Name:…………………….. Teacher……………………… Assessment No. ……………

PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

Mathematics Extension 1

Year 11 Yearly Examination, 2022

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- The HSC Reference Sheet will be provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations

	Marks
Section I: Multiple Choice	**/10**
Section II: Question 11	**/15**
Section II: Question 12	**/14**
Section II: Question 13	**/16**
Section II: Question 14	**/15**
Total	**/70**

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Assessment Number.............................

Section I: Multiple Choice (10 Marks)

Answer questions 1 - 10 on the **Multiple-choice Answer shee**t provided.
Allow about 15 minutes for this section.

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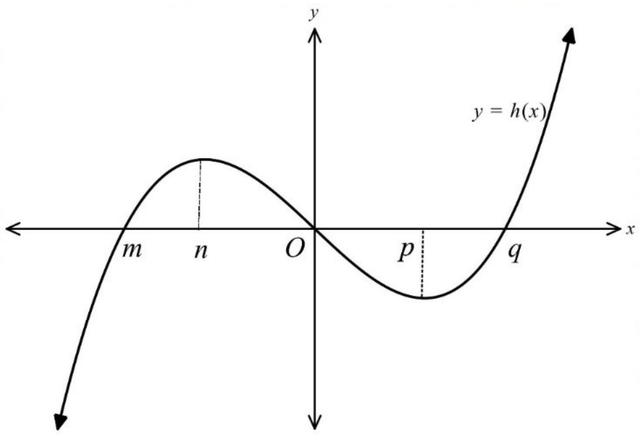
1. The graph of a cubic function $y = h(x)$ is shown below. The function has x-intercepts at $x = m$, $x = 0$ and $x = q$, and has turning points where $x = n$ and $x = p$. **1**



Which of the following restrictions on the domain would allow the function $y = h(x)$ to have an inverse function $y = h^{-1}(x)$?

(A) $[m, 0]$

(B) $[m, q]$

(C) $[0, \infty)$

(D) $[n, p]$

2. What is the highest degree of the remainder of a sixth degree polynomial when it is divided by a fourth degree polynomial? **1**

(A) zero

(B) one

(C) two

(D) three

3. Which of the following is a true statement? **1**

(A) $\sin 3x \sin 4y = \frac{1}{2}\left[\cos(3x-4y)-\cos(3x+4y)\right]$

(B) $\sin 3x \sin 4y = \frac{1}{2}\left[\cos(3x+4y)-\sin(3x-4y)\right]$

(C) $\sin 3x \sin 4y = \frac{1}{2}\left[\cos(3x+4y)-\cos(3x-4y)\right]$

(D) $\sin 3x \sin 4y = \frac{1}{2}\left[\cos(3x-4y)-\sin(3x+4y)\right]$

4. Which one of the following is the inverse function of $f(x) = x^2 - 4x + 2$, for $x \geq 2$? **1**

(A) $f^{-1}(x) = 2 + \sqrt{x+2}$

(B) $f^{-1}(x) = 2 - \sqrt{x+2}$

(C) $f^{-1}(x) = 2 + \sqrt{x-2}$

(D) $f^{-1}(x) = 2 - \sqrt{x-2}$

5. Let α and β be the roots of the polynomial $P(x) = 3x^2 - x + 2$. What is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$? **1**

(A) $-\frac{2}{3}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

6. If $A+B=45°$ and $\tan A=2$, what is the value of $\tan B$? **1**

(A) $-\frac{1}{2}$

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

7. If $\sin\theta=\frac{1}{\sqrt{3}}$, and $90°<\theta<180°$, what is the exact value of $\sin 2\theta$? **1**

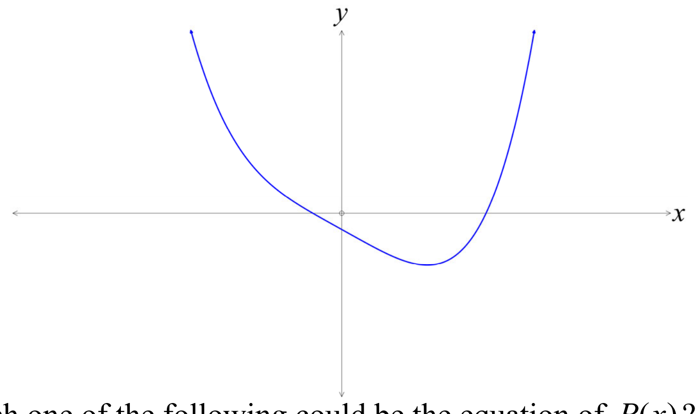
(A) $-\frac{2\sqrt{2}}{3}$

(B) $-\frac{2\sqrt{3}}{3}$

(C) $\frac{2\sqrt{2}}{3}$

(D) $\frac{2\sqrt{3}}{3}$

8. The graph of polynomial $y=P(x)$ is given below. **1**



Which one of the following could be the equation of $P(x)$?

(A) $P(x)=x^3-x-1$

(B) $P(x)=x^4-x-1$

(C) $P(x)=x^6-x+1$

(D) $P(x)=-x^8-x-1$

9. $P(x) = x^3 + 3x^2 - kx + 4$ has a remainder of k when divided by $x - 2$. What is the value of k? **1**

(A) -1

(B) 2

(C) 5

(D) 8

10. What is the domain of the function $y = 3\sin^{-1}(x)$? **1**

(A) $-\frac{1}{3} \le x \le \frac{1}{3}$

(B) $0 \le x \le \frac{1}{3}$

(C) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(D) $-1 \le x \le 1$

END OF SECTION I

Section II (60 Marks)
Start a new writing booklet for each question.

Attempt Questions 11 - 14.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculation.
Allow about 1 hour and 45 minutes for this section.

Question 11 (15 marks) **Start a new writing booklet.**

(a) Find the exact value of $\cos 75° \sin 45°$. **3**

(b) Solve the inequality $|x-4| \geq 2$. **2**

(c) Show that $\dfrac{1}{1+\sqrt{2}\sin x} + \dfrac{1}{1-\sqrt{2}\sin x} = 2\sec 2x$. **3**

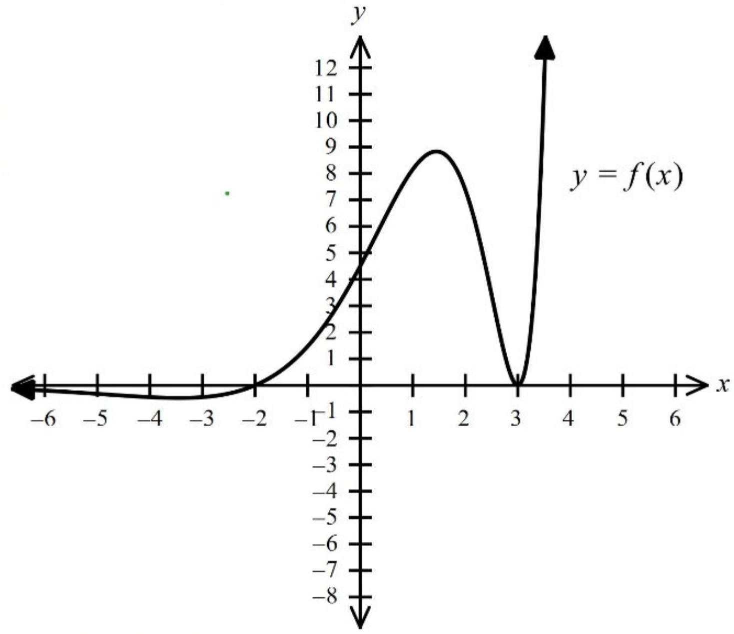
(d) If $t = \tan\dfrac{x}{2}$, show that $2\cos x - 5\sin x = -2\left(\dfrac{t^2+5t-1}{t^2+1}\right)$. **2**

(e) Solve $3x^2 - 8x - 7 \geq 2x^2 - 2x + 9$. **2**

This question is continued on the next page.

(f) **Use Insert A to answer this question. Draw your graphs directly on the insert, then put the insert inside your Question 11 writing booklet.**

The graph of $y = f(x)$ is shown below.



(i) Draw a sketch of $y = |(f(x))|$. Clearly indicate on your sketch all key values. **Use Insert A.** **1**

(ii) Draw a sketch of $y = \frac{1}{f(x)}$. Clearly indicate on your sketch all key values and possible asymptotes. **Use Insert A.** **2**

END OF QUESTION 11

Question 12 (14 marks) **Start a new writing booklet.**

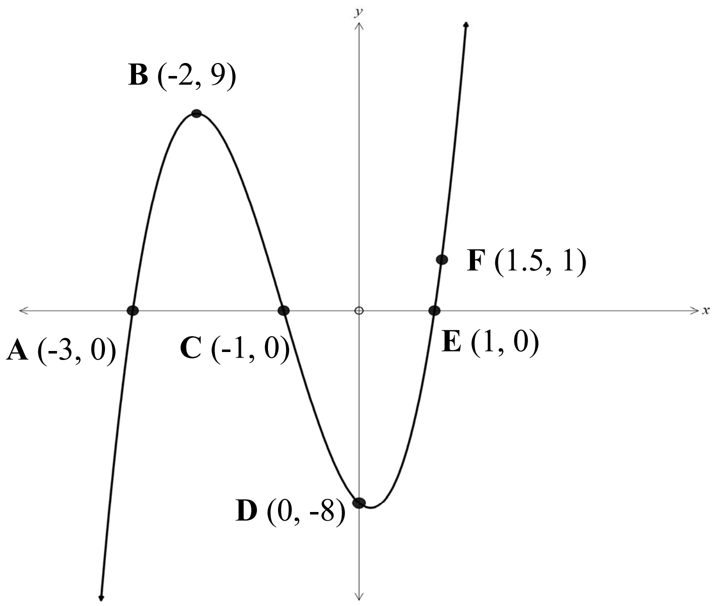
(a) A particle is moving along the x-axis such that its displacement (x metres) from a fixed-point O after t seconds is given by $x = 2t^3 - 12t^2$.

(i) Sketch the graph of displacement x against time t, for $t \geq 0$. Clearly indicate on your graph all co-ordinate intercepts. **2**

(ii) Find the velocity of the particle at 6 seconds. **1**

(b) **Use Insert B to answer this question. Draw your graph directly on the insert, then put the insert inside your Question 12 writing booklet.**

The graph below shows $y = f(x)$. **3**



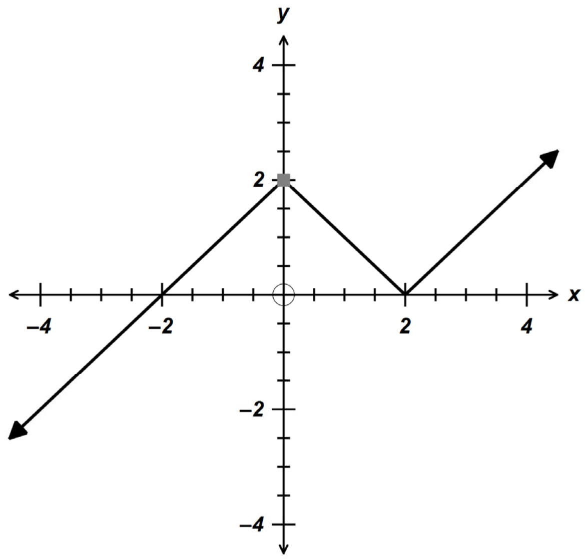
Sketch the graph of $y^2 = f(x)$ on **Insert B**. Indicate on your graph the new locations of points **A**, **B**, **C**, **D**, **E**, **F**, clearly stating their coordinates, where appropriate.

This question is continued on the next page.

(c) Express the polynomial $P(x) = x^3 + 4x^2 - 11x - 30$ as the product of three linear factors. **3**

(d) **Use Insert C to answer this question. Draw your graph directly on the insert, then put the insert inside your Question 12 writing booklet.**

The graph below shows $y = f(x)$ where $f(x) = \begin{bmatrix} x+2 \; for \; x<0 \\ |x-2| \; for \; x \geq 0 \end{bmatrix}$.



(i) Sketch the graph of $y = f(-x)$. Clearly indicate all intercepts on the co-ordinate axes. **Use Insert C.** **2**

(ii) Sketch the graph of $y = f(x) + f(-x) + 1$, carefully showing any intercepts on the co-ordinate axes. **Use Insert C.** **3**

END OF QUESTION 12

Question 13 (16 marks) **Start a new writing booklet.**

(a) Show that $\sec(\tan^{-1} p) = \sqrt{1+p^2}$. **2**

(b) Solve $P(x) = x^4 - 3x^3 - 6x^2 + 28x - 24 = 0$, given that $P(x) = 0$ has a root of multiplicity 3. **3**

(c) The polynomial equation $2x^3 - 4x^2 + 7x - 9 = 0$ has roots α, β and γ. Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **2**

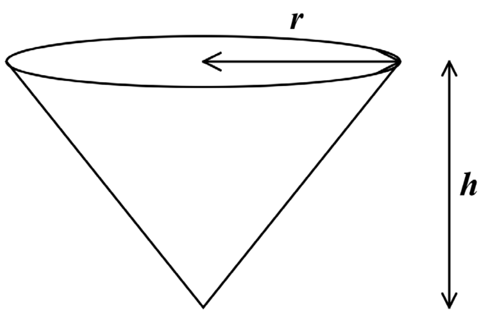
(d) A metal is heated to a temperature of 180° and placed in a room to cool. The temperature of the room is a constant 20°. The temperature of the metal, *T*, changes such that $\frac{dT}{dt} = -k\left(T - 20\right)$, where *k* is a constant and *t* is the time in minutes after the metal has been placed in the room.

(i) Show that $T = 20 + Ae^{-kt}$, where *A* is a constant, satisfies $\frac{dT}{dt} = -k\left(T - 20\right)$. **1**

(ii) The metal is initially cooling at a rate of 6° per minute. Show that $k = 0.0375$. **2**

This question is continued on the next page.

(e) A conical paper cup is equal in radius and height, where $r = h$. The cup is being filled with water so that the water level rises at a rate of 2cm/sec. Find the rate that water is being poured into the cup in cm^3/sec when the water level is 2cm. **3**



(f) A curve is defined by the parametric equations:

$$x = 3\cos(t) + 4 \text{ and } y = 3\sin(t) - 5$$

(i) The point $P(4, -2)$ lies on the curve. Find the value of t within the domain $[0,\ 2\pi]$ which corresponds to the point P. **1**

(ii) Find the Cartesian equation which describes the curve. **2**

END OF QUESTION 13

Question 14 (15 marks) **Start a new writing booklet.**

(a) The piecewise function $f(x)$ is defined below. **2**

$$f(x)\begin{cases} x-1, & x \le 0 \\ x+1, & 0 < x \le 3 \\ x+10 & x > 3 \end{cases}$$

Find the domain of $f^{-1}(x)$.

(b) Solve $\dfrac{x^2-2}{x-2} \ge 1$. **3**

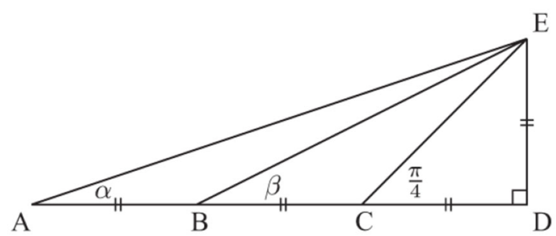
(c) Consider the function $f\left(x\right) = \cos^{-1}\left(x-1\right) + 2\sin^{-1}\left(x-2\right)$.

(i) Find the exact value of k given $f\left(1\right) = k\pi$. **1**

(ii) Find the domain of $f\left(x\right)$. **2**

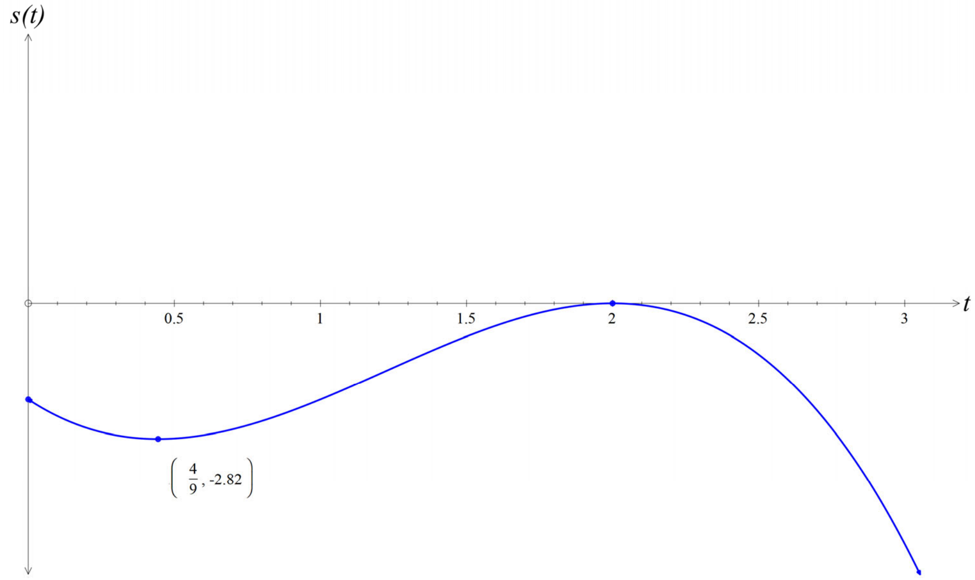
(d) Find the exact value of $\sin\left(\arctan\dfrac{4}{3} + 2\arctan\dfrac{1}{2}\right)$. Show all working. **3**

(e) Prove that, in the diagram below, $\alpha + \beta = \dfrac{\pi}{4}$. **2**



This question is continued on the next page.

(f) The displacement s(t) of a particle from the origin at a time t seconds is given by the graph below. Sketch the velocity function of the particle, clearly indicating all key values. **2**



END OF EXAM

Y11 Extension 1 Yearly Exam 2022 Term 3 – Solutions

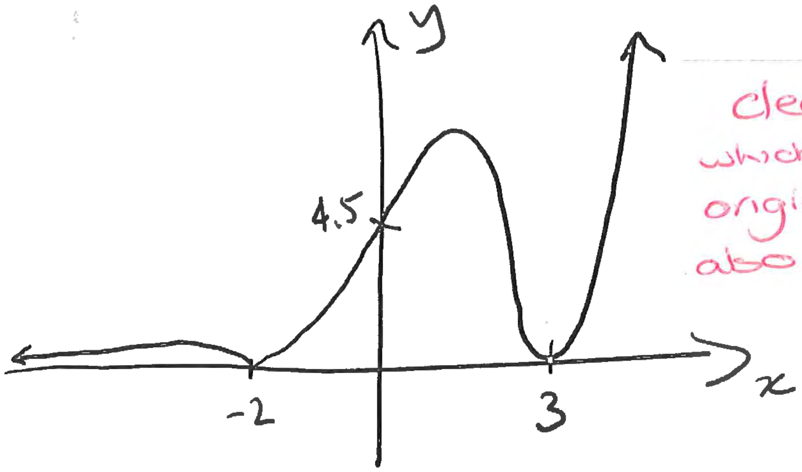
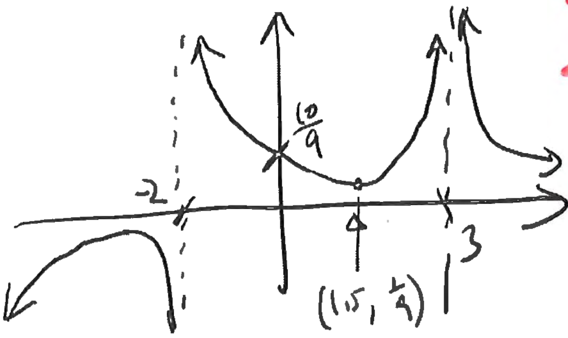
Section I – Multiple Choice

1	**D** As that is the only section where the function is monotonically decreasing.	
2	**D** Because if the remainder is of degree 3, it can no longer be divided by a divisor of degree 4.	
3	**A** By definition. Refer to the reference sheet.	
4	**A** $y=(x-2)^2-4+2$ $y+2=(x-2)^2$ $x-2=\sqrt{y+2}$ $x=2+\sqrt{y+2}$ $f^{-1}(x)=2+\sqrt{x+2}$	
5	**C** $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $\alpha+\beta=\frac{1}{2}$ $\alpha\beta=\frac{2}{3}$ $\therefore \frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$	
6	**B** $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ $\tan 45°=\frac{2+\tan B}{1-2\tan B}$ $1=\frac{2+\tan B}{1-2\tan B}$ $\tan B=-\frac{1}{3}$	

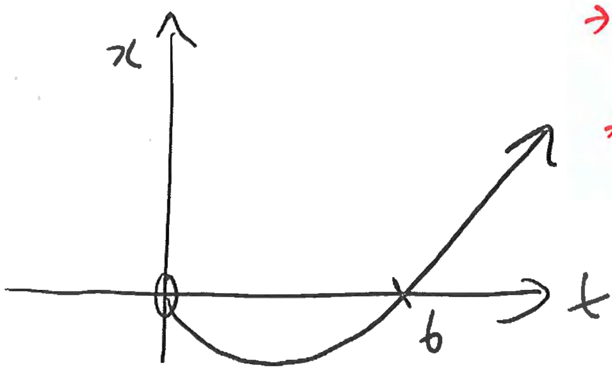
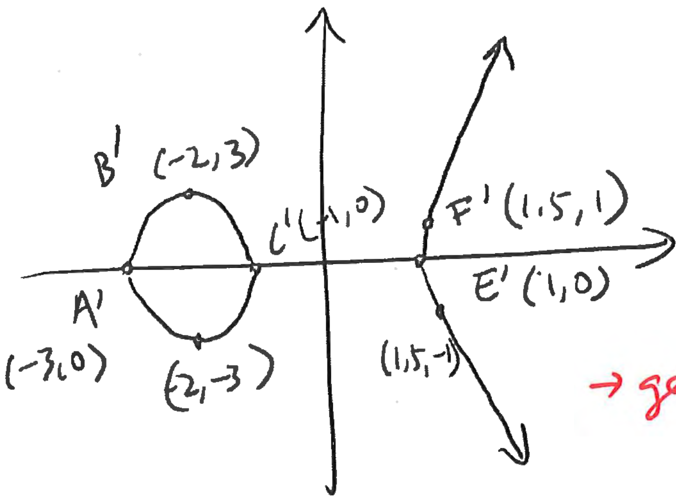
7	A $\because \sin\theta = \frac{1}{\sqrt{3}}, 90° < \theta < 180°,$ $\cos\theta = -\frac{\sqrt{2}}{\sqrt{3}}$ $\sin 2\theta = 2\cos\theta\sin\theta$ $= 2\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{\sqrt{2}}{\sqrt{3}}\right)$ $= -\frac{2\sqrt{2}}{3}$	
8	B As the power of the polynomial must be even, the leading coefficient must be positive, and the y-intercept must be negative.	
9	D $P(2) = (2)^3 + 3(2)^2 - k(2) + 4 = k$ $k = 24 - 3k$ $k = 8$	
10	D As the domain of $\sin^{-1}(x)$ is $-1 \le x \le 1$. The 3 stretches the graph vertically by a factor three and will affect the range of the function, but not the domain.	

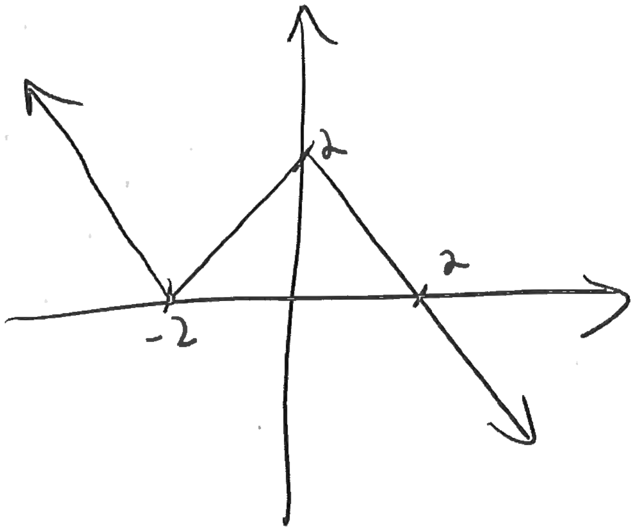
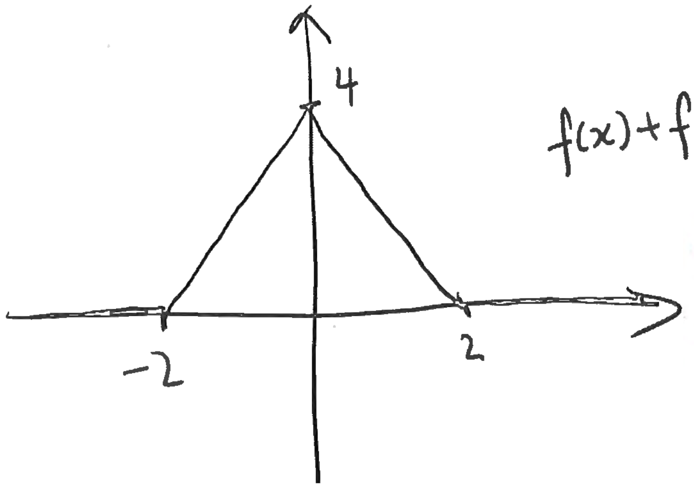
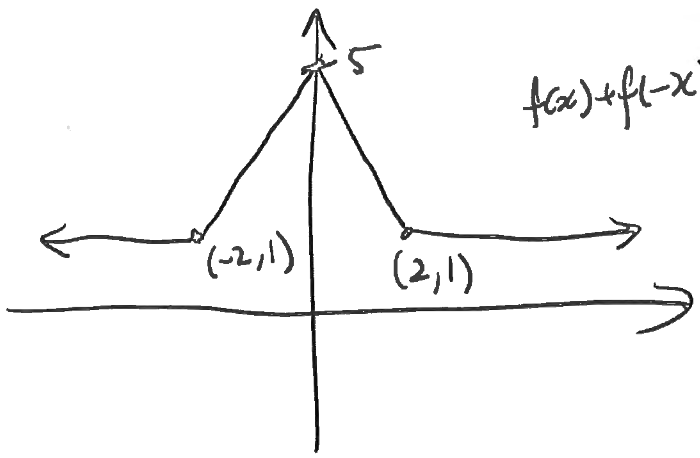
Section II- Question 11

(a)	$\cos 75° \sin 45° = \cos(45° + 30°) \times \frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{2}}(\cos 45° \cos 30° - \sin 45° \sin 30°)$ $= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$ $= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ $= \frac{\sqrt{3}-1}{4}$ Method 2. $\cos 75° \sin 45°$ $= \frac{1}{2}[\sin(75°+45°) - \sin(75°-45°)]$ refer to triangles on reference sheet for exact values.	3 marks
(b)	$\|x-4\| = 2$ $x - 4 = 2, x = 6$ $x - 4 = -2, x = 2$ $\therefore x \le 2, x \ge 6$ → cannot be written as $6 \le x \le 2$. method 2 Solve 2 cases ① $x - 4 = 2$ ② $-(x-4) = 2$.	2 marks
(c)	$\frac{1}{1+\sqrt{2}\sin x} + \frac{1}{1-\sqrt{2}\sin x} = \frac{(1-\sqrt{2}\sin x)+(1+\sqrt{2}\sin x)}{1-2\sin^2 x}$ $= \frac{2}{1-2(1-\cos^2 x)}$ $= \frac{2}{1-2+2\cos^2 x}$ $= \frac{2}{2\cos^2 x - 1}$ $= \frac{2}{\cos 2x}$ $= 2\sec 2x$ recognise denominator is difference of 2 squares Consider the number of marks allocated as an indication of amount of working to write.	3 marks
(d)	$2\cos x - 5\sin x = 2\left(\frac{1-t^2}{1+t^2}\right) - 5\left(\frac{2t}{1+t^2}\right)$ $= \frac{2-2t^2-10t}{1+t^2}$ $= -2\left(\frac{t^2+5t-1}{1+t^2}\right)$ Use reference sheet.	2 marks
(e)	$3x^2 - 8x - 7 \ge 2x^2 - 2x + 9$ $x^2 - 6x - 16 \ge 0$ $(x+2)(x-8) \ge 0$ $x \le -2, x \ge 8$ test point or draw parabola	2 marks

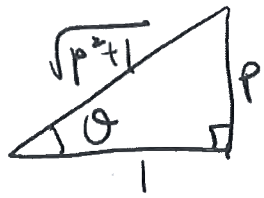
(f)	(i) 	1 marks Clearly indicate which parts of the original graph are also in new graph.
	(ii) 	2 marks • show new y intercepts • points of intersection of graphs • asymptotes.

Question 12

(a)	**(i)** $x=2t^2(t-6)$  → time t can not be negative. → Otherwise generally well Done	2 marks
	(ii) $v=\dfrac{dx}{dt}=6t^2-24t$ $v=6(6)^2-24(6)$ $=72ms^{-1}$ → Generally well Done	1 mark
(b)	 → generally well Done.	3 marks
(c)	$P(-2)=(-2)^3+4(-2)^2-11(-2)-30=0$ $\therefore P(x)=(x+2)(x^2+2x-15)$ $=(x+2)(x+5)(x-3)$ → Generally well Done. → Some students used long division → very time consuming!	3 marks

(d)	**(i)**  → Generally well Done	2 marks
	(ii)  $f(x)+f(-x)$  $f(x)+f(-x)+1$	3 marks → Poorly Done. → Use table of value for the y-values of $f(x)$ and $f(-x)$ to figure the new y-value out.

Question 13

(a)	Let $\theta=\tan^{-1}p$  $\sec\theta=\dfrac{1}{\cos\theta}$ $\because \cos\theta=\dfrac{1}{\sqrt{1+p^2}}$ $\therefore \sec\theta=\sqrt{1+p^2}$ You need to include a diagram will all the information to support your answer.	2 marks
(b)	$P'(x)=4x^3-9x^2-12x+28$ $P''(x)=12x^2-18x-12$ $0=12x^2-18x-12$ $0=6\left(2x^2-3x-2\right)$ $0=(2x+1)(x-2)$ $x=-\dfrac{1}{2},x=2$ $P(2)=0$ $\therefore x^4-3x^3-6x^2+28x-24=(x-2)^3(x+3)$ $x=2,x=-3$ Generally well done. Long division is another method. You need to solve, so don't stop in the factorised form	3 marks
(c)	$(\alpha+1)(\beta+1)(\gamma+1)$ $=(\alpha\beta+\alpha+\beta+1)(\gamma+1)$ $=\alpha\beta\gamma+\alpha\beta+\alpha\gamma+\alpha+\beta\gamma+\beta+\gamma+1$ $=(\alpha\beta\gamma)+(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)+1$ $=\left(\dfrac{9}{2}\right)+\left(\dfrac{7}{2}\right)+(2)+1$ $=11$ Generally well done. Some mistakes in expanding.	2 marks
(d)	**(i)** $T=20+Ae^{-kt}$ $\dfrac{dT}{dt}=-kAe^{-kt}$ $=-k\left(Ae^{-kt}+20-20\right)$ $=-k(T-20)$ It's a show question so you need to include all the necessary steps.	1 mark

	(ii) $-6=-k(180-20)$ $-6=-k\times 160$ $k=\frac{6}{160}=0.0375$ Most missed that $\frac{dT}{dt}=-6$. Some used average rate of change, not instantaneous.	2 marks	
(e)	$V=\frac{\pi r^2 h}{3}\ \because r=h, V=\frac{\pi r^3}{3}$ $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $=\pi r^2\times 2$ $=\pi(2)^2\times 2$ $=25.1\text{cm}^3/\text{sec}$ It was okay. Some used the wrong formula. Show all steps.	3 marks	
(f)	**(i)** $4=3\cos t+4$ $0=\cos t, t=\frac{\pi}{2}, t=\frac{3\pi}{2}$ $-2=3\sin t-5$ $1=\sin t, t=\frac{\pi}{2}$ $\therefore t=\frac{\pi}{2}$ That produces 2 solutions in the given domain, you need to give them both. Sin t then limits the answer to one.	1 mark	
	(ii) $\cos t=\frac{x-4}{3},\ \sin t=\frac{y+5}{3}$ $\sin^2 t+\cos^2 t=1$ $\left(\frac{x-4}{3}\right)^2+\left(\frac{y+5}{3}\right)^2=1$ $(x-4)^2+(y+5)^2=9$ It was generally well done. Some forgot to square 3.	2 marks	

Question 14

(a)	Range for f(x) is: For $x \le 0, y \in (-\infty, -1]$ For $0 < x \le 3, y \in (1, 4]$ For $x > 3, y \in (13, \infty)$ $\therefore$ for $f^{-1}(x)$, $x \in (-\infty, -1] \cup (1, 4] \cup (13, \infty)$ Draw the piecewise graph. Range of function is domain of inverse. Ensure it is easy to see difference between round and square brackets. $x \le -1, \; 1 < x \le 4, \; x > 13$	2 marks
(b)	$\dfrac{x^2-2}{x-2} = 1$ Key values: $x = 2$ $x^2 - 2 = x - 2$ $x^2 - x = 0$ $x(x-1) = 0 \therefore x = 0, x = 1$ Testing values: $x = -2$: $\dfrac{(-2)^2 - 2}{-2-2} = -\dfrac{1}{2} \ge 1$ is false $x = 0.5$ $\dfrac{(0.5)^2 - 2}{0.5-2} = \dfrac{7}{6} \ge 1$ is true $x = 1.5$ $\dfrac{(1.5)^2 - 2}{1.5-2} = -\dfrac{1}{2} \ge 1$ is false $x = 5$ $\dfrac{(5)^2 - 2}{5-2} = \dfrac{23}{3} \ge 1$ is true $\therefore 0 \le x \le 1, x > 2$ **METHOD 2**	3 marks

	$(x^2-2)(x-2)\geq(x-2)^2$ Keep in factorised form. $(x^2-2)(x-2)-(x-2)^2\geq 0$ Do not expand. $(x-2)\left[(x^2-2)-(x-2)\right]\geq 0$ $(x-2)(x^2-x)\geq 0$ test values or draw cubic. $x(x-2)(x-1)\geq 0$ do not divide by x as the solution $x=0$ is then lost. $\therefore 0\leq x\leq 1, x>2$	
(c)	**(i)** $f(1)=\cos^{-1}(0)+2\sin^{-1}(-1)$ $\sin^{-1}(-x)=-\sin^{-1}(x)$ $=\frac{\pi}{2}+2\left(-\frac{\pi}{2}\right)$ or consider range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $=-\frac{\pi}{2}\quad\therefore k=-\frac{1}{2}$	1 mark
	(ii) Domain of $\cos^{-1}(x-1)$ is $-1\leq x\leq 1$ $\therefore -1\leq x-1\leq 1,\ 0\leq x\leq 2$ Domain of $\sin^{-1}(x-2)$ is $-1\leq x\leq 1$ $\therefore -1\leq x-2\leq 1,\ 1\leq x\leq 3$ $\therefore x\in[1,2]$ overlap of domains.	2 marks

<table>
<tr><td>(d)</td><td>

Let $\alpha = \arctan\frac{4}{3}$ and $\beta = \arctan\frac{1}{2}$

Recommended to draw triangles

$\frac{4}{3} = \tan\alpha,\ \sin\alpha = \frac{4}{5}, \cos\alpha = \frac{3}{5}$

$\frac{1}{2} = \tan\beta,\ \sin\beta = \frac{1}{\sqrt{5}}, \cos\beta = \frac{2}{\sqrt{5}}$

$\therefore \sin\left(\arctan\frac{4}{3} + 2\arctan\frac{1}{2}\right)$

$= \sin(\alpha + 2\beta)$

note correct algebra.

$= \sin\alpha\cos 2\beta + \cos\alpha\sin 2\beta$

use reference sheet for correct expansion

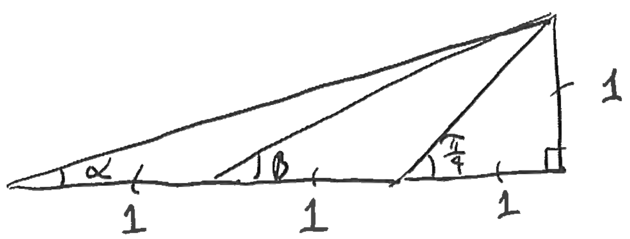
$= \left(\frac{4}{5}\right)(2\cos^2\beta - 1) + \left(\frac{3}{5}\right)(2\sin\beta\cos\beta)$

$= \left(\frac{4}{5}\right)\left(2\times\left(\frac{2}{\sqrt{5}}\right)^2 - 1\right) + \left(\frac{3}{5}\right)\left(2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}\right)$

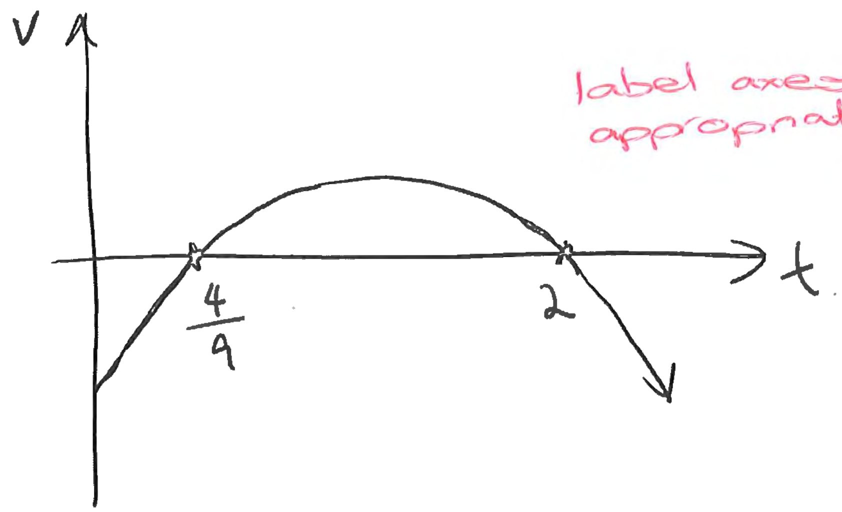
$= \left(\frac{4}{5}\times\frac{3}{5}\right) + \left(\frac{3}{5}\times\frac{4}{5}\right)$

$= \frac{24}{25}$

</td><td>3 marks</td></tr>
<tr><td>(e)</td><td>



</td><td>2 marks</td></tr>
</table>

	$\tan\beta = \frac{1}{2}, \tan\alpha = \frac{1}{3}$ $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ $= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$ $= \frac{5}{6} \div \frac{5}{6}$ $= 1$ $\therefore \alpha+\beta = \tan^{-1}(1) = \frac{\pi}{4}$ show question so write full solution.	
(f)	 label axes appropriately.	2 marks