that nothing over 40°C should be placed in them. After 4 minutes, the curry has cooled to 85°C. How long does Gordon have to wait from initially taking the curry off the stove before he can safely put the curry into his containers? Answer correct to the nearest minute. **3**

(c) Solve $\cos 7x - \cos 3x = \sin 5x$, $0 \leq x \leq \pi$ **4**

End of paper

Section I

10 marks
Attempt Questions 1 to 10
Use the multiple-choice answer sheet for Questions 1 to 10

---

1. Find the Cartesian equation for the curve with the parametric equations:

$$x = 4p - 6$$

$$y = 3p$$

$p = \frac{y}{3}$

(A) $y = \frac{1}{12}x + \frac{1}{2}$

(B) $y = \frac{3}{4}x + \frac{9}{2}$

(C) $y = \frac{1}{12}x - \frac{1}{2}$

(D) $y = \frac{3}{4}x + 6$

$x = 4\left(\frac{y}{3}\right) - 6$

$= \frac{4}{3}y - 6$

$\frac{3}{4}x = y - \frac{9}{2}$

$y = \frac{3}{4}x + \frac{9}{2}$

2. A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.

At a certain time $\dot{x} = -4$ m/s and $\ddot{x} = -5$ m/s^2.

Which statement describes the motion of the particle at that time?

(A) The particle is moving right with increasing speed.

(B) The particle is moving right with decreasing speed.

(C) The particle is moving left with increasing speed.

(D) The particle is moving left with decreasing speed.

$\dot{x}$ = velocity, negative means moving left.

velocity and acceleration both negative, ie. both moving left. ∴ increasing speed.

3. Let $t = \tan\frac{\theta}{2}$ where $0 < \theta < 180°$. Which of the following gives the correct expression for $\sec\theta + \tan\theta$?

$\sec\theta + \tan\theta$

(A) $\frac{1-t}{1+t}$

$= \frac{1}{\cos\theta} + \tan\theta$

(B) $\frac{1+2t+t^2}{1-t}$

$= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$

(C) $\frac{1+t}{1-t}$

$= \frac{t^2+2t+1}{1-t^2}$

(D) $\frac{t^2-2t-1}{1+t^2}$

$= \frac{(t+1)^2}{(t+1)(t-1)}$

4. Which expression is equivalent to $\cos 3x \cos x + \sin 3x \sin x$?

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

(A) $2\cos 4x \sin 4x$

$= \cos(3x - x)$

(B) $\cos 4x + \sin 4x$

$= \cos 2x$

(C) $\cos 4x$

(D) $\cos 2x$

5. Which of the following best describes $y = \arctan\theta$?

(A) Even function

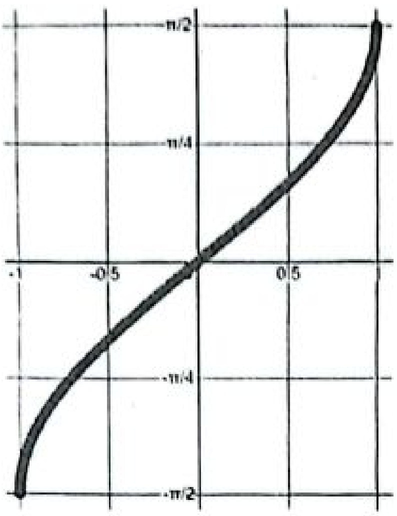
(B) Odd function

(C) Neither even nor odd function

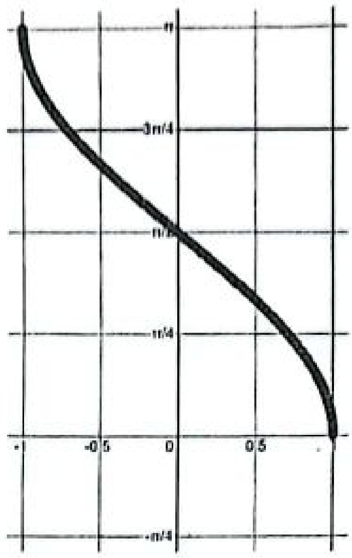
(D) Both even and odd function

6. Which graph best represents $y = \sin^{-1} x$?

(A)



(B)



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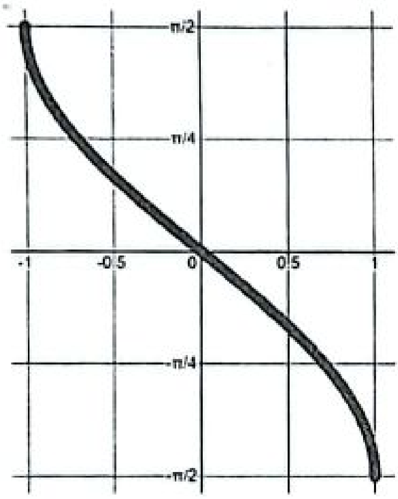
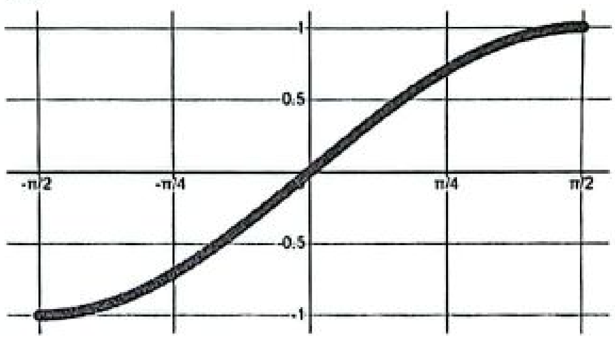


(D)



$a=4, b=-2, c=0, d=9, e=-10$

7. The roots of $4x^4 - 2x^3 + 9x - 10$ are α, β, γ and δ.

What is the value of $\alpha\beta\gamma\delta(\alpha + \beta + \gamma + \delta)$?

$\alpha+\beta+\gamma+\delta = -\frac{b}{a} = -\frac{(-2)}{4} = \frac{1}{2}$

$\alpha\beta\gamma\delta = \frac{e}{a} = \frac{-10}{4}$

(A) $\frac{5}{4}$

(B) $-\frac{5}{4}$

(C) $\frac{9}{4}$

(D) $-\frac{9}{4}$

$\therefore \frac{-10}{4} \times \frac{1}{2} = -\frac{5}{4}$

8. For a polynomial $P(x) = x^3 - 3x^2 + 1$, with roots α, β and γ it is known that:

$$\alpha^2 + \beta^2 + \gamma^2 = 9$$

What is the value of $\alpha^3 + \beta^3 + \gamma^3$?

Given α, β and γ are roots, then:

$$\alpha^3 - 3\alpha^2 + 1 = 0$$
$$\beta^3 - 3\beta^2 + 1 = 0$$
$$+ \quad \gamma^3 - 3\gamma^2 + 1 = 0$$
$$\alpha^3 + \beta^3 + \gamma^3 - 3(\alpha^2 + \beta^2 + \gamma^2) + 3 = 0$$
$$\alpha^3 + \beta^3 + \gamma^3 = 3(\alpha^2 + \beta^2 + \gamma^2) - 3$$
$$= 3(9) - 3$$
$$= 24$$

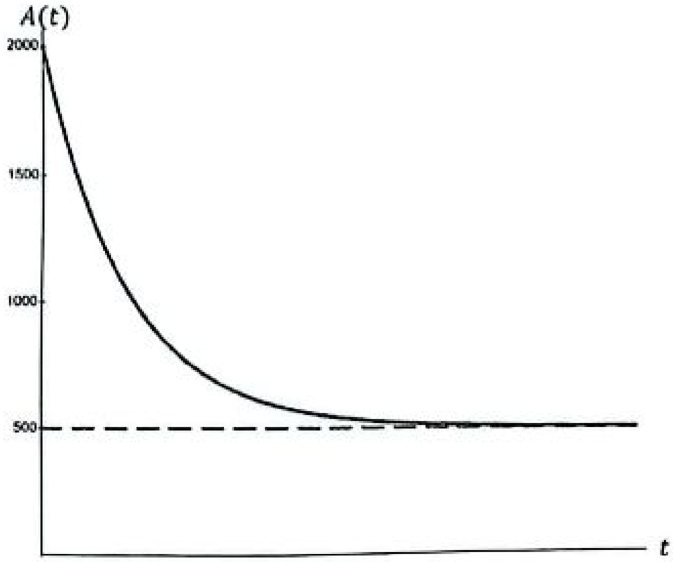
(A) 3

(B) 10

(C) 24

(D) 27

9. The population of ants $A(t)$, in a nest at time t is shown in the diagram below.



Which equation best represents this graph?

(A) $A(t) = 500 + 1500e^{-kt}$

(B) $A(t) = 500 - 2000e^{-kt}$

(C) $A(t) = 1500 + 500e^{-kt}$

(D) $A(t) = 2000 - 500e^{-kt}$

when $t = 0$, $A = 2000$ (from graph)

$t \to \infty$, $A \to 500$

TESTING (A)

$A(0) = 500 + 1500e^{-k(0)} = 2000$ ✓

$A(\infty) = 500 + 1500e^{-k(\infty)} = 500$ ✓

10. The Cartesian equation $y = 2x^2 - 5x + 3$, is being converted to parametric equations with $x = 5 - 3m$. Find the parametric equation for y in terms of m.

(A) $y = 18m^2 - 15m + 28$

(B) $y = 18m^2 - 45m + 28$

(C) $y = 18m^2 + 15m + 28$

(D) $y = 18m^2 + 45m + 28$

$$y = 2(5-3m)^2 - 5(5-3m) + 3$$
$$= 2(25 - 30m + 9m^2) - 25 + 15m + 3$$
$$= 50 - 60m + 18m^2 - 25 + 15m + 3$$
$$= 18m^2 - 45m + 28$$

End of Section I

Question 11

a) $\dfrac{\cos^2 x - \cos 2x}{\sin^2 x + \cos 2x} = \tan^2 x$

$$LHS = \frac{\cos^2 x - (\cos^2 x - \sin^2 x)}{\sin^2 x + \cos^2 x - \sin^2 x}$$ ✓

(mostly welldone)

$$= \frac{\sin^2 x}{\cos^2 x}$$ ✓

$$= \tan^2 x = RHS$$

b) $x = t^3 - 15t^2 + 54t - 40$

(i) when $t = 0$

$x = -40$ cm ✓ 1 mark

$\therefore$ the meerkat is 40cm to the left from Henry.

(ii) when $t = 1$, $x = 0$

$\therefore t - 1$ is a factor of $t^3 - 15t^2 + 54t - 40$

$$\begin{array}{r|l} & t^2 - 14t + 40 \\ \hline t-1 & t^3 - 15t^2 + 54t - 40 \\ & t^3 - t^2 \\ \hline & -14t^2 + 54t \\ & -14t^2 + 14t \\ \hline & 40t - 40 \\ & 40t - 40 \\ \hline & - \end{array}$$ ✓ 1 mark

Solution is required to achieve full marks.

If you use synthetic division, you have to mention it.

$t^2 - 14t + 40 = 0$ ✓ 1 mark

$(t - 10)(t - 4) = 0$

$\therefore t = 10, 4$ ✓ 1 mark

$\therefore$ the meerkat is in front of Henry at 4s and 10s.

(iii) $v=0$, solve for t

$\dot{x} = 3t^2 - 30t + 54$ ✓ 1 mark for the derivative

$3t^2 - 30t + 54 = 0$

$t^2 - 10t + 18 = 0$

$\Delta = (-10)^2 - 4(1)(18) = 28$

$t = \dfrac{10 \pm \sqrt{28}}{2}$

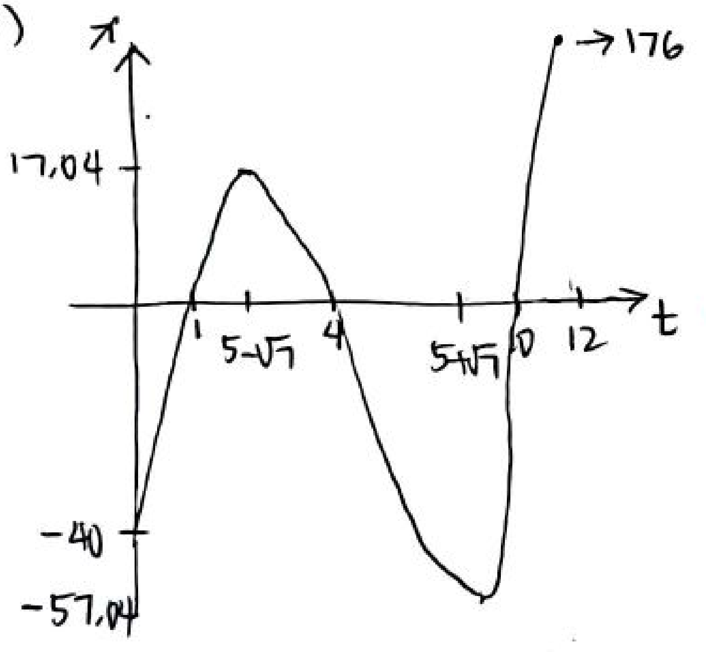
$t = \dfrac{10 \pm 2\sqrt{7}}{2}$

$t = 5 \pm \sqrt{7}$

1 mark for the correct answer.

$\therefore$ the meerkat stops moving when time is $(5-\sqrt{7})$ s and $(5+\sqrt{7})$ s.

(iv)



✓ 1 mark for showing correct application of distance

total distance

$= 40 + 2\times 17.04 + 2\times 57.04 + 176$

$= 364.16$ cm ✓ 1 mark for correct final answer.

poorly done

Question 12 (11 marks) Use a SEPARATE writing booklet.

Done well

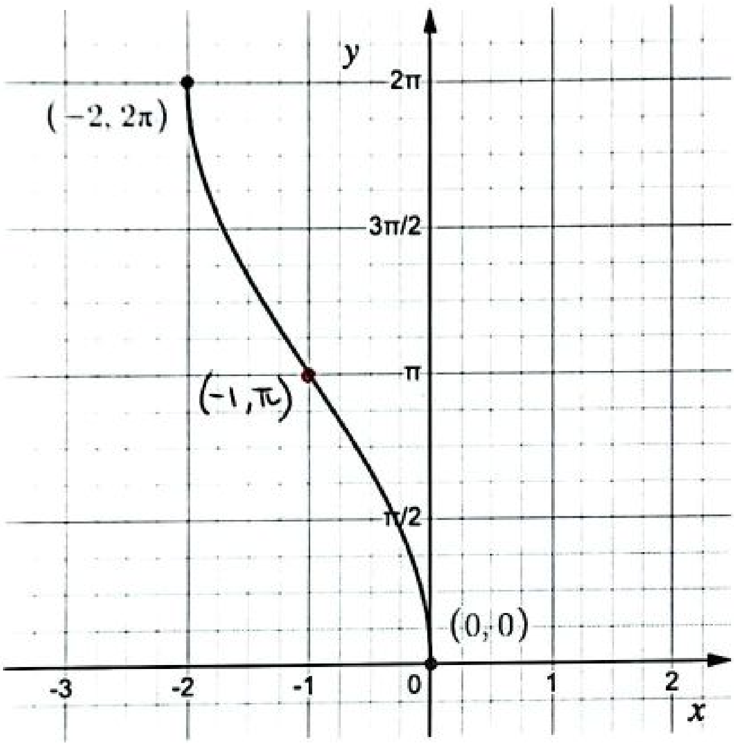
(a) (i) State the domain and range of $y = 2\cos^{-1}(x+1)$ in interval notation. **2**

D: $-1 \le x+1 \le 1$
$-2 \le x \le 0$
$\therefore [-2, 0]$ (1)

* Interval notation

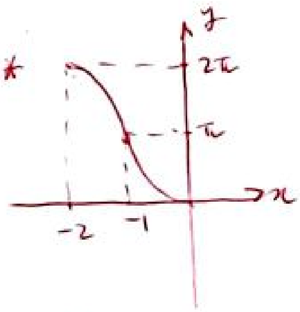
R: $0 \le \cos^{-1}(x+1) \le \pi$
$0 \le y \le 2\pi$
$\therefore [0, 2\pi]$ (1)

(ii) Sketch the graph of $y = 2\cos^{-1}(x+1)$, indicating clearly the coordinates of the endpoints of the graph. **2**



1 mark for the correct shape

1 mark for showing the correct end points



* (−1)

* Show the point of inflection

Done well

(b) Aimee notices a tap is turned on and water is flowing out of it. She quickly grabs an empty bucket with a capacity of 16 litres and puts it under the tap to collect the water. **2**

While the bucket is filling Aimee starts to turn the tap off. However, the tap is difficult to turn and the bucket fills up before she can finish turning the tap off.

The bucket is filling so that at time t seconds the volume of water V, in litres, is given by $V = -t^2 + 8t$.

By what percentage has Aimee reduced the flow rate of the tap in the first 3 seconds?

$V = -t^2 + 8t$

$\frac{dV}{dt} = -2t + 8$

At $t = 0$, $\frac{dV}{dt} = 8$ L/sec (1)

At $t = 3$, $\frac{dV}{dt} = -2(3) + 8$
$= 2$ L/sec

✗ Attempted to find the change in volumes instead of rates

Flow rate reduction $= 8 - 2$
$= 6$ L/sec

$\therefore$ Percentage $= \frac{6}{8} \times 100$
$= 75\ \%$ (1)

✗ $\frac{2}{8} \times 100 = 25\%$

Question 12 (11 marks) Use a SEPARATE writing booklet.

(c) Sketch the graph of $y = x^4 - 5x^3 - 3x^2 + 17x - 10$, given that it has a root at $x = 1$ which has multiplicity 2. **5**

Label all intercepts on your graph. Do NOT use calculus and do NOT calculate the coordinates of the vertices/turning points, just roughly sketch them appropriately, according to your scale and intercepts.

Sub $y = 0$ for the x-intercepts,
$0 = x^4 - 5x^3 - 3x^2 + 17x - 10$

Let the roots be 1, 1, α and β

$1 + 1 + \alpha + \beta = 5,$
$\alpha + \beta = 3,$
$\beta = 3 - \alpha \quad [1]$

$(1)(1)\alpha\beta = -10$
$\alpha\beta = -10 \quad [2]$
Sub. [1] into [2],
$\alpha(3 - \alpha) = -10$
$\alpha^2 - 3\alpha - 10 = 0$
$(\alpha - 5)(\alpha + 2) = 0$
$\alpha = 5 \text{ or } -2$
From [1], $\beta = -2 \text{ or } 5$

$\therefore$ The roots are -2, 1 and 5.

Sub $x = 0$ for the y-intercepts,
$y = -10$

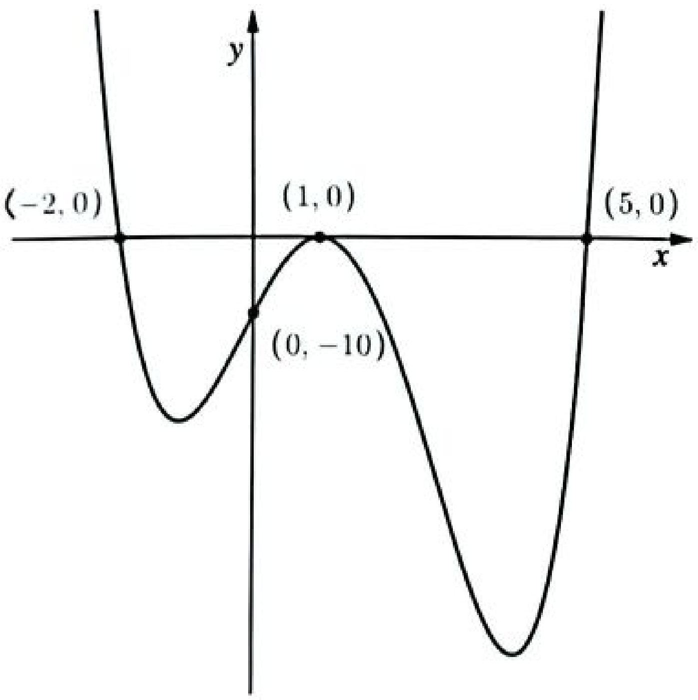
(2)

Alternative:

$(x-1)^2$ is a factor since $x=1$ is a double root.

$$\begin{array}{r|l} & x^2 - 3x - 10 \\ x^2 - 2x + 1 & x^4 - 5x^3 - 3x^2 + 17x - 10 \\ & x^4 - 2x^3 + x^2 \\ & -3x^3 - 4x^2 + 17x \\ & -3x^3 + 6x^2 - 3x \\ & -10x^2 + 20x - 10 \\ & -10x^2 + 20x - 10 \\ & 0 \end{array}$$

$y = (x-1)^2(x^2 - 3x - 10)$
$y = (x-1)(x-5)(x+2)$
For x-int, $0 = (x-1)(x-5)(x+2)$
$\therefore x = 1, 5, -2$



(3)

* Turning points on the y-axis −1

Question 13

i) $N(t) = 120e^{kt}$

at $t = 2$, $N = 270$

$$270 = 120e^{2k}$$

$$\frac{270}{120} = e^{2k}$$

$$\ln\left(\frac{270}{120}\right) = 2k$$

$$\frac{\ln\left(\frac{270}{120}\right)}{2} = k$$

$k \approx 0.405$ (3 d.p) ✓

* Done well

ii) at $t = 10$

$$N = 120e^{10k}$$

$$= 120e^{10 \times \frac{1}{2}\ln\left(\frac{270}{120}\right)}$$

$$\approx 6919.8\ldots$$

$\therefore$ 6919 or 6920 zombies ✓

* (both answers accepted)

* Do not use rounded answer from part i)

iii) $\frac{dN}{dt} = k(120e^{kt})$ ✓

at $t = 20$

* Done well

$$\frac{dN}{dt} = k(120e^{20k})$$

$= 161793.07$ ✓ zombies/day

b) i) $x^2 + 4x + y^2 - 5y - 20 = 0$

$$x^2 + 4x + 4 + y^2 - 5y + \left(\frac{5}{2}\right)^2 = 20 + 4 + \left(\frac{5}{2}\right)^2$$

(completing the square)

$$(x+2)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{121}{4}$$ ✓

$$(x+2)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{11}{2}\right)^2$$

centre of circle is $(-2, 5/2)$

* Done reasonably well
* Students lost marks because of poor algebra.
* Skipping too many lines won't get you the mark.

ii)

$$x = \frac{11\cos\theta - 4}{2} \qquad y = \frac{11\sin\theta + 5}{2}$$

$$2x = 11\cos\theta - 4 \qquad 2y = 11\sin\theta + 5$$

$$2x + 4 = 11\cos\theta \qquad 2y - 5 = 11\sin\theta$$

$$\frac{2x+4}{11} = \cos\theta \qquad \frac{2y-5}{11} = \sin\theta$$ 1

using the identity: $\sin^2\theta + \cos^2\theta = 1$ 1

$$\Rightarrow \left(\frac{2x+4}{11}\right)^2 + \left(\frac{2y-5}{11}\right)^2 = 1$$

$$\frac{(2x+4)^2}{11^2} + \frac{(2y-5)^2}{11^2} = 1$$

$$(2x+4)^2 + (2y-5)^2 = 11^2$$

$$4(x+4)^2 + 4(y-5)^2 = 11^2$$

$$(x+4)^2 + (y-5)^2 = \frac{11^2}{2^2}$$ 1

which gives us the original equation $\therefore$ The parametric equations represent where Elena is.

* Done, not well
* Major issues because of poor algebra.
* Need to mention the identity.
* Need to find the eq. of circle properly.

iii) $\theta = 135°$ $\quad \cos(135°) = -\frac{1}{\sqrt{2}}$, $\sin(135°) = \frac{1}{\sqrt{2}}$

$$x = \frac{\frac{-11}{\sqrt{2}} - 4}{2\sqrt{2}} \qquad y = \frac{\frac{11}{\sqrt{2}} + 5}{2\sqrt{2}}$$

$$= \frac{-11 - 4\sqrt{2}}{2\sqrt{2}} \begin{matrix} \times\sqrt{2} \\ \times\sqrt{2} \end{matrix} \qquad = \frac{11 + 5\sqrt{2}}{2\sqrt{2}} \begin{matrix} \times\sqrt{2} \\ \times\sqrt{2} \end{matrix}$$

$$= \frac{-11\sqrt{2} - 8}{4} \qquad = \frac{11\sqrt{2} + 10}{4}$$

$$\therefore \text{ Position } \left(\frac{-11\sqrt{2}-8}{4} , \frac{11\sqrt{2}+10}{4} \right)$$

* Done poorly
* Biggest issue was x coord: $\frac{11\sqrt{2} - 8}{\cancel{2}4}$
* ↳ This happens if using the new calculator and copying $-\frac{8 + 11\sqrt{2}}{4}$ incorrectly.

Q14. (a) Show that $\tan(\cos^{-1}(-2/3)) = -\frac{\sqrt{5}}{2}$

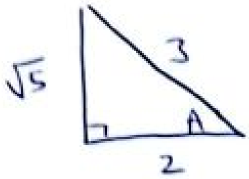
$LHS = \tan(\cos^{-1}(-2/3))$

Let $A = \cos^{-1}(-2/3)$

$\therefore \cos A = -2/3 \quad 0 \le A \le \pi$

but $\cos A < 0 \quad \therefore \frac{\pi}{2} < A < \pi$

$\therefore$ In the 2nd quadrant



so tan is negative too

$\therefore \tan A = -\frac{\sqrt{5}}{2}$

* students had to explain why it is negative for the 2nd mark.
* Q2 is not correct notation for 2nd quadrant.
* Students who only drew a triangle and found tan A, got 1 mark only

(b) (i) $T = 25 + 95e^{-kt} \implies T - 25 = 95e^{-kt}$ ... (1)

$$\frac{dT}{dt} = -k \times 95e^{-kt}$$
$$= -k(T-25) \quad (\text{as } T - 25 = 95e^{-kt})$$

(1)

* verify initial conditions

$$T = 25 + 95e^{-k(0)}$$
$$= 25 + 95e^{0}$$
$$= 25 + 95$$
$$= 120$$

(1)

$\therefore$ The initial conditions are satisfied.

A lot of students forgot to do this.

If you just wrote this, then you didn't get the mark.

(iii) when $t=4$, $T=85$, $k=??$

$$85 = 25 + 95e^{-4k}$$
$$60 = 95e^{-4k}$$
$$60/95 = e^{-4k}$$
$$-4k = \ln\left(\frac{60}{95}\right)$$
$$-4k = \ln\left(12/19\right)$$
$$k = -\frac{1}{4}\ln\left(12/19\right) \quad \text{......} \quad 1$$

when $T=40$, $t=?$

$$40 = 25 + 95e^{-kt}$$
$$15 = 95e^{-kt}$$
$$-kt = \ln\left(\frac{15}{95}\right)$$
$$t = -\frac{1}{k}\ln\left(3/19\right)$$
$$= \frac{4\ln(3/19)}{\ln(12/19)} \quad 1$$
$$= 16.06 \text{ minutes}$$

but Gordan needs to wait for the temperature to be under 40°C according to the safety warnings, So he must 17 minutes, as at 16 minutes the temperature is more than 40°C. 1

(c) $\cos 7x - \cos 3x = \sin 5x \qquad 0 \le x \le \pi$

$\cos 3x - \cos 7x = -\sin 5x$ 1

$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$

$\cos(5x - 2x) - \cos(5x + 2x) = -\sin 5x$

$2\sin 5x . \sin 2x = -\sin 5x$

$2\sin 5x . \sin 2x + \sin 5x = 0$ 1

$\sin 5x(2\sin 2x + 1) = 0$

$\sin 5x = 0 \quad$ or $\quad \sin 2x = -\frac{1}{2}$

$5x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi,$

$x = 0, \pi/5, 2\pi/5, \frac{3\pi}{5}, \frac{4\pi}{5}, \frac{5\pi}{5},$

$= 0, \pi/5, 2\pi/5, \frac{3\pi}{5}, 4\pi/5, \pi$ 1

or// $\sin 2x = -1/2$

$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}$ 1

* students who divided by $\sin 5x$, could only get a maximum of 3 marks, but most only received 2 marks as they went from ... 1 straight to their answers.