
Student Number

SCEGGS Darlinghurst

2021 | **Year 11**
Task 3

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black pen.
- Write your student number at the top of each page
- NESA approved calculators may be used.
- A Reference Sheet is provided at the end of this task.
- For Questions 5-7, show relevant mathematical reasoning and/or calculations.

Total marks

/35

Multiple Choice: 4 marks

- Attempt Question 1-4
- Allow about 8 minutes for the Multiple choice.
- Write your answer to each Multiple choice question on your own A4 paper.

Extended Response: 31 marks

- Attempt Question 5-7
- Allow about 52 minutes for the Extended Response.
- Answer each question on a new piece of A4 paper.

At the conclusion of the assessment, upload your responses in three separate sections I, II, III.	
Section I	Multiple choice Q1-4 and Q5
Section II	Q6
Section III	Q7

Section I

Multiple choice

4 marks
Attempt Questions 1–4
Allow about 8 minutes for the Multiple choice.

Write the answer to each Multiple choice question on your own paper.

1. Which of the following is an expression for $\sin(A - B) - \sin(A + B)$?

   A. $-2\sin A\cos B$

   B. $-2\cos A\sin B$

   C. $2\sin A\cos B$

   D. $2\cos A\sin B$

2. Libby owns five non-identical black dresses and four non-identical white dresses. In how many ways can she arrange the dresses in a line in her cupboard if the black and white dresses alternate?

   A. $5!4!$

   B. $2!5!4!$

   C. $\frac{9!}{5!4!}$

   D. $\frac{5!4!}{2}$

3. What is the exact value of $\tan 75^\circ$?

   A. $-2 + \sqrt{3}$

   B. $-2 - \sqrt{3}$

   C. $2 + \sqrt{3}$

   D. $2 - \sqrt{3}$

4. What is the domain and range of $y = 2\sin^{-1}\left(\frac{x}{3}\right)$?

   A. $-1 \leq x \leq 3$ and $-3 \leq y \leq 3$

   B. $-1 \leq x \leq 3$ and $-\pi \leq y \leq \pi$

   C. $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$

   D. $-3 \leq x \leq 3$ and $-\pi \leq y \leq \pi$

End of Multiple choice

Extended response

31 marks
Attempt Questions 5-7
Allow about 52 minutes for this section

Answer each question on a new piece of A4 paper.

In Questions 5-7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (10 marks) Start a new piece of A4 paper.

(a) Evaluate $\tan^{-1}(-1.75)$ correct to 3 significant figures. **1**

(b) In a class of 23 students, there are 12 girls and 11 boys.

(i) The class selects a captain and vice-captain. In how many ways is this possible if there are no restrictions? **1**

(ii) A group of 4 students consisting of 2 girls and 2 boys is chosen at random from the 23 students to go on the student representative council.

(α) In how many ways could this be done? **1**

(β) What is the probability that a certain girl, Alana, and a certain boy, Brian, have been included in the student representative council group? **2**

(c) Find the exact value of $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right)$. **1**

(d) SCEGGS has six houses. Badham, Barton, Beck, Christian, Docker, Langley. How many students must be chosen from the whole school to ensure that at least 100 students are from at least one of the houses? (Explain your reasoning) **2**

(e) Use factorial notation to show that $\binom{8}{5} = \binom{7}{4} + \binom{7}{5}$ **2**

End of Question 5

End of Section I

Section II

Start a new piece of A4 paper

Question 6 (10 marks)

(a) If $\sin\theta = \frac{5}{\sqrt{34}}$ for $\frac{\pi}{2} \le \theta \le \pi$, what is the exact value of $\sin 2\theta$? **2**

(b) Show, without a calculator, that:

$$2\cos 40^\circ \cos 20^\circ - 2\sin 25^\circ \sin 5^\circ = \frac{1}{2}\left(\sqrt{3} + 1\right)$$ **2**

(c) At a staff meeting, four Maths teachers, three History teachers and two Art teachers are randomly seated around a circular table.

(i) How many seating arrangements are there without restriction? **1**

(ii) How many arrangements are there if the History teachers sit together? **1**

(iii) How many arrangements are there if the teachers are seated together in groups according to their departments? **1**

(d) Consider the function $f(x) = 4\tan^{-1}(x-1)$.

Sketch the function $y = f(x)$. Show all features and intercepts with the axes. **3**

End of Question 6 and Section II

Section III

Start a new piece of A4 paper

Question 7 (11 marks)

(a) Prove that $\cos\left(\sin^{-1}\left(\frac{1}{4}\right) - \tan^{-1}\left(-\frac{1}{2}\right)\right) = \frac{10\sqrt{3}-\sqrt{5}}{20}$. **3**

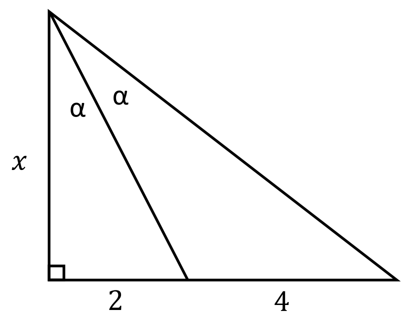
(b) The letters in the word SCREENTIME are arranged randomly.

(i) How many arrangements are there with no restrictions? **1**

(ii) How many arrangements are there if the first letter is a consonant and the last letter is a vowel? **2**

(c) Using $t = \tan\left(\frac{\theta}{2}\right)$, show that $\frac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta} = \frac{1+\tan\left(\frac{\theta}{2}\right)}{1-tan\left(\frac{\theta}{2}\right)}$ **2**

(d) Find the exact value of the angle, α, shown in the diagram below. **3**



Not to scale

End of Question 7 and Section III

End of assessment

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

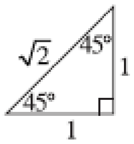
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

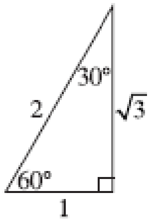
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

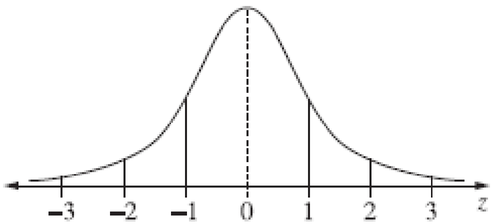
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

– 2 –

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

– 3 –

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 Task 3 11Ext 1 Yearly solutions

1. Which of the following is an expression for $\sin(A-B)-\sin(A+B)$?

A. $-2\sin A\cos B$

B. $-2\cos A\sin B$

C. $2\sin A\cos B$

D. $2\cos A\sin B$

$\sin(A-B) - \sin(A+B)$
$= \sin A\cos B - \cos A\sin B - (\sin A\cos B + \cos A\sin B)$
$= \sin A\cos B - \cos A\sin B - \sin A\cos B - \cos A\sin B$
$= -2\cos A\sin B$

B

2. Libby owns five non-identical black dresses and four non-identical white dresses.
In how many ways can she arrange the dresses in a line in her cupboard if the black and white dresses alternate?

A. 5!4!

B. 2!5!4!

C. $\frac{9!}{5!4!}$

D. $\frac{5!4!}{2}$

B W B W B W B W B

$\underset{\text{arrange black}}{5!}\cdot\underset{\text{arrange white}}{4!}$

A

3. What is the exact value of $\tan 75^\circ$?

can't be negative

A. ~~$-2+\sqrt{3}$~~

B. ~~$-2-\sqrt{3}$~~

C. $2+\sqrt{3}$

D. $2-\sqrt{3}$

Quick way

You could just test this on your calculator.

Long way

$\tan 75^\circ = \tan(30^\circ + 45^\circ)$

$= \dfrac{\tan 30^\circ + \tan 45^\circ}{1-\tan 30^\circ \tan 45^\circ}$

$= \dfrac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}\times 1} \quad \times\frac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1}$

rationalise

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{3+2\sqrt{3}+1}{3-1}$

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$

C

4. What is the domain and range of $y = 2\sin^{-1}\left(\frac{x}{3}\right)$?

A. $-1 \le x \le 3$ and $-3 \le y \le 3$

B. $-1 \le x \le 3$ and $-\pi \le y \le \pi$

C. $-3 \le x \le 3$ and $-3 \le y \le 3$

D. $-3 \le x \le 3$ and $-\pi \le y \le \pi$

domain

$-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$

range

$-\frac{\pi}{2} \le \frac{y}{2} \le \frac{\pi}{2}$

$-\pi \le y \le \pi$

D

Question 5 (10 marks) Start a new piece of A4 paper.

(a) Evaluate $\tan^{-1}(-1.75)$ correct to 3 significant figures. **1**

Calculator in radians -1.05 (3 sf) ✓

Calculator in degrees -60.3° (3 sf) (must state degrees)

(b) In a class of 23 students, there are 12 girls and 11 boys.

(i) The class selects a captain and vice-captain. In how many ways is this possible if there are no restrictions? **1**

(ii) A group of 4 students consisting of 2 girls and 2 boys is chosen at random from the 23 students to go on the student representative council.

(α) In how many ways could this be done? **1**

(β) What is the probability that a certain girl, Alana, and a certain boy, Brian, have been included in the student representative council group? **2**

i) $\underset{\text{Capt}}{\underline{23}} \times \underset{\text{Vice}}{\underline{22}} = 506$ or ${}^{23}P_2$ ✓

ii) α) ${}^{12}C_2 \times {}^{11}C_2 = 3630$ ✓

β) $\underset{\text{Alana}}{1} \times \underset{\text{Brian}}{1} \times \underset{\text{girl}}{{}^{11}C_1} \times \underset{\text{boy}}{{}^{10}C_1} = 110$ ✓

probability $= \dfrac{110}{3630}$ ✓

$= \dfrac{1}{33}$

(c) Find the exact value of $\arctan\left(\tan\left(\frac{2\pi}{3}\right)\right)$. **1**

$\alpha = \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

$= \tan^{-1}(-\sqrt{3})$ where $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$= -\frac{\pi}{3}$ ✓

(d) SCEGGS has six houses. Badham, Barton, Beck, Christian, Docker, Langley.
How many students must be chosen from the whole school to ensure that
at least 100 students are from at least one of the houses? (Explain your reasoning) **2**

By the pigeonhole principle

Badham Barton Beck Christian Docker Langley

the 6 houses are the pigeonholes

progress

The pigeons are the minimum number of students

$n = 6 \times 99 + 1$

$= 595$

$\therefore$ 595 students have to be chosen to have at least 100 students in at least one house.

(e) Use factorial notation to show that $\binom{8}{5} = \binom{7}{4} + \binom{7}{5}$ **2**

$\text{LHS} = \binom{8}{5}$

$= \dfrac{8!}{5!\,3!}$

$\text{RHS} = \binom{7}{4} + \binom{7}{5}$

$= \dfrac{7!}{4!\,3!} + \dfrac{7!}{5!\,2!}$

make a common denominator.

$\begin{cases} 5! = 5 \times 4! \\ 3! = 3 \times 2! \end{cases}$

$= \dfrac{5 \times 7!}{5 \times 4!\,3!} + \dfrac{3 \times 7!}{5! \times 3 \times 2!}$

factorise top

$= \dfrac{7!\,(5+3)}{5!\,3!}$

$= \dfrac{7! \times 8}{5!\,3!}$

$= \dfrac{8!}{5!\,3!}$

$= \binom{8}{5}$

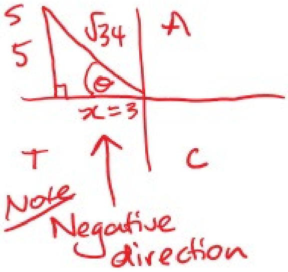
$\therefore$ LHS = RHS

$\binom{8}{5} = \binom{7}{4} + \binom{7}{5}$

Question 6 (10 marks)

(a) If $\sin\theta = \frac{5}{\sqrt{34}}$ for $\frac{\pi}{2} \leq \theta \leq \pi$, what is the exact value of $\sin 2\theta$? **2**

$\sin\theta = \frac{5}{\sqrt{34}}$, $\frac{\pi}{2} \leq \theta \leq \pi$ $\quad \therefore \theta$ lies in Quadrant 2



By pythagoras $x = \sqrt{(\sqrt{34})^2 - 5^2}$

$= \sqrt{34-25}$

$= \sqrt{9}$

$= 3$ ✓

$\therefore \cos\theta = -\frac{3}{\sqrt{34}}$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{5}{\sqrt{34}} \times -\frac{3}{\sqrt{34}}$

$= -\frac{30}{34}$ ✓ double angle result.

$= -\frac{15}{17}$

(b) Show, without a calculator, that:

$$2\cos 40^\circ \cos 20^\circ - 2\sin 25^\circ \sin 5^\circ = \frac{1}{2}(\sqrt{3}+1)$$ **2**

reference sheet

$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

LHS $= 2\cos 40^\circ \cos 20^\circ - 2\sin 25^\circ \sin 5^\circ$

$= 2 \times \frac{1}{2}[\cos(40-20)^\circ + \cos(40+20)^\circ] - 2 \times \frac{1}{2}[\cos(25-5)^\circ - \cos(25+5)^\circ]$

$= \cos 20^\circ + \cos 60^\circ - (\cos 20^\circ - \cos 30^\circ)$ ✓

$= \cos 60^\circ + \cos 30^\circ$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$ ✓

$= \frac{1}{2}(1+\sqrt{3})$

$=$ RHS

(c) At a staff meeting, four Maths teachers, three History teachers and two Art teachers are randomly seated around a circular table.

(i) How many seating arrangements are there without restriction? **1**

(ii) How many arrangements are there if the History teachers sit together? **1**

(iii) How many arrangements are there if the teachers are seated together in groups according to their departments? **1**

i) No restrictions seating 9 people around a table

$\underbrace{1}_{\text{seat one person}} \times \underbrace{8!}_{\text{seat the rest}} = 40320$ ✓

ii) 

$\underbrace{1}_{\text{Seat history teachers}} \times \underbrace{6!}_{\text{seat the rest}} \times \underbrace{3!}_{\text{History teachers can swap}} = 4320$ ✓

iii) 

$\underbrace{1}_{\text{seat one group}} \times \underbrace{2!}_{\text{seat other 2 groups}} \times \underbrace{2!}_{\text{arrange Art teachers}} \times \underbrace{3!}_{\text{arrange History}} \times \underbrace{4!}_{\text{arrange maths}} = 576$ ✓

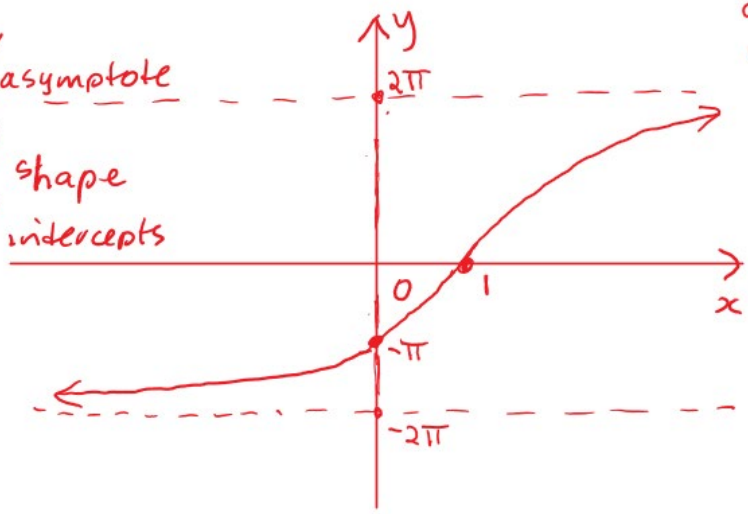
(d) Consider the function $f(x) = 4\tan^{-1}(x - 1)$.

Sketch the function $y = f(x)$. Show all features and intercepts with the axes. **3**

✓ asymptote

✓ shape

✓ intercepts



d: $-\infty < x < \infty$

r: $-\frac{\pi}{2} < \frac{y}{4} < \frac{\pi}{2}$

$-\frac{4\pi}{2} < y < \frac{4\pi}{2}$

$-2\pi < y < 2\pi$

x intercept $y=0$

$0 = 4\tan^{-1}(x-1)$

$0 = \tan^{-1}(x-1)$

$\tan 0 = x-1$

$0 = x-1$

$x = 1$

y intercept $x=0$

$y = 4\tan^{-1}(-1)$

$= 4 \times -\frac{\pi}{4}$

$= -\pi$

Question 7 (11 marks)

(a) Prove that $\cos\left(\sin^{-1}\left(\frac{1}{4}\right) - \tan^{-1}\left(-\frac{1}{2}\right)\right) = \frac{10\sqrt{3}-\sqrt{5}}{20}$. **3**

$\cos\left(\sin^{-1}\left(\frac{1}{4}\right) - \tan^{-1}\left(-\frac{1}{2}\right)\right)$

$= \cos(A - B)$

Let $A = \sin^{-1}\frac{1}{4}$

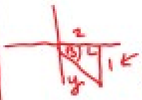
$\sin A = \frac{1}{4}$ where $-\frac{\pi}{2} \le A \le \frac{\pi}{2}$

By pythagoras
$x = \sqrt{4^2 - 1^2}$
$= \sqrt{15}$

$\therefore \cos A = \frac{\sqrt{15}}{4}$

Let $B = \tan^{-1}(-\frac{1}{2})$

$\tan B = -\frac{1}{2}$ where $-\frac{\pi}{2} < B < \frac{\pi}{2}$



Negative direction

By pythagoras $y = \sqrt{1^2 + 2^2}$
$= \sqrt{5}$

$\sin B = \frac{-1}{\sqrt{5}}$

$\cos B = \frac{2}{\sqrt{5}}$

✓ correct set up.

$\therefore \cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \frac{\sqrt{15}}{4} \times \frac{2}{\sqrt{5}} + \frac{1}{4} \times -\frac{1}{\sqrt{5}}$

✓ identity work

$= \frac{2\sqrt{15} - 1}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

rationalise denominator

$= \frac{2\sqrt{75} - \sqrt{5}}{20}$

$= \frac{2 \times 5\sqrt{3} - \sqrt{5}}{20}$

$= \frac{10\sqrt{3} - \sqrt{5}}{20}$

✓ surds work

$= RHS$.

(b) The letters in the word SCREENTIME are arranged randomly.

(i) How many arrangements are there with no restrictions? **1**

(ii) How many arrangements are there if the first letter is a consonant and the last letter is a vowel? **2**

SCRENTIM
E
E

10 letters, 3 repeated E

i) No restriction $\frac{10!}{3!} = 604800$ ✓

ii) □ - - - - - - - - □

consonant
SCRNTM

vowel
EEEI

$\left({}^{6}C_{1} \times {}^{4}C_{1} \times 8! \right) \div 3!$

choose consonant, choose vowel, arrange rest, repeated E

✓ ✓

$= 161280$

(c) Using $t = \tan\left(\frac{\theta}{2}\right)$, show that $\frac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta} = \frac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}$ **2**

$$LHS = \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta}$$

$$= \frac{\frac{2t}{1+t^2} + 1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2} - 1 + \frac{1-t^2}{1+t^2}} \quad \begin{matrix}\times(1+t^2)\\ \times(1+t^2)\end{matrix}$$ ✓

$$= \frac{2t + (1+t^2) - (1-t^2)}{2t - (1+t^2) + (1-t^2)}$$

$$= \frac{2t + \cancel{1} + t^2 - \cancel{1} + t^2}{2t - 1 - t^2 + 1 - t^2}$$ ✓

$$= \frac{2t + 2t^2}{2t - 2t^2}$$

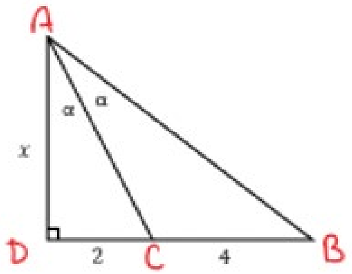
$$= \frac{2t(1+t)}{2t(1-t)}$$

$$= \frac{1+t}{1-t}$$

$$= \frac{1 + \tan\frac{\theta}{2}}{1 - \tan\frac{\theta}{2}}$$

$= RHS.$

(d) Find the exact value of the angle, α, shown in the diagram below. **3**



Not to scale

in $\triangle ABD$ $\boxed{\tan 2\alpha = \frac{6}{x}}$

in $\triangle ACD$ $\boxed{\tan\alpha = \frac{2}{x}}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-(\tan\alpha)^2}$$

$$\frac{6}{x} = \frac{2\times\frac{2}{x}}{1-\left(\frac{2}{x}\right)^2}$$

$$\frac{6}{x} = \frac{\frac{4}{x}}{1-\frac{4}{x^2}} \quad \frac{\times x^2}{\times x^2}$$

↑ multiply top and bottom by x^2 to simplify

$$\frac{6}{x} = \frac{4x}{x^2-4}$$

$$6(x^2-4) = 4x^2$$

✓equation

$$6x^2-24 = 4x^2$$
$$2x^2 = 24$$
$$x^2 = 12$$
$$x = \pm\sqrt{12}$$
$$x = \pm 2\sqrt{3}$$

$x = 2\sqrt{3}$ only since $x > 0$ for length.

✓solution's validity.

$$\tan\alpha = \frac{2}{x}$$
$$= \frac{2}{2\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$

S	A ✓
✓T	C

$\alpha = 30^\circ, 210^\circ, 390^\circ, \ldots$

Since angles in a triangle add to 180°,
$\alpha = 30^\circ$ (or $\frac{\pi}{6}$) is the only valid solution