Ms Davis
Mrs Greenberg
Mrs Millar

Mr Feng
Ms Lau
Mrs Squires

Name:..

Teacher's Name:.....................................

Pymble Ladies' College

YEAR 11
YEARLY EXAMINATION
2019

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Erasable pens are **not** to be used
- NESA approved calculators may be used
- Reference sheets are provided

Total Marks: 70

Section I 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II 60 marks

- Attempt all Questions 11 - 30
- Allow about 1 hours and 45 minutes for this section

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

AutoMarque Answer Sheet ©

Student's name ______________________________

Yr 11 Extension Mathematics

Student Number

Hundreds:	0	1	2	3	4	5	6	7	8	9
Tens:	0	1	2	3	4	5	6	7	8	9
Units:	0	1	2	3	4	5	6	7	8	9

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

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Section I

Multiple Choice (10 marks)

Use the Multiple Choice Answer Sheet for questions 1 to 10.

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1 What is the remainder when $P(x) = x^3 - x^2 + x + 3$ is divided by $(x-1)$?

(A) 0

(B) 2

(C) 3

(D) 4

2 Given that $\sin 2\alpha = \frac{7}{10}$ and $\sin\alpha = \frac{2}{5}$, what is the value of $\cos\alpha$?

(A) $\frac{-7}{4}$

(B) $\frac{7}{8}$

(C) $\frac{7}{25}$

(D) $\frac{14}{25}$

3 A function has the parametric equations $x = \frac{2}{t}$ and $y = 2t^2$.

What is the Cartesian equation of this curve?

(A) $y = \frac{4}{x}$

(B) $y = \frac{8}{x}$

(C) $y = \frac{4}{x^2}$

(D) $y = \frac{8}{x^2}$

4 Given α, β and γ are the roots of $x^3 - 4x - 1 = 0$, what does $\alpha\beta + \alpha\gamma + \beta\gamma$ equal?

(A) -4

(B) -1

(C) 0

(D) 4

5 The polynomial $P(x) = x^3 + 2x + k$ has $(x-2)$ as a factor.
What is the value of k?

(A) -12

(B) -10

(C) 10

(D) 12

6 Which of the following is a simplified expression for $\dfrac{\sin 2x}{1 - \cos 2x}$?

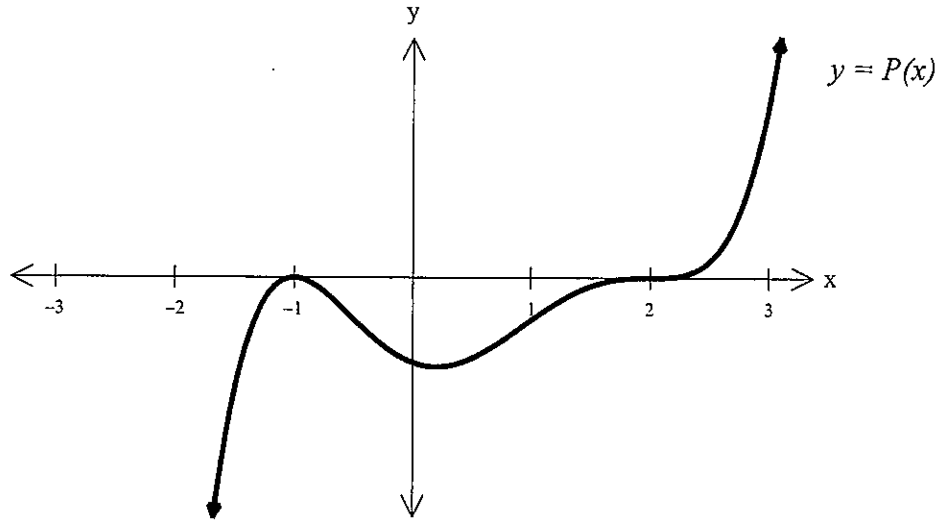
(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) $\cot x$

7 The diagram shows the graph of a polynomial.



Which of the following could be the equation of this polynomial $P(x)$?

(A) $P(x)=(x-1)(x+2)$

(B) $P(x)=(x-1)^2(x+2)^3$

(C) $P(x)=(x+1)^2(x-2)^3$

(D) $P(x)=(x+1)(x-2)^3$

8 If $\cos^{-1}x+\sin^{-1}x=\dfrac{\pi}{2}$ and $\cos^{-1}x-\sin^{-1}x=\dfrac{\pi}{4}$, what is the value of $\cos^{-1}x$?

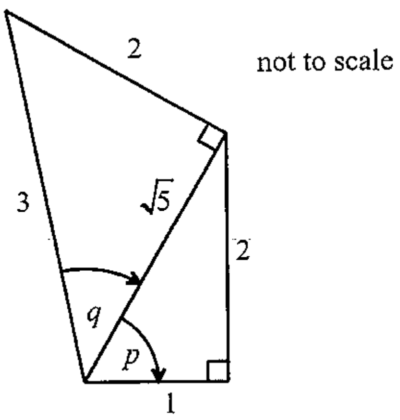
(A) $\dfrac{5\pi}{16}$

(B) $\dfrac{3\pi}{8}$

(C) $\dfrac{7\pi}{16}$

(D) $\dfrac{5\pi}{8}$

9 The diagram shows two right-angled triangles with sides and angles given.



What is the value of $\sin(p+q)$?

(A) $\frac{2}{\sqrt{5}}+\frac{2}{3}$

(B) $\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{3}$

(C) $\frac{2}{3\sqrt{5}}+\frac{2}{3}$

(D) $\frac{4}{3\sqrt{5}}+\frac{1}{3}$

10 What is the value of $\sin^{-1}\left(\sin\frac{5\pi}{7}\right)$?

(A) $\frac{5\pi}{7}$

(B) $\frac{2\pi}{7}$

(C) $\frac{-5\pi}{7}$

(D) $\frac{-2\pi}{7}$

Section II - Part A (16 marks)

Name..............................

Teacher: PL MM JS BD NF DG

11 Find the exact value of $\sin 75° \cos 15°$. **1**

12 If $t = \tan A$, then find $\sin 2A + \cos 2A$ as a single fraction in terms of t. **2**

13 Find the exact value of $\sin 15°$. **3**

14 Solve $\sin\theta = \sin 2\theta$ for $0 \le \theta \le 2\pi$. **2**

15 A curve is represented by the parametric equations $x = 2\cos\theta$ and $y = 2\sin\theta$. **2**
Find the Cartesian equation of this curve and describe it geometrically in words.

Section II - Part B (15 marks)

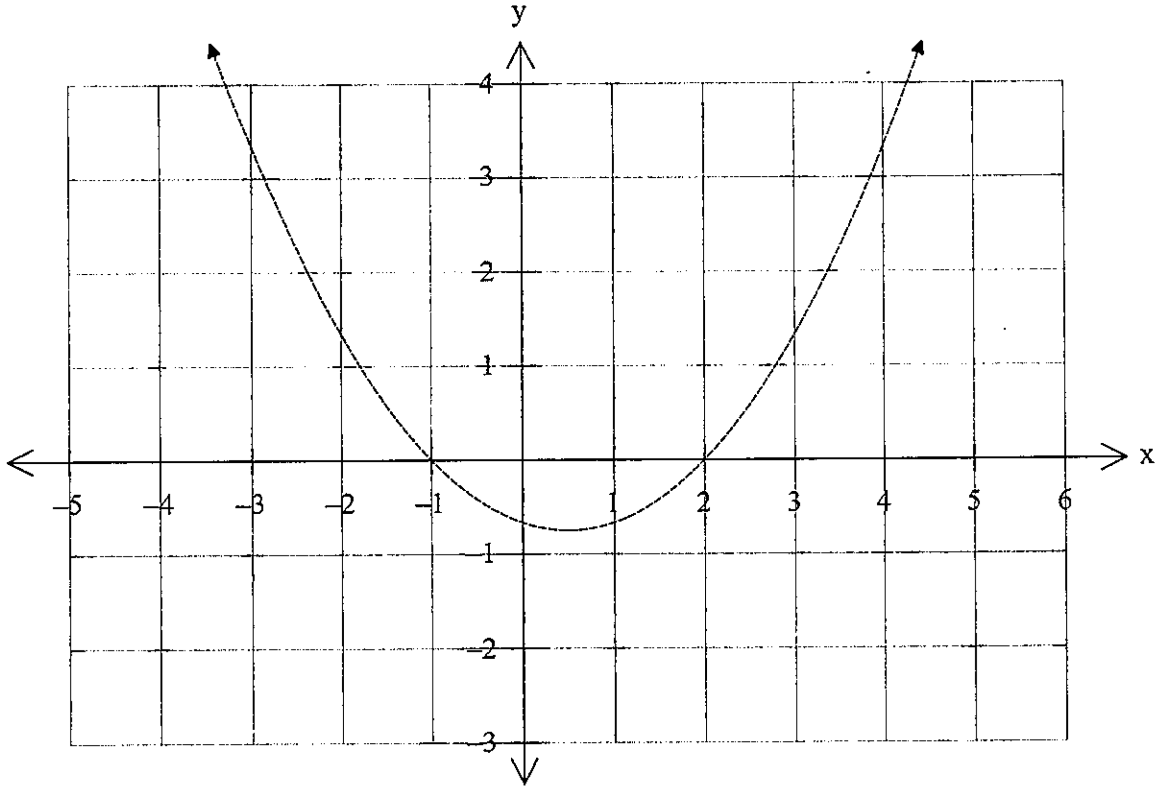
Name……………………………

Teacher: PL MM JS BD NF DG

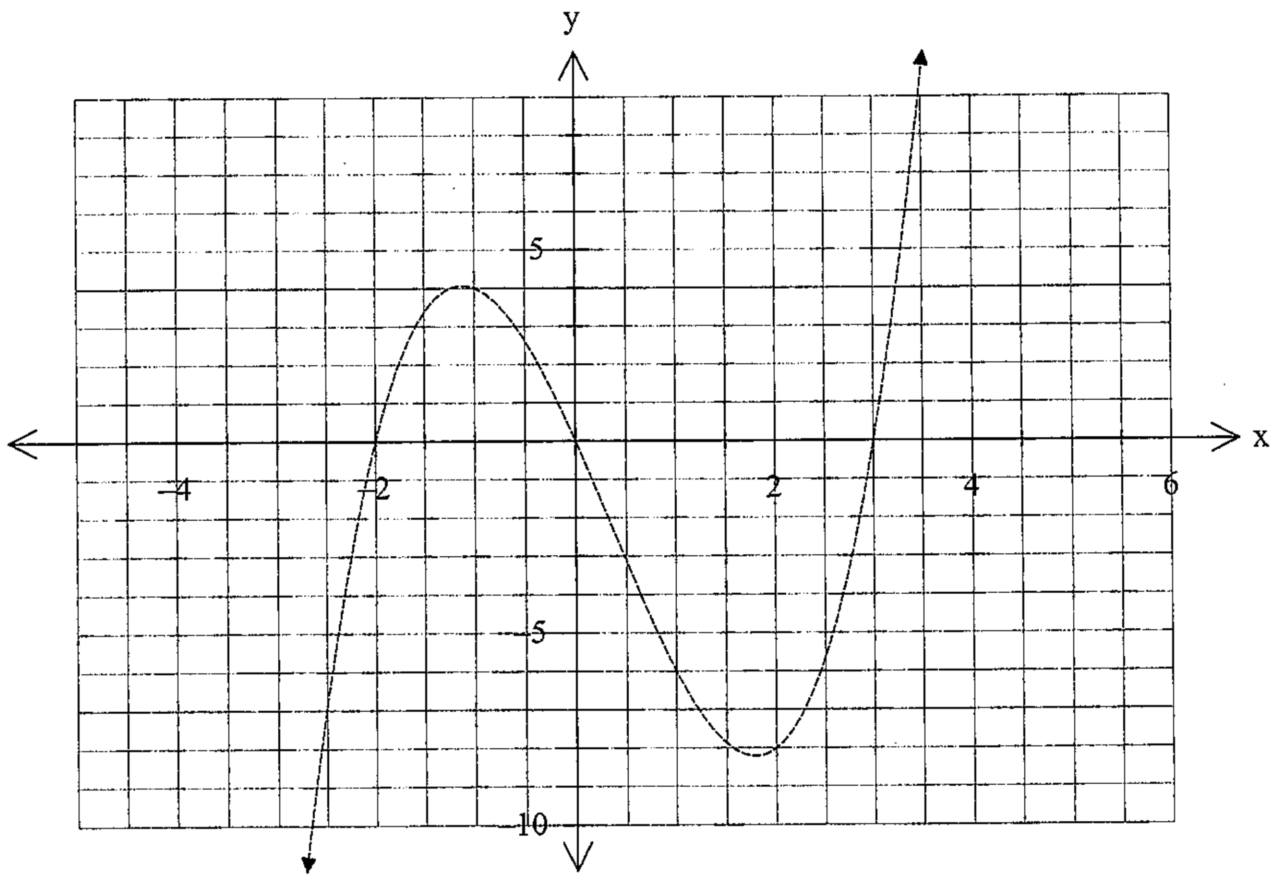
18 Two of the roots of $x^3 - px^2 - qx + 30 = 0$ are 3 and 5. Find the value of p and q. **3**

19 If $P(x) = x^3 - x^2 - 8x + 12$, show that $P(x)$ has a zero of multiplicity 2. **2**

20 (a) The diagram shows the graph of $y=f(x)$. Sketch the graph of $y=\sqrt{f(x)}$. **3**



(b) The diagram shows the graph of $y=f(x)$. Sketch the graph of $y=f(|x|)$. **2**

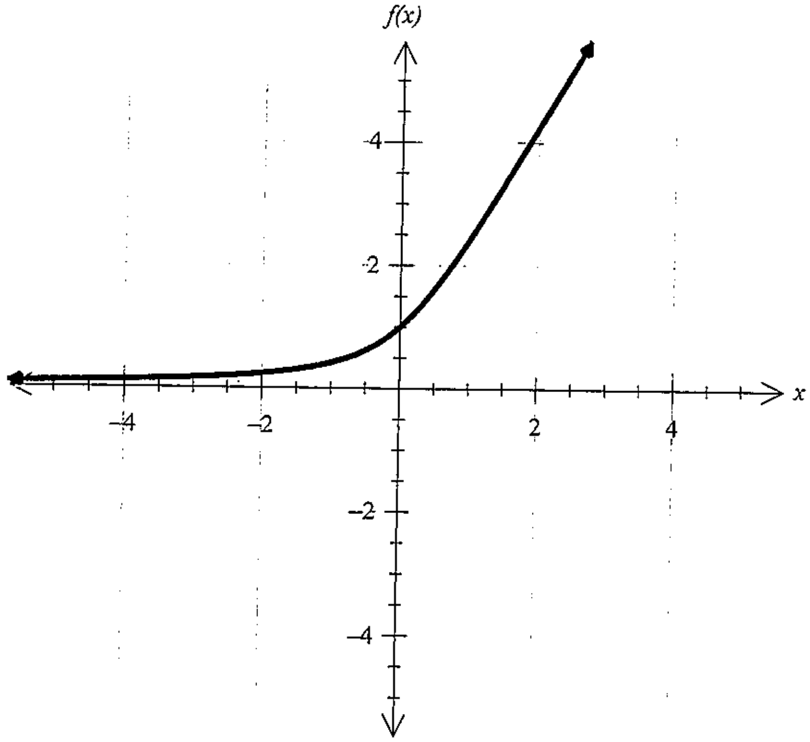


Section II - Part C (16 marks)

Name..............................

Teacher: PL MM JS BD NF DG

23 Consider the function $f(x) = x + \sqrt{x^2 + 1}$.



(a) On the graph above, draw the inverse function of $f(x)$. **2**

(b) Show that $f^{-1}(x) = \frac{1}{2}(x - \frac{1}{x})$ **3**

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24 Find the exact value of $\cos\left(\tan^{-1}\left(\frac{-1}{3}\right)\right)$. **2**

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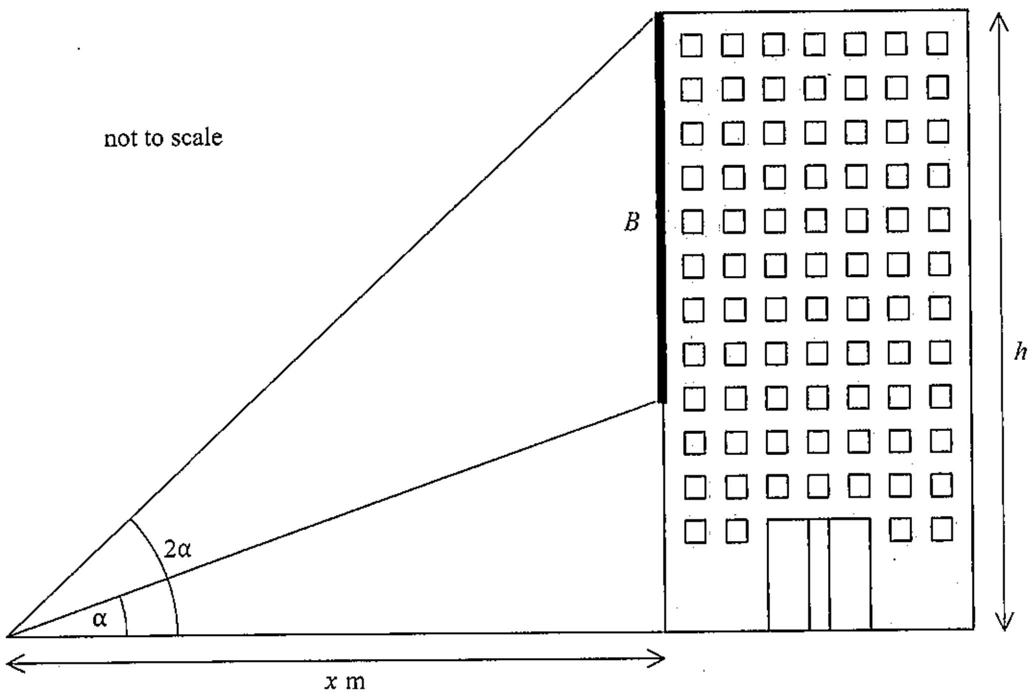
25 A large advertising banner (B) is hung from the top of a building. The angle of elevation from an observer x metres from the base of the building to the top of the banner is 2α ,while the angle of elevation to the bottom of the banner is α .



(a) Show that the height, h, of the building is $h = x\tan 2\alpha$. **1**

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Section II - Part D (13 marks) Name..............................

Teacher: PL MM JS BD NF DG

27 A Cartesian equation can be defined by the parametric equations $x=\dfrac{t}{t+1}$ and $y=\dfrac{t}{t-1}$.

(a) Show that $x=\dfrac{t}{t+1}$ can be rewritten as $t=\dfrac{x}{1-x}$. **2**

(b) Hence, find the Cartesian equation defined by the parametric equations $x=\dfrac{t}{t+1}$ and $y=\dfrac{t}{t-1}$. **2**

28 Find the inverse function of $y = \log_2(x-3)$. **2**

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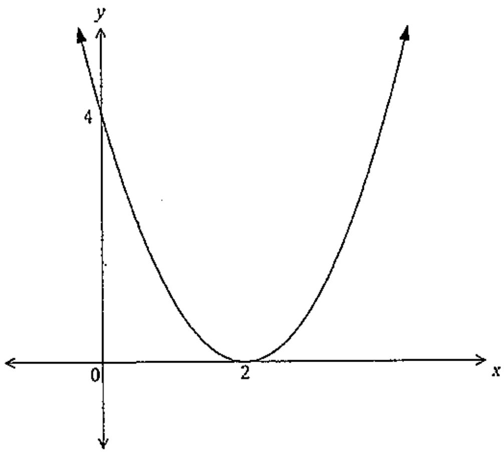
29 The graph shows the parabola $y = f(x)$, given $f(x) = (x-2)^2$.



(a) Explain why an inverse function $y = f^{-1}(x)$ exists when $x \geq 2$. **1**

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(b) State the domain and range of this inverse function. **2**

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(c) At what point will $f(x) = f^{-1}(x)$? **1**

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30 Prove that $\dfrac{\cos x-\cos(x+2\theta)}{2\sin\theta}=\sin(x+\theta)$. **3**

End of Paper

Extra writing space

If you use this space, clearly indicate which question you are answering.

Question 25 continued

(b) Using $t = \tan\alpha$, or otherwise, show that the height, B, of the banner is $B = x\tan\alpha\sec 2\alpha$ metres. **3**

Section II Part C question 26 is on back page of this booklet.

26 Given $f(x)=x^3-1$, $g(x)=3x+1$ and $h(x)=4x-5$,

(a) show that $g(f(x))+xh(x)=3x^3+4x^2-5x-2$, **2**

(b) solve $g(f(x))+xh(x)=0$. **3**

21 Solve $x^2(x-1)(5-x)<0$. **2**

22 If $f(x)=x^2-x-6$, sketch both $y=f(x)$ and $y=\dfrac{1}{f(x)}$, on the same number plane. **3**

Show all key features and clearly label both graphs.

Extra writing space

If you use this space, clearly indicate which question you are answering.

16 Find the values of x which satisfy $\frac{1}{x+1} \geq 3$. **3**

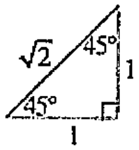
17 Given $f(x)=2x+3$ and $g(x)=|x-1|$, find the value(s) of x such that $f(x)<g(x)$. **3**

Extra writing space

If you use this space, clearly indicate which question you are answering.

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

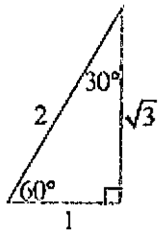


$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

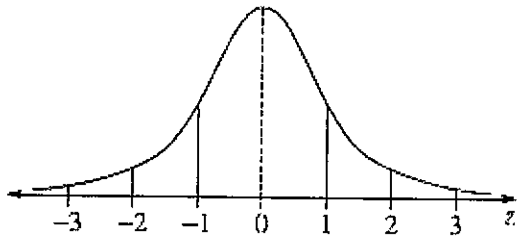
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$= \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Solutions

Section I

Multiple Choice (10 marks)

Use the Multiple Choice Answer Sheet for questions 1 to 10.

---

1 What is the remainder when $P(x)=x^3-x^2+x+3$ is divided by $(x-1)$?

(A) 0

(B) 2

(C) 3

(D) 4

$P(1) = (1)^3-(1)^2+(1)+3$
$= 4$

2 Given that $\sin 2\alpha = \frac{7}{10}$ and $\sin\alpha = \frac{2}{5}$, what is the value of $\cos\alpha$?

(A) $\frac{-7}{4}$

(B) $\frac{7}{8}$

(C) $\frac{7}{25}$

(D) $\frac{14}{25}$

$\sin 2\alpha = \frac{7}{10}$
$\therefore 2\sin\alpha\cos\alpha = \frac{7}{10}$
$2\left(\frac{2}{5}\right)\cos\alpha = \frac{7}{10}$
$\frac{4}{5}\cos\alpha = \frac{7}{10}$
$\cos\alpha = \frac{7}{10}\times\frac{5}{4}$
$\cos\alpha = \frac{7}{8}$

3 A function has the parametric equations $x=\frac{2}{t}$ and $y=2t^2$.
What is the Cartesian equation of this curve?

(A) $y=\frac{4}{x}$

(B) $y=\frac{8}{x}$

(C) $y=\frac{4}{x^2}$

(D) $y=\frac{8}{x^2}$

$x = \frac{2}{t}$
$t = \frac{2}{x}$; $y = 2t^2$
$y = 2.\left(\frac{2}{x}\right)^2$
$\therefore y = \frac{8}{x^2}$

4 Given α, β and γ are the roots of $x^3-4x-1=0$, what does $\alpha\beta+\alpha\gamma+\beta\gamma$ equal?

(A) -4

(B) -1

(C) 0

(D) 4

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$
$= \frac{-4}{1}$
$= -4$

5 The polynomial $P(x)=x^3+2x+k$ has $(x-2)$ as a factor.
What is the value of k?

(A) -12

(B) -10

(C) 10

(D) 12

$P(2) = 0$
$0 = (2)^3+2(2)+k$
$0 = 12+k$
$k = -12$

6 Which of the following is a simplified expression for $\frac{\sin 2x}{1-\cos 2x}$?

(A) $\sin x$

(B) $\cos x$

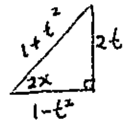
(C) $\tan x$

(D) $\cot x$

$= \frac{2\sin x\cos x}{1-(1-2\sin^2 x)}$
$= \frac{2\sin x\cos x}{2\sin^2 x}$
$= \frac{\cos x}{\sin x}$
$= \cot x$

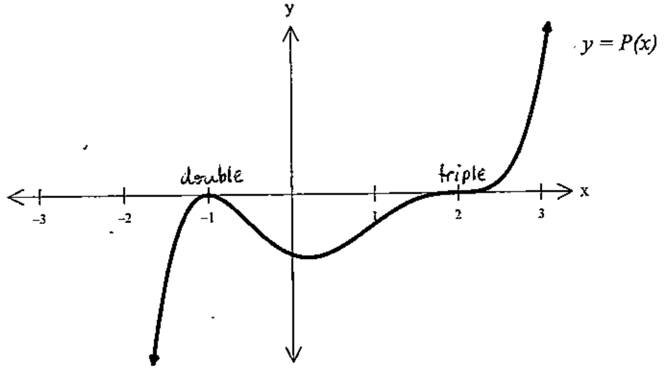
OR

Let $t = \tan x$



$\therefore \frac{\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}} \times \frac{1+t}{1+t}$
$= \frac{2t}{1+t^2-1+t^2}$
$= \frac{2t}{2t^2}$
$= \frac{1}{t}$
$= \cot x$

7 The diagram shows the graph of a polynomial.



Which of the following could be the equation of this polynomial $P(x)$?

(A) $P(x)=(x-1)(x+2)$

(B) $P(x)=(x-1)^2(x+2)^3$

(C) $P(x)=(x+1)^2(x-2)^3$

(D) $P(x)=(x+1)(x-2)^3$

8 If $\cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}$ and $\cos^{-1}x-\sin^{-1}x=\frac{\pi}{4}$, what is the value of $\cos^{-1}x$?

(A) $\frac{5\pi}{16}$

(B) $\frac{3\pi}{8}$

(C) $\frac{7\pi}{16}$

(D) $\frac{5\pi}{8}$

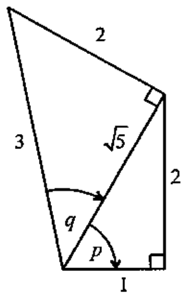
$$\overset{A}{\cos^{-1}x}+\overset{B}{\sin^{-1}x}=\frac{\pi}{2} \quad -\boxed{1}$$

$$\overset{A}{\cos^{-1}x}-\overset{B}{\sin^{-1}x}=\frac{\pi}{4} \quad -\boxed{2}$$

$$\boxed{1}+\boxed{2}: \quad 2\cos^{-1}x=\frac{3\pi}{4}$$

$$\cos^{-1}x=\frac{3\pi}{8}$$

9 The diagram shows two right-angled triangles with sides and angles given.



not to scale

$\sin p=\frac{2}{\sqrt5}$ $\quad\cos p=\frac{1}{\sqrt5}$

$\sin q=\frac{2}{3}$ $\quad\cos q=\frac{\sqrt5}{3}$

$$\sin(p+q)=\sin p.\cos q+\cos p.\sin q$$
$$=\frac{2}{\sqrt5}\cdot\frac{\sqrt5}{3}+\frac{1}{\sqrt5}\cdot\frac{2}{3}$$
$$=\frac{2}{3}+\frac{2}{3\sqrt5}$$

What is the value of $\sin(p+q)$?

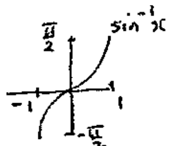
(A) $\frac{2}{\sqrt5}+\frac{2}{3}$

(B) $\frac{2}{\sqrt5}+\frac{\sqrt5}{3}$

(C) $\frac{2}{3\sqrt5}+\frac{2}{3}$

(D) $\frac{4}{3\sqrt5}+\frac{1}{3}$

$-1\le x\le 1$



$\boxed{-\frac{\pi}{2}\le y\le\frac{\pi}{2}}$

10 What is the value of $\sin^{-1}\left(\sin\frac{5\pi}{7}\right)$? $\quad A=\sin^{-1}\left(\sin\frac{5\pi}{7}\right)$

(A) $\frac{5\pi}{7}$

(B) $\frac{2\pi}{7}$

(C) $\frac{-5\pi}{7}$

(D) $\frac{-2\pi}{7}$

$$\sin A=\sin\frac{5\pi}{7}$$
$$\sin A=\sin\left(\pi-\frac{2\pi}{7}\right)$$
$$\sin A=\sin\frac{2\pi}{7}$$

Section II - Part A (16 marks)   Name..............................

Teacher: PL MM JS BD NF DG

11 Find the exact value of $\sin 75° \cos 15°$. 1

$\sin 75° \cos 15° = \frac{1}{2}\left[\sin 90° + \sin 60°\right]$

$= \frac{1}{2}\left[1 + \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{2} \cdot \frac{(2+\sqrt{3})}{2}$

$= \frac{2+\sqrt{3}}{4}$

(*) students who used a product of compound results were less successful.
eg $\sin(30°+45°)\cos(45°-30°)$ due to alg. errors.

1	RW

- students were expected to write as a sum & substitute correctly
- ISE

12 If $t = \tan A$, then find $\sin 2A + \cos 2A$ as a single fraction in terms of t. 2

$\sin 2A + \cos 2A$, $t = \tan A$

$= \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}$

$= \frac{2t+1-t^2}{1+t^2}$

$= \frac{-t^2+2t+1}{1+t^2}$

2	correct soln • ISE
1	identify half angle result & substitute

- final answer had to be a single simplified fraction in terms of t.
- students who did not use $t = \tan A$ were given an ecf in some cases
  → no errors in algebra
  → relationship easily identified

(*) many students factorised incorrectly $-t^2+2t+1 \Rightarrow (t+1)^2$ or $(t-1)^2$
NO! NO! NO!

13 Find the exact value of $\sin 15°$. 3

$\sin 15° = \sin(60° - 45°)$

$= \sin 60° \cos 45° - \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

OR

$\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

(*) many students did not achieve 3 because they did not rationalise

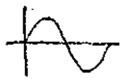
3	correct solution • MUST be rationalised
2	substitute correctly
1	expanded using compound angle result

14 Solve $\sin\theta = \sin 2\theta$ for $0 \le \theta \le 2\pi$. 2

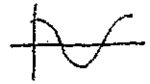
$\sin\theta = 2\sin\theta\cos\theta$

$0 = 2\sin\theta\cos\theta - \sin\theta$

$0 = \sin\theta(2\cos\theta - 1)$



ie. $\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$



$\theta = 0, \pi, 2\pi$

$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Sol'n: $\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

2	• correct solutions (5)
1	• one correct set of solutions or one error made

(*) many students not using radians but converting from degrees
→ errors made in conversion

(*) many students divided by $\sin\theta$ and lost a set of solutions.

15 A curve is represented by the parametric equations $x = 2\cos\theta$ and $y = 2\sin\theta$. Find the Cartesian equation of this curve and describe it geometrically in words. 2

$x = 2\cos\theta$, $\cos\theta = \frac{x}{2}$

$y = 2\sin\theta$, $\sin\theta = \frac{y}{2}$

$\cos^2\theta + \sin^2\theta = 1$ (Pythag. Id)

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

$x^2 + y^2 = 4$

Circle centre $(0,0)$, radius = 2 units

2	correct solution • students were expected to describe shape & features correctly in words
1	equation in cartesian form

16 Find the values of x which satisfy $\frac{1}{x+1} \geq 3$. **3**

$$(x+1)^2 \times \frac{1}{x+1} \geq 3 \times (x+1)^2 \quad ; x \neq -1$$

$$x+1 \geq 3(x+1)^2 \quad ; x \neq -1$$

$$0 \geq 3(x+1)^2 - (x+1) \quad ; x \neq -1$$

$$0 \geq (x+1)\left[3(x+1) - 1\right] \quad ; x \neq -1$$

$$0 \geq (x+1)(3x+2) \quad ; x \neq -1$$

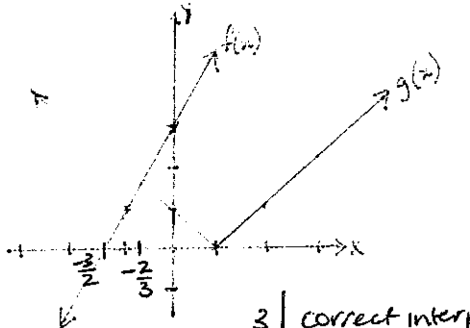
Sol'n : $-1 < x \leq -\frac{2}{3}$

3	correct Solution solution needed to exclude $x = -1$
2	simplified quadratic inequality
1	multiply by $(x+1)^2$ both sides

(*) one student sketched a hyperbola and solved, after finding the point of intersection.

(*) students who did not sketch the quadratic were often unsuccessful.

17 Given $f(x) = 2x+3$ and $g(x) = |x-1|$, find the value(s) of x such that $f(x) < g(x)$. **3**



$$f(x) < g(x)$$

$$2x+3 < -x+1$$

$$3x < -2$$

$$x < -\frac{2}{3}$$

Sol'n $x < -\frac{2}{3}$

3	correct interpretation of solution to inequalities.
2	inequalities solved correctly.
1	inequality correctly set up with abs value.

(*) working was difficult to read with many students leaving two regions without reasoning / testing

(*) students are encouraged to sketch neat diagrams that are labelled. This reduced the amount of work required to solve the problem.

(*) many students were unable to determine why $x < -4$ was not the correct solution

(*) lots of errors with manipulating negatives around inequalities

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If you use this space, clearly indicate which question you are answering.

$\alpha=3,\ \beta=5,\ \gamma$

$\alpha+\beta+\gamma = -\frac{b}{a}$

$8+\gamma = p$ —①

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$

$15+3\gamma+5\gamma = -q$

$15+8\gamma = -q$ —②

$\alpha\beta\gamma = -\frac{d}{a}$

$15\gamma = -30$

$\gamma = -2$

Subst. $\alpha = -2$ into ①: $p = 8+(-2)$

$\boxed{p=6}$

Subst. $\alpha=-2$ into ②: $-q = 15+8(-2)$

$-q = -1$

$\boxed{q=1}$

OR ⟵

Section II - Part B (15 marks) Name..............................

Teacher: PL MM JS BD NF DG

18 Two of the roots of $x^3 - px^2 - qx + 30 = 0$ are 3 and 5. Find the value of p and q. 3

$P(3)=0$

$(3)^3 - p(3)^2 - q(3) + 30 = 0$

$27 - 9p - 3q + 30 = 0$

$9p + 3q = 57$

$(\div 3)$ $3p + q = 19$ —— $\boxed{1}$

$P(5)=0$

$(5)^3 - p(5)^2 - q(5) + 30 = 0$

$125 - 25p - 5q + 30 = 0$

$25p + 5q = 155$

$(\div 5)$ $5p + q = 31$ —— $\boxed{2}$

$\boxed{2} - \boxed{1}$: $2p = 12$, $p = 6$

Subst. $p=6$ into $\boxed{1}$: $3(6) + q = 19$, $q = 1$

Sol'n: $\begin{cases} p=6 \\ q=1 \end{cases}$

1	Either $\boxed{1}$ or $\boxed{2}$
2	Solve for either p or q
3	Correct soln

19 If $P(x) = x^3 - x^2 - 8x + 12$, show that $P(x)$ has a zero of multiplicity 2. 2

$P(x) = x^3 - x^2 - 8x + 12$

$P'(x) = 3x^2 - 2x - 8$

$0 = 3x^2 - 2x - 8$

$= (3x+4)(x-2)$

$x = -\frac{4}{3}$ or 2

$P(2) = (2)^3 - (2)^2 - 8(2) + 12$

$= 0$

$P(2) = P'(2) = 0$

$\therefore$ Zero of multiplicity 2.

1	Find $P(2)=0$
2	Find $P(2)=P'(2)=0$ or Find fully factorised form

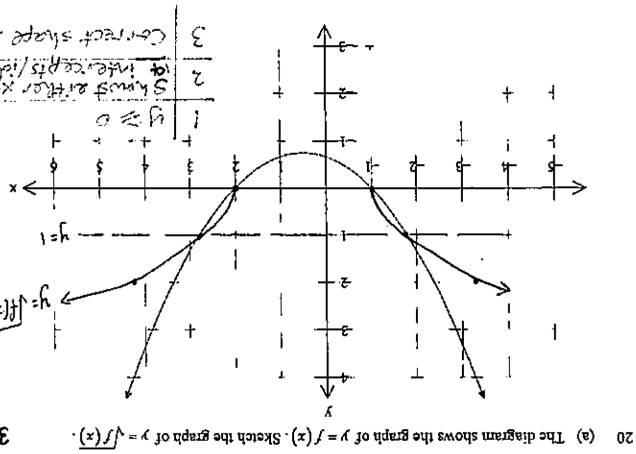
OR $P(x) = (x-2)(x^2+x-6)$

$= (x-2)(x+3)(x-2)$

$= (x-2)^2(x+3)$

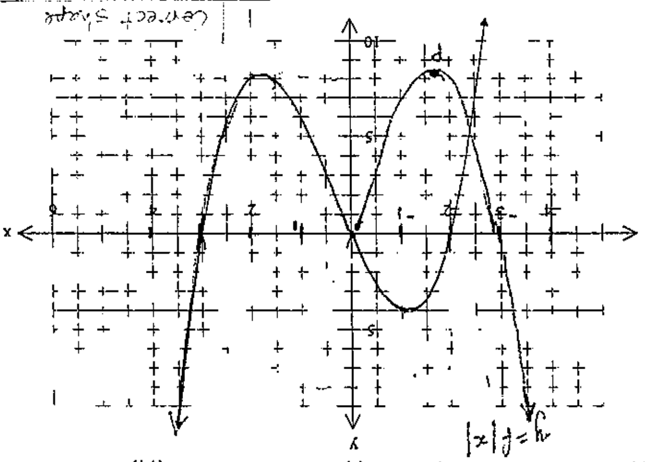
$$\begin{array}{r} x^2+x-6 \\ x-2\overline{)x^3-x^2-8x+12} \\ \underline{x^3-2x^2} \\ x^2-8x \\ \underline{x^2-2x} \\ -6x+12 \\ \underline{-6x+12} \end{array}$$

20 (a) The diagram shows the graph of $y=f(x)$. Sketch the graph of $y=\sqrt{f(x)}$. 3



1	$y \geq 0$
2	Shows either x-intercepts or intercepts/idea y=1
3	Correct shape.

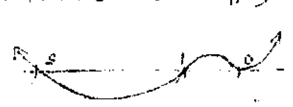
(b) The diagram shows the graph of $y=f(x)$. Sketch the graph of $y=f(|x|)$. 2



1	Correct shape
2	Correct point P & x-intercepts

21 Solve $x^2(x-1)(5-x) < 0$. 2

Zeros at $x=0, 1, 5$ Leading term: $-x^4$



Sol'n: $x<0$, $0<x<1$, $x>5$

1	Correct zeros
2	Correct sol'n.

22 If $f(x) = x^2 - x - 6$, sketch both $y=f(x)$ and $y=\frac{1}{f(x)}$, on the same number plane. 3

Show all key features and clearly label both graphs.

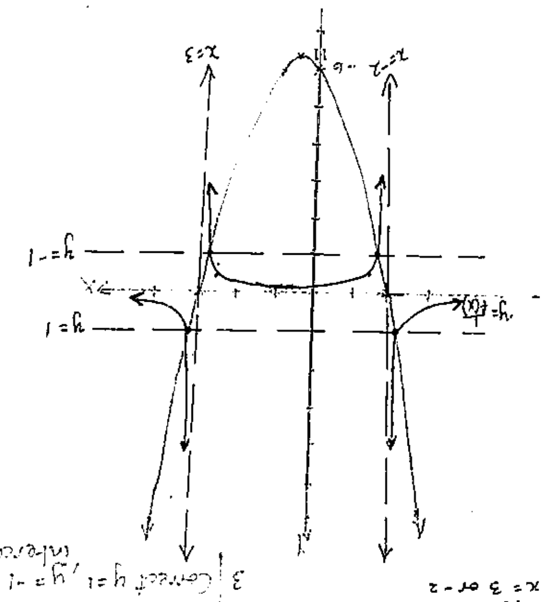
$f(x) = x^2 - x - 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or -2

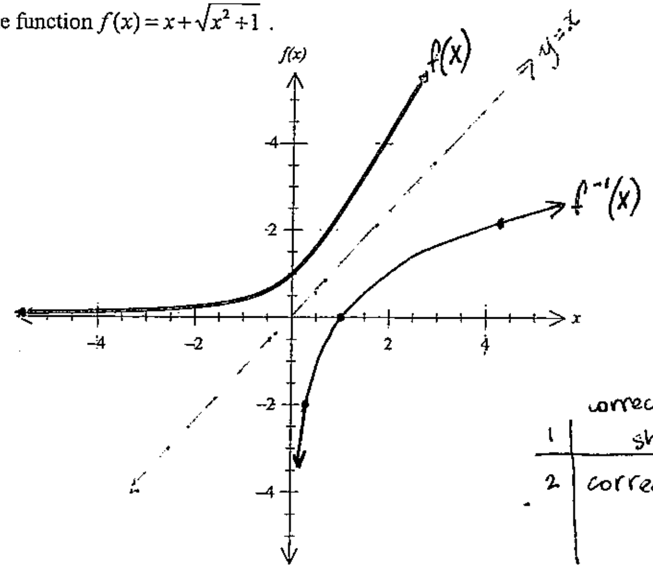
1	Vertical asymptotes.
2	Correct shape
3	Correct $y=1$, $y=-1$ intercepts.



Section II - Part C (16 marks) Name.............................

Teacher: PL MM JS BD NF DG

23 Consider the function $f(x) = x + \sqrt{x^2+1}$.



	correct x-int or shape
2	correct soln

(a) On the graph above, draw the inverse function of $f(x)$. **2**

(b) Show that $f^{-1}(x) = \frac{1}{2}(x - \frac{1}{x})$ **3**

Let $y = x + \sqrt{x^2+1}$

f^{-1}: $x = y + \sqrt{y^2+1}$

$x - y = \sqrt{y^2+1}$

$(x-y)^2 = y^2 + 1$

$x^2 - 2xy + y^2 = y^2 + 1$

$x^2 - 2xy = 1$

$x^2 - 1 = 2xy$

$\frac{x^2-1}{2x} = y \quad ; x \neq 0$

$y = \frac{x^2}{2x} - \frac{1}{2x}$

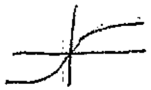
$y = \frac{x}{2} - \frac{1}{2x}$

i.e. $f^{-1}(x) = \frac{1}{2}\left(x - \frac{1}{x}\right)$

1	$x = y + \sqrt{y^2+1}$
2	works towards $x^2 - 2xy + y^2 = y^2 + 1$
3	$f^{-1}(x) = \frac{x}{2} - \frac{1}{2x}$

24 Find the exact value of $\cos\left(\tan^{-1}\left(\frac{-1}{3}\right)\right)$. **2**

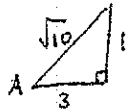
A



$A = \tan^{-1}\left(-\frac{1}{3}\right)$

$\therefore \tan A = \frac{-1}{3}$

S | A
T | C



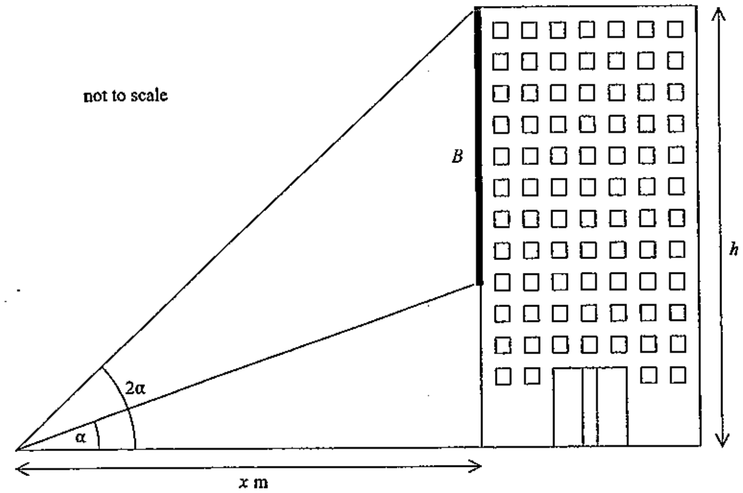
tan -ve in Q4

$\therefore \cos A = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

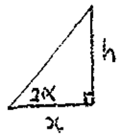
$\cos A = \frac{3\sqrt{10}}{10}$

1	$\tan A = -\frac{1}{3}$
2	correct soln $\frac{3}{\sqrt{10}}$ or $\frac{3\sqrt{10}}{10}$

25 A large advertising banner (B) is hung from the top of a building. The angle of elevation from an observer x metres from the base of the building to the top of the banner is 2α ,while the angle of elevation to the bottom of the banner is α.



(a) Show that the height, h, of the building is $h = x\tan 2\alpha$. **1**



$\tan 2\alpha = \frac{h}{x}$

$h = x . \tan 2\alpha$ ... As req'd.

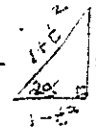
1	correct soln

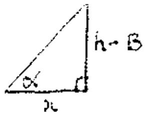
Question 25 continued

(b) Using $t = \tan\alpha$, or otherwise, show that the height, B, of the banner is $B = x\tan\alpha\sec 2\alpha$ metres. **3**

$t = \tan\alpha$

• $t = \tan\alpha$ • $h = x\tan 2\alpha$ • $\tan 2\alpha = \dfrac{2t}{1-t^2}$





$\tan\alpha = \dfrac{h-B}{x}$

$$\begin{aligned}
h - B &= x\tan\alpha \\
B &= h - x\tan\alpha \\
&= x\tan 2\alpha - x\tan\alpha \\
&= x \times \frac{2t}{1-t^2} - xt \\
&= xt\left(\frac{2}{1-t^2} - 1\right) \\
&= xt\left(\frac{2-(1-t^2)}{1-t^2}\right) \\
&= xt\left(\frac{1+t^2}{1-t^2}\right) \\
&= xt\left(\frac{1}{\cos 2\alpha}\right) \\
&= x.\tan\alpha.\sec 2\alpha \quad \text{... As req'd.}
\end{aligned}$$

1	$\tan 2\alpha = \dfrac{2t}{1-t^2}$ or $\tan\alpha = \dfrac{h-b}{x}$
2	$B = h - x.\tan\alpha$ $\therefore B = x.\tan 2\alpha - x.\tan\alpha$ attempts to replace $\tan 2\alpha$ and $\tan\alpha$ with t-formulae
3	correct conclusion

Section II Part C question 26 is on back page of this booklet.

26 Given $f(x) = x^3 - 1$, $g(x) = 3x + 1$ and $h(x) = 4x - 5$,

(a) show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$, **2**

$$\begin{aligned}
g(f(x)) &= g(x^3-1) \\
&= 3(x^3-1)+1 \\
&= 3x^3 - 2
\end{aligned}$$

$$\begin{aligned}
g(f(x)) + x.h(x) &= 3x^3 - 2 + x(4x-5) \\
&= 3x^3 - 2 + 4x^2 - 5x \\
&= 3x^3 + 4x^2 - 5x - 2
\end{aligned}$$

1	$g(f(x))$ $= 3(x^3-1)+1$
2	correct soln

(b) solve $g(f(x)) + xh(x) = 0$. **3**

$3x^3 + 4x^2 - 5x - 2 = 0$

$$\begin{aligned}
P(1) &= 3(1)^3 + 4(1)^2 - 5(1) - 2 \\
&= 3 + 4 - 5 - 2 \\
&= 0
\end{aligned}$$

$\therefore (x-1)$ is a factor.

$$\begin{array}{r}
3x^2 + 7x + 2 \\
x - 1 \,\big)\, \overline{3x^3 + 4x^2 - 5x - 2} \\
\underline{3x^3 - 3x^2} \quad\quad\quad\; \\
7x^2 - 5x \quad\;\; \\
\underline{7x^2 - 7x} \quad\;\; \\
2x - 2 \\
\underline{2x - 2}
\end{array}$$

ie. $3x^3 + 4x^2 - 5x - 2 = 0$

$(x-1)(3x^2 + 7x + 2) = 0$

$(x-1)(3x+1)(x+2) = 0$

$x = -2, -\frac{1}{3}, 1$

$3x$ 1
x 2

1	$P(1) = 0$ or $P(-2) = 0$
2	Finds other facto by long ÷ or roots are $-2, 1, \alpha$ $\therefore \alpha - 1 = \frac{4}{3}$ $\alpha = -\frac{1}{3}$
3	soln: $x = -2$ or $-\frac{1}{3}$ or 1

Section II - Part D (13 marks) Name............................

Teacher: PL MM JS BD NF DG

27 A Cartesian equation can be defined by the parametric equations $x = \frac{t}{t+1}$ and $y = \frac{t}{t-1}$.

(a) Show that $x = \frac{t}{t+1}$ can be rewritten as $t = \frac{x}{1-x}$. 2

$x = \frac{t}{t+1}$

$x(t+1) = t$

$xt + x = t$ 1

$x = t - xt$

$x = t(1-x)$ 1

$\therefore \quad t = \frac{x}{1-x}$ ... As req'd 2

(b) Hence, find the Cartesian equation defined by the parametric equations $x = \frac{t}{t+1}$ and $y = \frac{t}{t-1}$. 2

Subst. $t = \frac{x}{1-x}$ into $y = \frac{t}{t-1}$

$= \dfrac{\frac{x}{1-x}}{\frac{x}{1-x} - 1}$ 1

$= \dfrac{\frac{x}{1-x}}{\frac{x-(1-x)}{1-x}} \times \frac{1-x}{1-x}$

$y = \frac{x}{2x-1}$ 1

2

2

28 Find the inverse function of $y = \log_2(x-3)$.

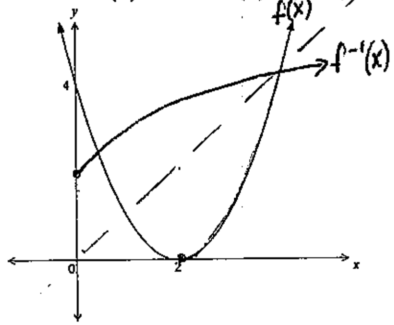
Inverse fn.: $x = \log_2(y-3)$ 1

$y - 3 = 2^x$

$y = 2^x + 3$ 1

2

29 The graph shows the parabola $y = f(x)$, given $f(x) = (x-2)^2$.



(a) Explain why an inverse function $y = f^{-1}(x)$ exists when $x \geq 2$. 1

For $x \geq 2$, $f(x)$ is monotonically increasing,

i.e. It passes the horizontal line test.

$\therefore$ When $x \geq 2$, $y = f^{-1}(x)$ exists 1

(b) State the domain and range of this inverse function. 2

$f^{-1}(x)$: D $x : x \geq 0, x \in \mathbb{R}$ 1

R $y : y \geq 2, y \in \mathbb{R}$ 1

2

(c) At what point will $f(x) = f^{-1}(x)$? 1

Meet on line of reflection $y = x$:

$y = (x-2)^2$

i.e. $x = (x-2)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 1$ or 4 1

For $x \geq 2$, Point $= (4, 4)$

30 Prove that $\dfrac{\cos x-\cos(x+2\theta)}{2\sin\theta}=\sin(x+\theta)$. 3

$$\text{LHS} = \frac{\cos x - \cos(x+2\theta)}{2\sin\theta}$$

$$= \frac{\cos x - \cos x\cos 2\theta + \sin x\sin 2\theta}{2\sin\theta} \quad 1$$

$$= \frac{\cos x - \cos x(1-2\sin^2\theta) + \sin x(2\sin\theta\cos\theta)}{2\sin\theta} \quad 1$$

$$= \frac{\cos x - \cos x + 2\cos x\sin^2\theta + 2\sin x\sin\theta\cos\theta}{2\sin\theta}$$

$$= \frac{2\cos x\sin^2\theta + 2\sin x\sin\theta\cos\theta}{2\sin\theta}$$

$$= \frac{2\sin\theta(\cos x\sin\theta + \sin x\cos\theta)}{2\sin\theta} \quad 1$$

$$= \sin x\cos\theta + \cos x\sin\theta$$

$$= \sin(x+\theta)$$

$= \text{RHS}$ ... As req'd

3

* Do not cross out when simplifying

End of Paper

Extra writing space

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