Name: ……………………… Teacher: ………………… Assessment No. …………………

PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

Mathematics Extension 1

Year 11 Yearly Examination, 2023

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- The HSC Reference Sheet will be provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

	Marks
Section I: Multiple Choice	**/10**
Section II: Question 11	**/15**
Section II: Question 12	**/15**
Section II: Question 13	**/15**
Section II: Question 14	**/15**
Total	**/70**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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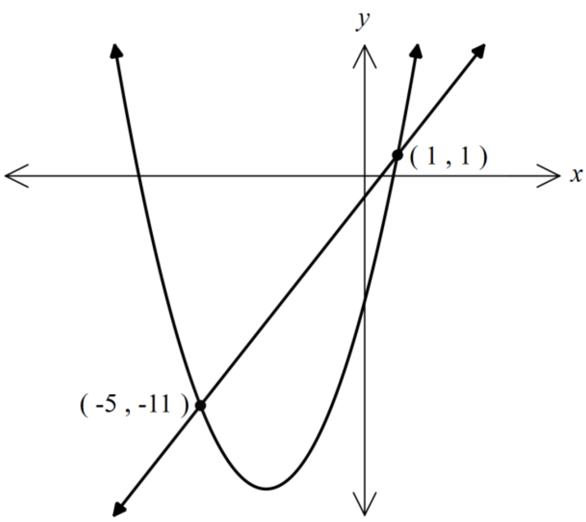
1. The function, $f(x)$, is one-to-one and has domain $[0,5)$ and range $(-2,5]$. **1**

Which of the following correctly describes the domain and range of its inverse function, $f^{-1}(x)$?

	Domain	Range
(A)	$(-2,5]$	$[0,5)$
(B)	$[-2,5)$	$(0,5]$
(C)	$[0,5)$	$(-2,5]$
(D)	$(0,5]$	$[-2,5)$

2. The diagram below shows $y = x^2 + 6x - 6$ and $y = 2x - 1$. **1**



Using the graph, or otherwise, for what values of x is $x^2 + 4x - 5 \leq 0$?

(A) $[-11,1]$

(B) $(-\infty, -11] \cup [1, \infty)$

(C) $[-5,1]$

(D) $(-\infty, -5] \cup [1, \infty)$

3. What is the remainder when $x^4 - 3x^3 + x$ is divided by $(x + 1)$? **1**

(A) -5

(B) -1

(C) 1

(D) 3

4. Solve **1**

$$|x + 1| < 2$$

(A) $-3 < x < 1$

(B) $x < -3$ or $x > 1$

(C) $-1 < x < 3$

(D) $x < -1$ or $x > 3$

5. There are 500 students in a school. **1**

Which of the following MUST be correct?

(A) There is at least 1 month where at least 41 students have their birthday.

(B) There are at least 2 months where at least 41 students have their birthday.

(C) There is at least 1 month where at least 42 students have their birthday.

(D) There are at least 2 months where at least 42 students have their birthday.

6. What is the coefficient of the term containing x^2 in the expansion of $\left(x - \frac{1}{x}\right)^8$? **1**

(A) $\binom{8}{2}$

(B) $-\binom{8}{2}$

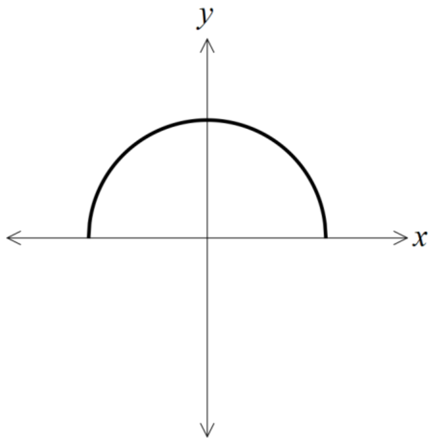
(C) $\binom{8}{5}$

(D) $-\binom{8}{5}$

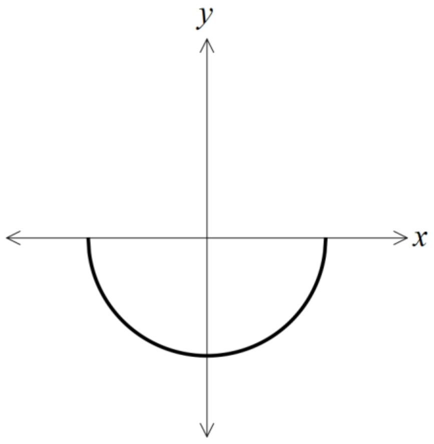
7. A curve on the xy-plane is defined parametrically by $x = \sin\theta$ and $y = \cos\theta$. **1**

Which of the following best represents the curve for $0 \leq \theta \leq \pi$?

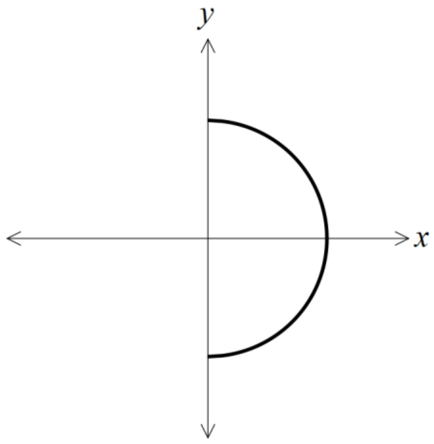
(A)



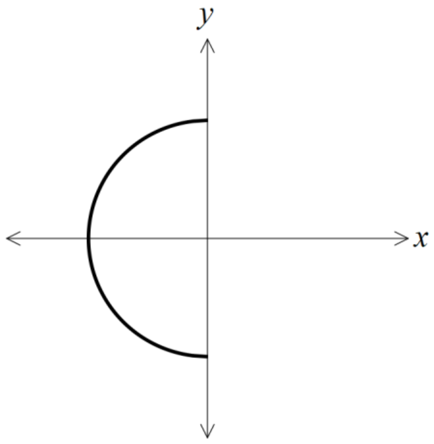
(B)



(C)



(D)



8. Let $P(x) = ax^3 + bx^2 + bx + a$ where a and b are constants and $a \neq 0$. **1**

The three distinct zeroes are -1, α and β.

Which of the following is a true statement?

(A) $\beta = -\dfrac{1}{\alpha}$

(B) $\beta = \dfrac{1}{\alpha}$

(C) $\beta = \alpha - 1$

(D) $\beta = 1 - \alpha$

9. In the school playground, 2 girls and 4 boys are seated in a circle. How many arrangements are possible if the girls *cannot* sit next to each other? **1**

(A) 18

(B) 24

(C) 60

(D) 72

10. The polynomial $P(x)$ has degree 5 and $P(x)$ is an odd function. **1**

Which of the following *could* be true?

I: The sum of the real roots of $P(x)$ is positive.
II: $P(x) = 0$ has one distinct solution.
III: $P(x)$ has a double root.

(A) I and II

(B) II only

(C) II and III

(D) I and III

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Start a new writing booklet for each question. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) Suppose $P(x) = x^3 - 4x^2 + x + 6$.

(i) Show that $(x + 1)$ is a factor of $P(x)$. **1**

(ii) Hence, write $P(x)$ as a product of linear factors. **2**

(b) A particle's displacement in metres after t seconds is given by $x = t^2 - 4t + 3$.

(i) Show that the particle is at rest after 2 seconds. **2**

(ii) Sketch the particle's displacement-time graph. **1**

(iii) What is the total distance travelled by the particle during the first 5 seconds? **1**

(c) Solve $\frac{x}{x+1} \leq 2$. **3**

(d) A curve in the xy-plane is defined parametrically by $x = 8t$ and $y = 4t^2$, where t is the parameter.

(i) Draw the curve for $-3 \leq t \leq 3$, showing the co-ordinates of the endpoints and any intercepts. **2**

(ii) For what value(s) of t will the curve intersect the line $y = 10 - \frac{3}{4}x$? **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet

(a) (i) In how many ways can the letters in the word CAMPING be arranged? **1**

(ii) In how many ways can the letters in the word EXAMINATION be arranged if the X must come first? **2**

(b) Simplify **2**

$$\frac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x}$$

(c) The function, $f(x) = e^{x^3}$, is one-to-one for all real values of x. **2**

Find the equation of the inverse function, $f^{-1}(x)$.

(d) A large ice cube is melting in such a way that its surface area, A, is decreasing at a rate of 6 cm^2 per minute.

(i) Show that the side length of the cube, x, decreases at a rate of $\frac{1}{2x}$ cm/min. **2**

(ii) At what rate is the side length decreasing when the ice cube's surface area is 600 cm^2? **2**

(e) Neatly sketch $y = 3\cos^{-1}\left(\frac{x}{2}\right)$ over its natural domain. **2**

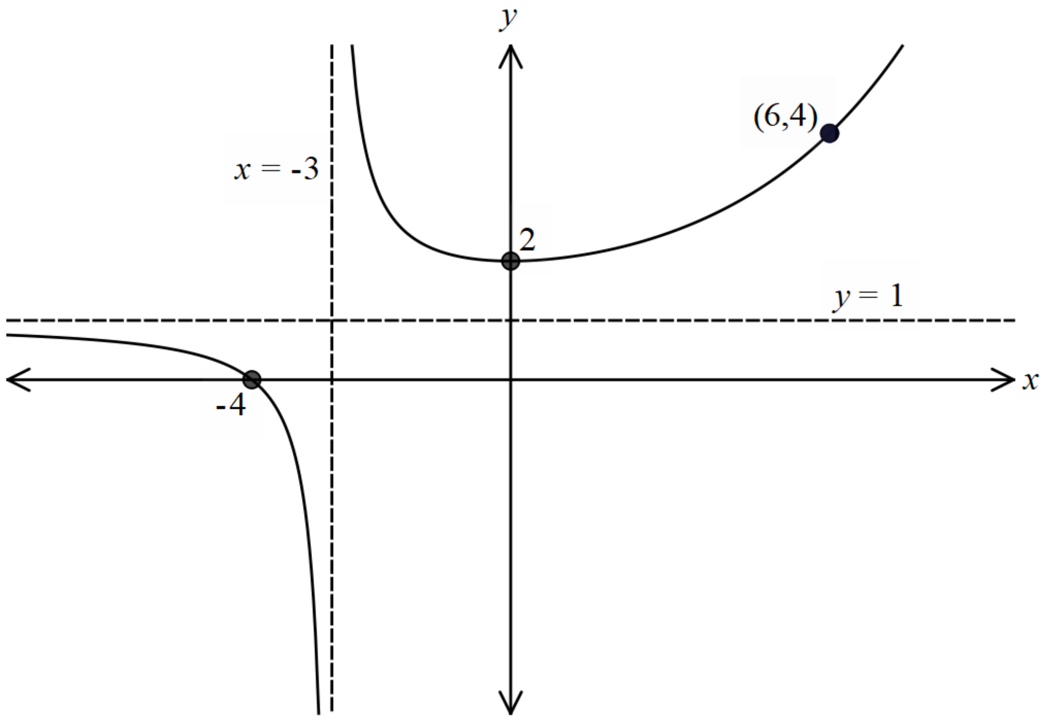
(f) The polynomial $P(x) = x^3 - kx^2 - (k+1)x$ has a double root at $x = -1$. **2**

Find the possible value(s) of k.

End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) Consider the graph of $y = f(x)$ below.



Answer parts (i) – (iii) on the **Q13a Insert** provided. Place the **Insert** inside your Question 13 writing booklet.

Sketch the following functions, showing all relevant information.

(i) $y = |f(x)|$ **1**

(ii) $y = \sqrt{f(x)}$ **2**

(iii) $y = \frac{1}{f(x)}$ **2**

(b) In a football season with sixteen teams, there are 121 games. Each game is played between two teams. **2**

Show that some pair of teams must have played each other more than once.

Question 13 continues on page 9

(c) An island is found to be contaminated with the substance strontium-90 after an accident at its nuclear power plant. The rate at which the amount in kilograms, A, of strontium-90 decays t years after the accident is given by

$$\frac{dA}{dt} = -kA$$

where k is a constant.

(i) Show that $A = A_0 e^{-kt}$ satisfies the differential equation, where A_0 is the amount of strontium-90 contaminating the island immediately after the accident. **1**

(ii) 29 years after the accident, scientists found that the amount of strontium-90 contaminating the island had halved. **2**

Show that $k = \frac{1}{29} \ln 2$.

(iii) The scientists declare that the island will become inhabitable again when the amount of strontium-90 falls below 5% of the initial amount. **2**

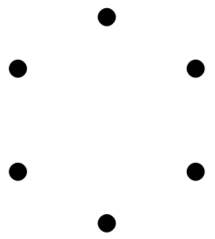
How many years after the accident will the island become inhabitable?

(d) Given that $\sin\theta = \frac{1}{4}$ and $\cos\theta < 0$, find the exact value of $\tan 2\theta$. **3**

End of Question 13

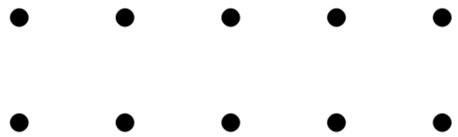
Question 14 (15 marks) Start a new writing booklet

(a) (i) Consider the dots below. **1**



Explain why ${}^6C_3$ distinct triangles can be formed using the dots as vertices.

(ii) How many distinct triangles can be formed using the dots below as vertices? **2**



(b) Show that $\sin(2\arccos x) = 2x\sqrt{1-x^2}$ for $-1 \leq x \leq 1$. **2**

(c) Joe is studying a superconductor that has been cooled to its critical temperature. Unfortunately, he does not know the critical temperature because his laser thermometer cannot measure temperatures below -50°C. **4**

Luckily, Joe knows that the superconductor will warm according to the differential equation

$$\frac{dT}{dt} = k(B - T)$$

which has a solution, $T = B - Ae^{-kt}$ (Do NOT prove this), where B is the temperature of the surrounding environment, A and k are constants and T is the temperature in °C after t minutes.

He places the superconductor, already at its critical temperature, in a 25°C laboratory. After half an hour, the laser thermometer shows that the temperature of the superconductor is -35°C. After a further half hour, its temperature is 5°C. What is the critical temperature of the superconductor?

Question 14 continues on page 11

(d) (i) If $t = \tan\theta$, show that **3**

$$\tan 4\theta = \frac{4t(1-t^2)}{1-6t^2+t^4}$$

(ii) Given that $\theta = \frac{\pi}{10}$ and $\frac{3\pi}{10}$ are the roots of $\tan 4\theta = \cot\theta$ for $0 < \theta < \frac{\pi}{2}$, find the exact value of $\tan\frac{\pi}{10}$. **3**

End of paper

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Section I

① $f(x)$ has $D: [0,5)$ and $R: (-2,5]$

So $f^{-1}(x)$ has $D: (-2,5]$ and $R: [0,5)$ $\rightarrow$ (A)

② $x^2+4x-5 \le 0$

$x^2+6x-6-2x+1 \le 0$

$x^2+6x-6 \le 2x-1$

$y=x^2+6x-6$ is below/on $y=2x-1$ for $-5 \le x \le 1$ $\rightarrow$ (C)

③ If $P(x) = x^4 - 3x^3 + x$ is divided by $(x+1)$,

by the Rem. Theorem, the remainder is

$$P(-1) = (-1)^4 - 3(-1)^3 + (-1)$$
$$= 1 + 3 - 1$$
$$= 3 \quad \rightarrow \text{(D)}$$

④ $|x+1| < 2$

$-2 < x+1 < 2$

$-3 < x < 1$ $\rightarrow$ (A)

⑤ 500 pigeons into 12 (months) holes:

$\frac{500}{12} = 41.6\ldots$ $\quad (500 = 41 \times 12 + 8)$

$\therefore$ At least one pigeonhole must contain at least 42 pigeons $\rightarrow$ (C)

⑥ $\left[x + (-x^{-1})\right]^8$ has general term:

$$^8C_r\, x^{8-r} (-x^{-1})^r = {}^8C_r\, x^{8-r} (-1)^r x^{-r}$$

$$= (-1)^r\, {}^8C_r\, x^{8-2r}$$

let $8-2r = 2 \rightarrow r = 3$:

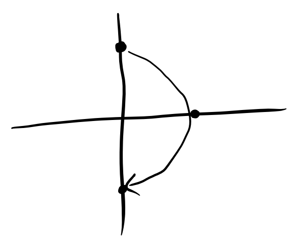
$$= (-1)^3\, {}^8C_3\, x^2$$

$$= \boxed{-{}^8C_3}\, x^2 \rightarrow \text{D}$$

⑦

θ	0	$\frac{\pi}{2}$	π
x	0	1	0
y	1	0	-1

$\downarrow$ $(0,1)$ $\quad \downarrow$ $(1,0)$ $\quad \downarrow$ $(0,-1)$



$\rightarrow$ C

⑧ $P(x) = ax^3 + bx^2 + bx + a$

let zeroes be $\alpha, \beta, -1$

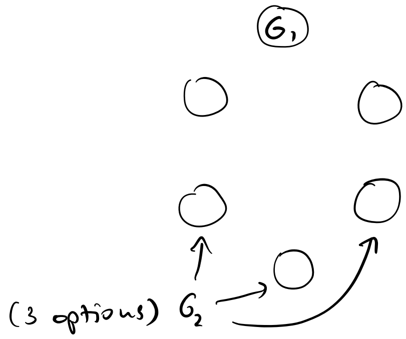
Product of zeroes: $\alpha \cdot \beta \cdot (-1) = -\frac{a}{a}$

$$-\alpha\beta = -1$$

$$\beta = \frac{1}{\alpha} \rightarrow \text{B}$$

(9) $\boxed{1} \times \boxed{3} \times \boxed{4!} = 72 \rightarrow$ (D)

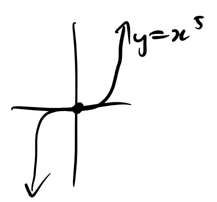
Place 1st girl | Place 2nd girl | Arrange boys



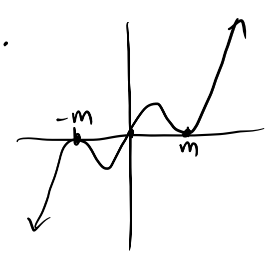
(10) ✗ I: For each positive root of $P(x)$, there must exist an opposite negative root to preserve point symmetry

$\therefore$ The sum of roots must equal <u>zero</u>.

✓ II: E.g. $x^5 = 0$ has one distinct soln, $x = 0$



✓ III: E.g.



$P(x) = x(x-m)^2(x+m)^2$

So (C)

Section II

Question 11

(a) $P(x) = x^3 - 4x^2 + x + 6$

(i) $P(-1) = (-1)^3 - 4(-1)^2 + (-1) + 6$

$= -1 - 4 - 1 + 6$

$= 0$

$\therefore$ By Factor Theorem $(x+1)$ is a factor of $P(x)$.

(ii)

$$\begin{array}{r l}
 & x^2 - 5x + 6 \\
x+1 \overline{)} & x^3 - 4x^2 + x + 6 \\
 & \underline{x^3 + x^2} \\
 & -5x^2 + x \\
 & \underline{-5x^2 - 5x} \\
 & 6x + 6 \\
 & \underline{6x + 6} \\
 & 0
\end{array}$$

$\rightarrow P(x) = (x+1)(x^2 - 5x + 6)$

$\therefore P(x) = (x+1)(x-2)(x-3)$

(b) (i) 'at rest' $\rightarrow v = 0$

$x = t^2 - 4t + 3$

$v = \frac{dx}{dt} = 2t - 4$

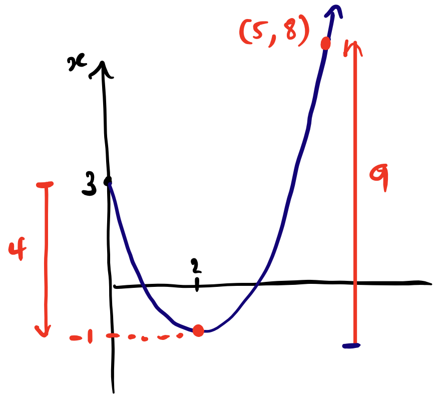
let $2t - 4 = 0$

$2t = 4$

$t = 2$

$\therefore$ The particle is at rest after 2 seconds.

(ii)



(iii) Consider endpoints and turning pts:

at $t=0$, $x=3$

$t=2$, $x = 2^2 - 4(2) + 3$

$= -1$

$t=5$, $x = 5^2 - 4(5) + 3$

$= 8$

4

9

Total distance travelled $= 4 + 9$

$= 13\text{m}$.

(c) $\dfrac{x}{x+1} \leq 2$ (Note $x \neq -1$)

$x(x+1) \leq 2(x+1)^2$ (Multiplying by $(x+1)^2$)

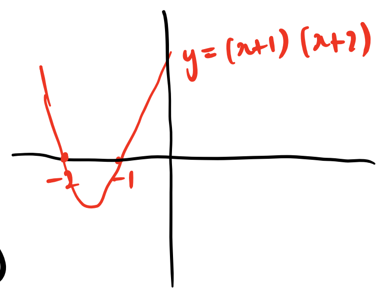
$x(x+1) - 2(x+1)^2 \leq 0$

$(x+1)[x - 2(x+1)] \leq 0$

$(x+1)(x - 2x - 2) \leq 0$

$(x+1)(-x-2) \leq 0$

$(x+1)(x+2) \geq 0$ (Dividing by -1)



$\therefore$ $x \leq -2$, $x > -1$ $(x \neq -1)$

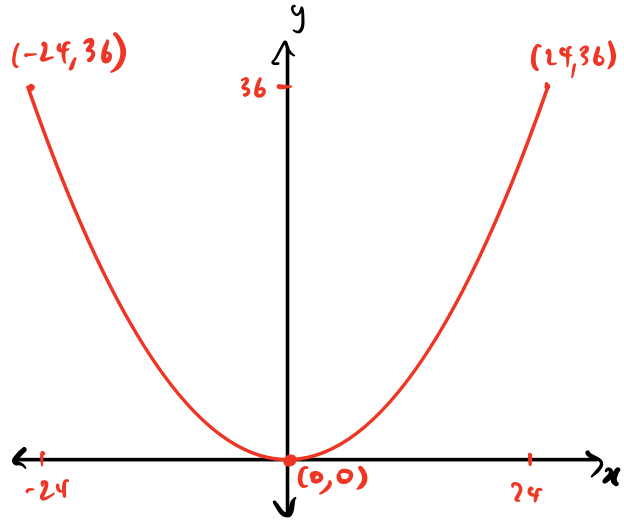
(d) $x = 8t$, $y = 4t^2$

(i) $\frac{x}{4} = 2t$, $y = (2t)^2$

$y = \left(\frac{x}{4}\right)^2$

$\therefore y = \frac{1}{16}x^2$

t	-3	0	3
x	-24	0	24
y	36	0	36



(ii) $y = 10 - \frac{3}{4}x$

Sub $x = 8t$, $y = 4t^2$:

$4t^2 = 10 - \frac{3}{4}(8t)$

$4t^2 = 10 - 6t$

$4t^2 + 6t - 10 = 0$

$2t^2 + 3t - 5 = 0$

$2t^2 - 2t + 5t - 5 = 0$

$2t(t-1) + 5(t-1) = 0$

$(t-1)(2t+5) = 0$

$t = 1, -\frac{5}{2}$

$\therefore$ $y = 6 - 2x$ and the curve intersect for $t = 1$ and $t = -\frac{5}{2}$.

Question 12

(a) (i) 7 distinct objects

$= 7!$ ways or 5040 ways

(ii)

$\boxed{1} \times \boxed{N}$

place X — arrange remaining 10.

→ 10 objects, 3 repeated pairs: A's, I's, N's

$\rightarrow N = \dfrac{10!}{2!2!2!} = 453600$

$\therefore$ There are 453600 ways.

(b)

$$\frac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x} = \frac{\cos(\overset{A}{4x} - \overset{B}{x}) - \cos(\overset{A}{4x} + \overset{B}{x})}{\sin(\overset{A}{4x} - \overset{B}{x}) + \sin(\overset{A}{4x} + \overset{B}{x})}$$

$$= \frac{\cancel{2}\cancel{\sin 4x}\sin x}{\cancel{2}\cancel{\sin 4x}\cos x} \quad \text{(from Ref. Sheet)}$$

$$= \tan x$$

(c)

$f: \ y = e^{x^3}$

$f^{-1}: \ x = e^{y^3}$

$y^3 = \ln x$

$y = \sqrt[3]{\ln x}$

$\therefore f^{-1}(x) = \sqrt[3]{\ln x}$

(d) (i) $\frac{dA}{dt} = -6$

$A = 6x^2$ (6 square faces)

$\frac{dA}{dx} = 12x$

$\frac{dx}{dt} = \frac{dx}{dA} \times \frac{dA}{dt}$

$= \frac{1}{12x} \times -6$

$\therefore \frac{dx}{dt} = -\frac{1}{2x} \longrightarrow$ ∴ The side length is decreasing at a rate of $\frac{1}{2x}$ cm/min.

(ii) If $A = 600$

$6x^2 = 600$

$x^2 = 100$

then $x = 10$ cm (>0)

If $x = 10$, $\frac{dx}{dt} = -\frac{1}{2(10)}$

$= -\frac{1}{20} \longrightarrow$ ∴ When $A = 600\text{cm}^2$, the side length is decreasing at $\frac{1}{20}$ cm/min.

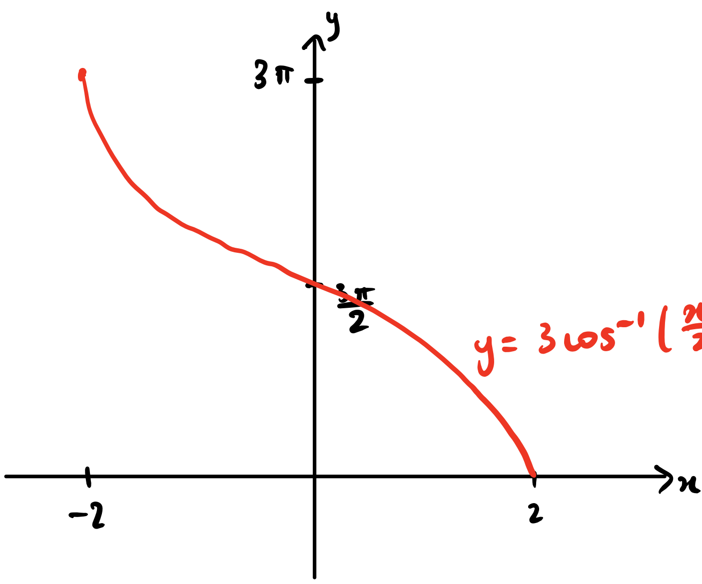
(e) $y = 3\cos^{-1}\left(\frac{x}{2}\right)$

D: $-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2$

R: $0 \leq \cos^{-1}\left(\frac{x}{2}\right) \leq \pi$

$0 \leq 3\cos^{-1}\left(\frac{x}{2}\right) \leq 3\pi$



(f) If $x=-1$ is a double root of $P(x)$, then:

$P(-1) = P'(-1) = 0$

$P(-1) = (-1)^3 - k(-1)^2 - (k+1)(-1)$

$= -1 - k + k + 1$

$= 0$ (not helpful to find k)

$P'(x) = 3x^2 - 2kx - (k+1)$

$P'(-1) = 3(-1)^2 - 2k(-1) - k - 1$

$0 = 3 + 2k - k - 1$

$0 = 2 + k$

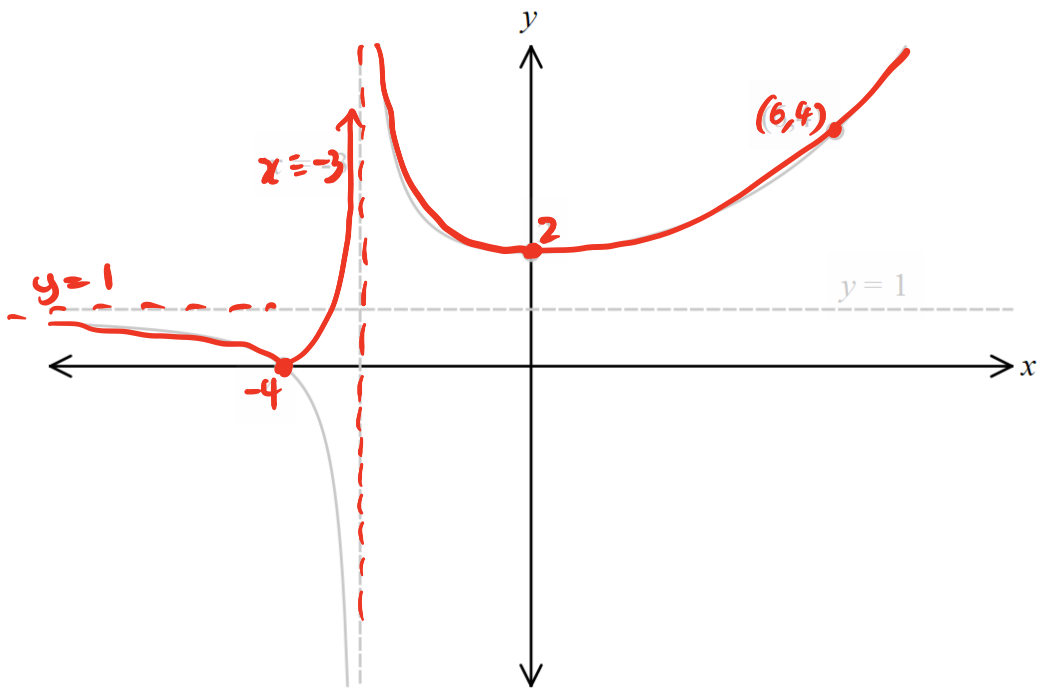
$\therefore k = -2$

Assessment No. SOLS

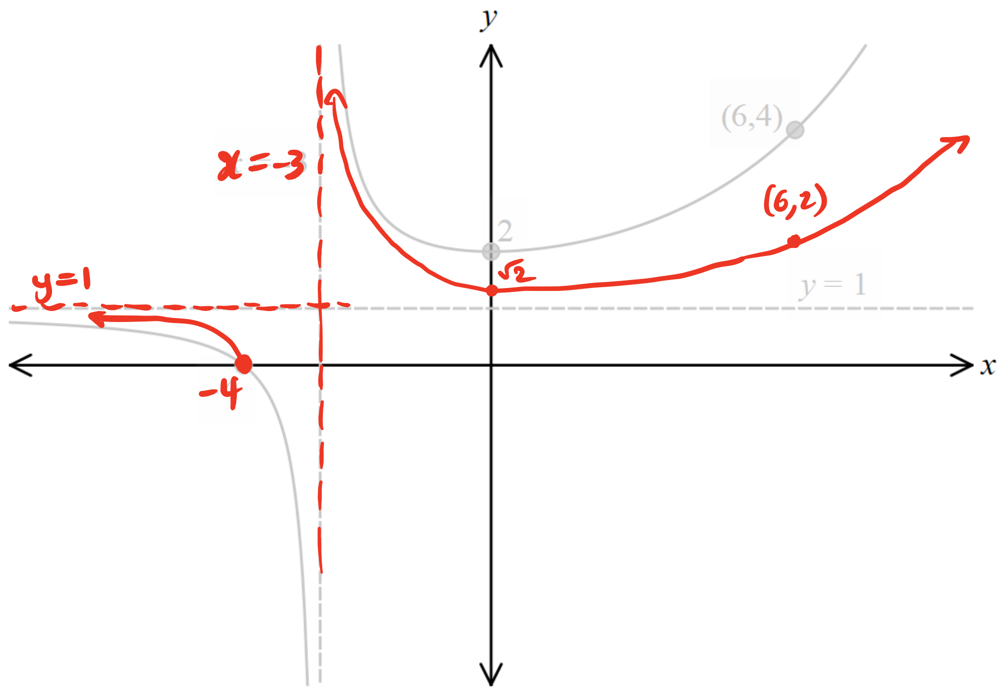
Q13a Insert

Note: If the transformed function shares features with $y = f(x)$, these features should be redrawn/relabelled.

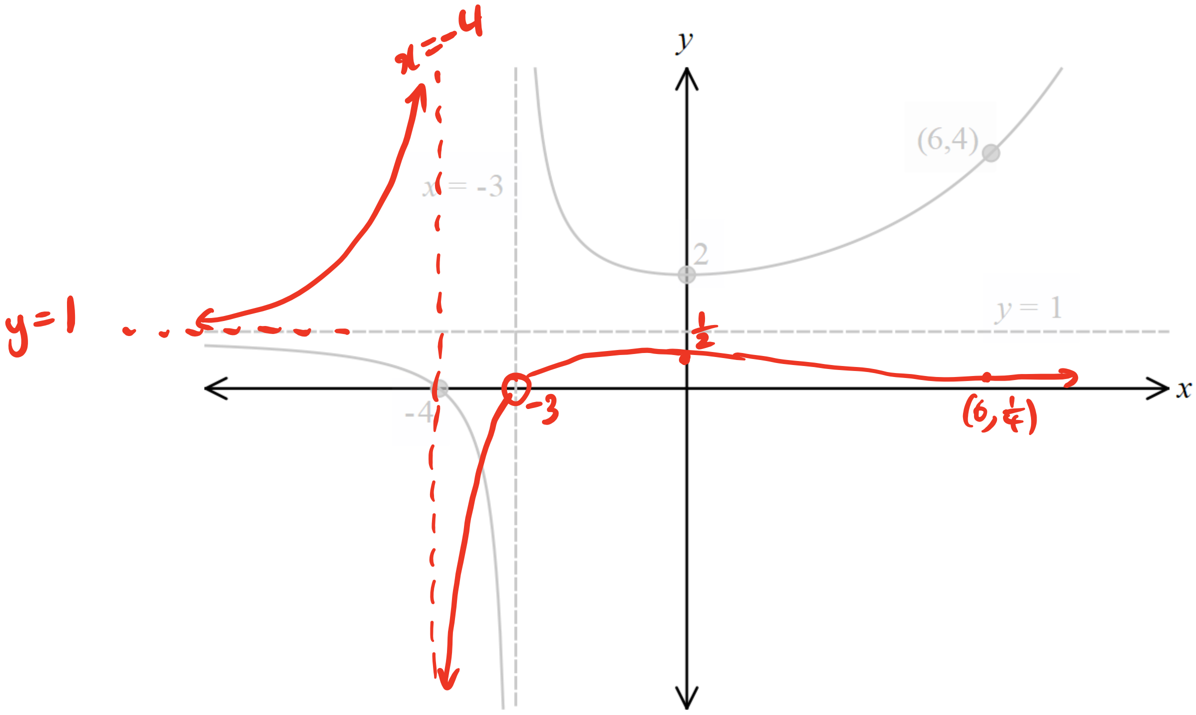
(i) $y = |f(x)|$



(ii) $y = \sqrt{f(x)}$



(iii) $y = \frac{1}{f(x)}$



Write your Assessment Number on the front page of this insert and place in your Question 13 writing booklet.

Question 13

(b) Ways of selecting 2 teams from 16 (no repeat fixtures) $= {}^{16}C_2 = 120$

Placing 121 pigeons (# games) into 120 holes (distinct games)

means one hole must contain at least two pigeons.

$\therefore$ At least one pair of teams will play each other more than once.

(c) (i) $A = A_0 e^{-kt}$

$$\frac{dA}{dt} = -kA_0e^{-kt}$$

$$\frac{dA}{dt} = -kA$$

$\therefore A = A_0e^{-kt}$ satisfies the differential equation.

(ii) at $t = 29$, $A = \frac{1}{2}A_0$:

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0}e^{-29k}$$

$$-29k = \ln\frac{1}{2}$$

$$-29k = -\ln 2$$

$$\therefore k = \frac{1}{29}\ln 2$$

(iii) let $A < 0.05A_0$

$$\cancel{A_0}e^{-kt} < 0.05\cancel{A_0} \qquad (A_0 > 0)$$

$$\ln(e^{-kt}) < \ln 0.05$$

$$-kt < \ln 0.05$$

$$t > \frac{\ln 0.05}{-\frac{1}{29}\ln 2} \qquad (k > 0 \text{ so } -k < 0)$$

$$t > 125.3\ldots$$

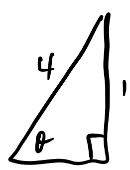
$\therefore$ The island will be inhabitable after 126 years (next whole year).

(d) $\sin\theta = \frac{1}{4} > 0$, $\cos\theta < 0$

S	A
T	C

$\therefore \tan\theta < 0$

With $\sin\theta = \frac{1}{4}$



$\sqrt{4^2-1^2} = \sqrt{15}$

$\rightarrow \tan\theta = -\frac{1}{\sqrt{15}}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(-\frac{1}{\sqrt{15}}\right)}{1-\left(-\frac{1}{\sqrt{15}}\right)^2}$$

$$= \frac{-\frac{2}{\sqrt{15}}}{1-\frac{1}{15}}$$

$$= \frac{-\frac{2}{\sqrt{15}}}{\frac{14}{15}}$$

$$= -\frac{15}{7\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$$

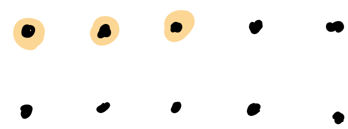
$$= -\frac{\sqrt{15}}{7}$$

Question 14

(i) Each triangle is formed by choosing 3 distinct dots as vertices from 6 options.

6 options, choose 3 $\rightarrow$ ${}^6C_3$ triangles.

(ii) Degenerate triangles must be ignored, such as the 'triangle' formed by the choice of these points:



Method 1

Each triangle must be formed from two dots in one row and one in the other:

$$\underbrace{\boxed{{}^2C_1}}_{\text{choose row for 2 dots}} \times \underbrace{\boxed{{}^5C_2}}_{\text{choose 2 dots}} \times \underbrace{\boxed{{}^5C_1}}_{\text{choose a dot in the other row}} = 100 \text{ triangles.}$$

Method 2

Remove degenerate triangles:

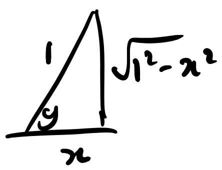
$$\underbrace{\boxed{{}^2C_1}}_{\text{choose row for deg. }\Delta} \times \underbrace{\boxed{{}^5C_3}}_{\text{choose 3 dots}} = 20$$

$$\underbrace{{}^{10}C_3}_{\text{total choices of 3 dots}} - 20 = 100 \text{ triangles.}$$

(b) let $y = \cos^{-1}x$

$$\sin(2\cos^{-1}x) = \sin 2y$$
$$= 2\sin y \cos y$$

If $y = \cos^{-1}x$, $x = \cos y$



so $\sin y = \sqrt{1-x^2}$

so $2\sin y \cos y = 2 \cdot \sqrt{1-x^2} \cdot x$

$$\therefore \sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}.$$

(c) $B = 25$ (surrounding environment)

at $t = 30$, $T = -35$ $\rightarrow$ $-35 = 25 - Ae^{-30k}$

$$Ae^{-30k} = 60 \quad (1)$$

at $t = 60$, $T = 5$ $\rightarrow$ $5 = 25 - Ae^{-60k}$

$$Ae^{-60k} = 20 \quad (2)$$

$(1)^2 \div (2)$: $\dfrac{A^{\cancel{2}}\cancel{e^{-60k}}}{\cancel{A}\cancel{e^{-60k}}} = \dfrac{3600}{20}$

$$A = 180$$

at $t = 0$,

$$T = B - Ae^{0}$$
$$= B - A$$
$$= 25 - 180$$
$$= -155^{\circ}C$$

$\therefore$ The critical temperature of the superconductor is $-155^{\circ}C$.

(d) (i) If $t = \tan\theta$, $\tan 2\theta = \frac{2t}{1-t^2}$ (t formula)

$$LHS = \tan 4\theta$$
$$= \tan(2 \cdot 2\theta)$$
$$= \frac{2\tan 2\theta}{1-\tan^2 2\theta}$$
$$= \frac{2 \cdot \frac{2t}{1-t^2}}{1-\left(\frac{2t}{1-t^2}\right)^2}$$
$$= \frac{\frac{4t}{\cancel{1-t^2}}}{\frac{(1-t^2)^2}{\cancel{(1-t^2)^2}} - \frac{4t^2}{\cancel{(1-t^2)^2}}} \times \frac{(1-t^2)^{\cancel{2}}}{\cancel{(1-t^2)^2}}$$
$$= \frac{4t(1-t^2)}{(1-t^2)^2-4t^2}$$
$$= \frac{4t(1-t^2)}{1-2t^2+t^2-4t^2}$$
$$= \frac{4t(1-t^2)}{1-6t^2+t^4}$$
$$= RHS$$

(ii) $\theta = \frac{\pi}{10}$ and $\frac{3\pi}{10}$ are solutions to $\tan 4\theta = \cot\theta$

If $t = \tan\theta$, the equation becomes:

$$\frac{4t(1-t^2)}{1-6t^2+t^4} = \frac{1}{t} \qquad \left(\text{from (i) and } \cot\theta = \frac{1}{\tan\theta}\right)$$

$$4t^2(1-t^2) = 1-6t^2+t^4$$

$$4t^2 - 4t^4 = 1-6t^2+t^4$$

$$5t^4 - 10t^2 + 1 = 0$$

Using quadratic formula:

$$t^2 = \frac{10 \pm \sqrt{100-4(5)(1)}}{10}$$

$$t^2 = \frac{10 \pm \sqrt{80}}{10}$$

$$t^2 = \frac{10 \pm 4\sqrt{5}}{10} = \frac{5 \pm 2\sqrt{5}}{5}$$

$$t = \pm\sqrt{\frac{5 \pm 2\sqrt{5}}{5}}$$

Since $\tan\frac{\pi}{10}$, $\tan\frac{3\pi}{10} > 0$ (1st Quad), ignore negative values:

$$t = \sqrt{\frac{5 \pm 2\sqrt{5}}{5}}$$

$$t = \sqrt{\frac{5-2\sqrt{5}}{5}} \quad \text{or} \quad \sqrt{\frac{5+2\sqrt{5}}{5}}$$

Since $\tan\theta$ is increasing, the smaller value corresponds to the solution $\tan\frac{\pi}{10}$.

$$\therefore \tan\frac{\pi}{10} = \sqrt{\frac{5-2\sqrt{5}}{5}}$$