Sydney Girls High School

MATHEMATICS EXTENSION 1

Year 11 Yearly

Assessment Task 3

2016

Time Allowed: 60 minutes + 5 minutes reading time

Total: 56 marks

Instructions:

- There are FOUR (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- All answer must be written to the simplest exact form.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number at the top of each question and clearly number each question.

Student 's Name:______________________Teacher's Name:______________________

Question 1 (14 marks)

a) Find the remainder when $P(x) = x^4 - 7x^3 + 5x^2 + 3x - 9$ is divided by $x - 5$. 1

b) Solve for x: $|2x+5| = 3x+9$. 3

c) Solve for x: $x^2 - 6x > 0$. 2

d) Simplify $(a+b)^2 - \dfrac{a^3 - b^3}{a-b}$. 2

e) Find the value of k for which $(x+2)$ is a factor of $2x^3 + kx^2 + 5x - 2$. 2

f) A coin is tossed and a die is rolled. Find the probability of getting a multiple of 3 and a tail. 2

g) Let $f(x) = x^3 + 5x^2 + 17x - 10$. The equation $f(x) = 0$ has only one real root.

i) Show that the root lies between 0 and 2. 1

ii) Use one application of halving the interval method to find a smaller interval containing the root. 1

Question 2 (14 marks)

a) Solve: $\dfrac{5}{x+2} \leq 1$. 2

b) Find the point $P(x, y)$ dividing the interval from $A(1,5)$ to $B(-1,2)$ externally in the ratio $3:2$. 2

c) A bag contains one green, two blue and three red marbles. Jane selects one marble from the bag and puts it aside. She then selects a second marble from the bag.

i. Find the probability that both the selected balls are blue. 1

ii. Find the probability that at least one of the selected balls is blue. 2

d) For the curve $y = 2x^3 - 6x^2 - 18x - 1$

i) Find the stationary points and determine their nature. 3

ii) Find the coordinates of any inflexion point. 2

iii) Sketch the curve in the domain $-2 \leq x \leq 5$. 2

Question 3 (14 marks)

a) The polynomial $P(x)=x^4-3x^3+ax^2+bx-6$ leaves a remainder of *8* when divided by $(x+1)$ and $(x-3)$ is a factor of $P(x)$. Find the values of a and b. 3

b) If α, β and γ are the roots of $x^3-3x^2+x-9=0$, find the value of:

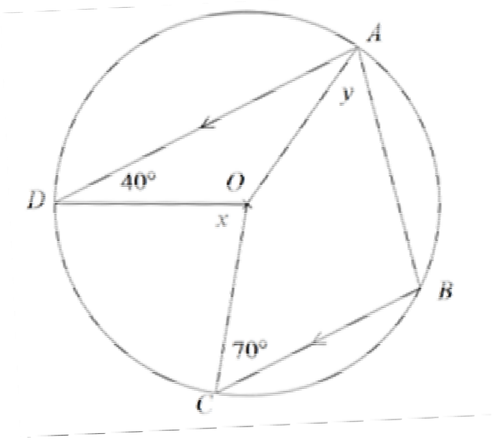
i) $\alpha\beta\gamma$ 1

ii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$ 2

iii) $\alpha^2+\beta^2+\gamma^2$ 2

c) Find the equation of the locus of a point $P(x, y)$ that moves so that the line PA is perpendicular to the line PB, given A and B have coordinates $(3,1)$ and $(5,-4)$ respectively. 2

d) Consider the circle with centre at O. Points A, B, C and D are on the circumference.



i. Find the value of x, giving reasons. 2

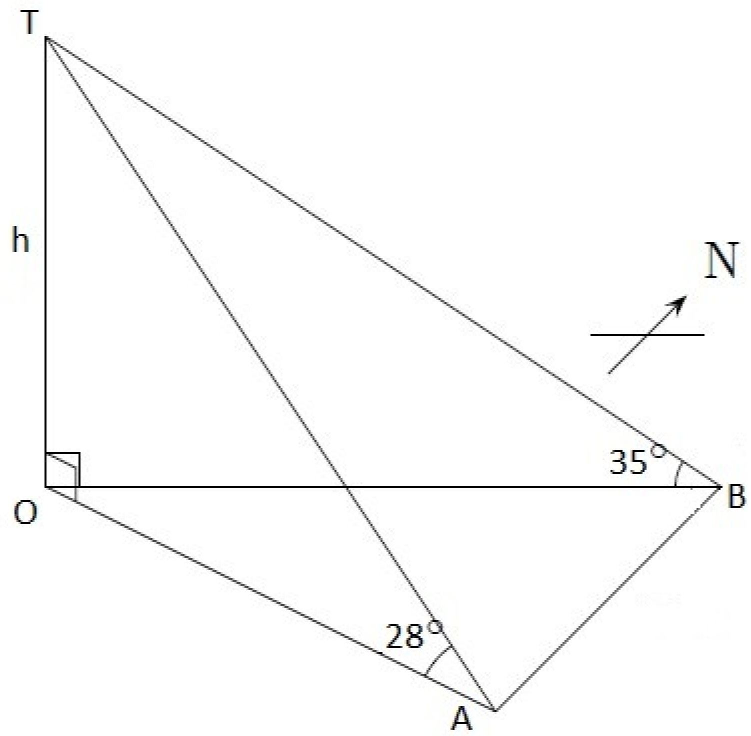
ii. Find the value of y, giving reasons. 2

Question 4 (14 marks)

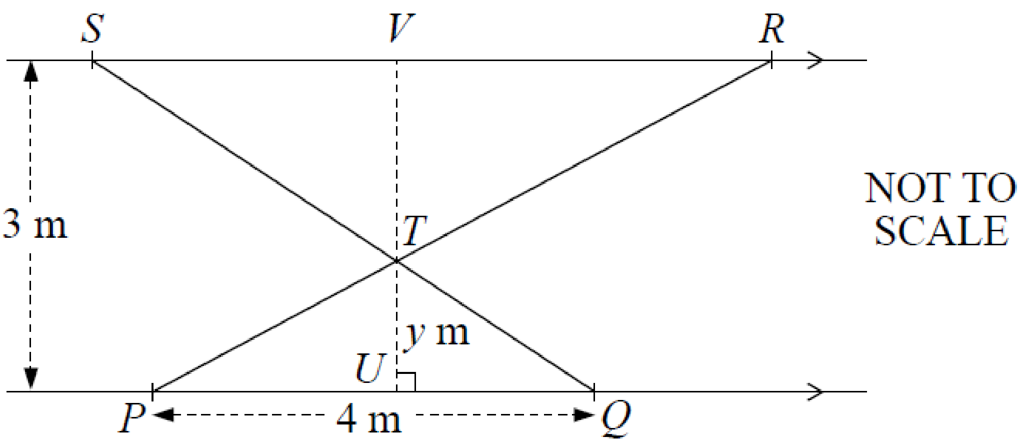
a) The graph of $y = f(x)$ passes through the point $(1,4)$ and $f'(x) = 2x + 7$. Find $f(x)$. 2

b) The polynomial $6x^3 - 7x^2 + ax + b$ has one root at $x = -1$ and the other two roots are reciprocals of each other. Find the values of *a* and *b*. 3

c) From a point *B* due east of a tower, the angle of elevation of the top of the tower *h* is 35°. From another point *A*, 120 m south-west of *B*, the angle of elevation of the top of the tower is 28°. Find the height of tower *h* to the nearest metre. 3



d) In the diagram, PQ and SR are parallel railings which are 3 metres apart.
The points P and Q are fixed 4 metres apart on the lower railing.
Two crossbars PR and QS intersect at T as shown in the diagram.
The line through T perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.



i. By using similar triangles, or otherwise, show that $\dfrac{SR}{PQ}=\dfrac{VT}{UT}$. **2**

ii. Show that $SR=\dfrac{12}{y}-4$. **1**

iii. Hence show that the total area A of ΔPTQ and ΔRTS is $A=4y-12+\dfrac{18}{y}$. **1**

iv. Find the value of y that minimises A. Justify your answer. **2**

The End.

Q1

a) $p(5) = 5^4 - 7(5)^3 + 5(5)^2 + 3(5) - 9$

$= -119$

$\therefore$ Remainder $= -119$ ✓

b) Solve $|2x + 5| = 3x + 9$

$$\left[\begin{array}{l} 2x + 5 = 3x + 9 \\ 2x + 5 = -3x - 9 \end{array}\right.$$ ✓ ✓

$$\left[\begin{array}{l} x = -4 \\ x = -\frac{14}{5} \end{array}\right.$$

Checking: with $x = -4$: $3 \neq -3$

$\therefore x = -4$ (Reject)

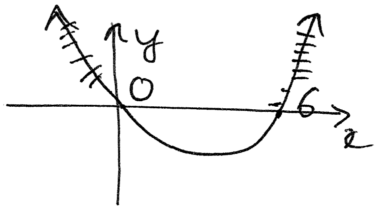
with $x = -\frac{14}{5}$: $\frac{3}{5} = \frac{3}{5}$

Final Solution : $x = -\frac{14}{5}$ ✓

* Some students did not check if both solutions are satisfied. Checking step is necesarily.

c) $x^2 - 6x > 0$

$x(x - 6) > 0$



$$\left[\begin{array}{l} x < 0 \\ x > 6 \end{array}\right.$$ OR ✓ ✓

d) $= \dfrac{(a-b)(a^2 + 2ab + b^2) - (a-b)(a^2 + ab + b^2)}{a-b}$ ✓

$= \dfrac{(a-b)[a^2 + 2ab + b^2 - a^2 - ab - b^2]}{(a-b)} = ab$ ✓

$\boxed{Q1}$ $f(x) = 2x^3 + kx^2 + 5x - 2$

e) $f(-2) = 0$ ✓

$4k - 28 = 0$

$k = 7$ ✓

f) $p(\text{multiple of } 3) = \frac{1}{3}$ ✓

$p(\text{ tail }) = \frac{1}{2}$

$p(\text{multiple of 3 and a tail}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ ✓

g) $f(x) = x^3 + 5x^2 + 17x - 10$

i) $f(0) = -10 < 0$ (below the x-axis) ✓

$f(2) = 52 > 0$ (above the x-axis)

Since $f(x)$ is a continuous function and $f(0) < 0$, $f(2) > 0$, there is at least one root between $x = 0$ and $x = 2$.

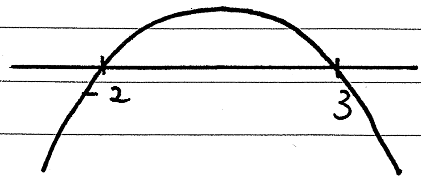
ii) $f\left(\frac{0+2}{2}\right) = f(1) = 13 > 0$ (above the x-axis)

Thus the root is now lies between $x = 0$ and $x = 1$ $\therefore$ $[0, 1]$ ✓

Question 2

(a) $\dfrac{5}{x+2} \leq 1$ Note: $x \neq -2$

$$5(x+2) \leq (x+2)^2$$
$$(x+2)(5-(x+2)) \leq 0$$
$$(x+2)(3-x) \leq 0$$



$\therefore x < -2$ or $x \geq 3$

Note: Common error was not excluding $x = -2$ from the solution set.

(b) $A(1,5)$ $B(-1,2)$

$3 : -2$

$$P = \left(\frac{3(-1) + 1(-2)}{3-2}, \frac{3(2) + 5(-2)}{3-2} \right)$$

$\therefore P = (-5, -4)$

Note: Read the question carefully (external, not internal division) and know how to use the formula correctly

(c) 1 G, 2 B, 3 R

(i) P(both blue) $= \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

(ii) P(at least one blue) $= 1 - P(\text{no blue})$

$$= 1 - \frac{4}{6} \times \frac{3}{5}$$

$$= \frac{3}{5}$$

Note: Common errors included not recognising the 1st ball is not replaced and not considering all cases in (ii) when adding all cases with at least 1 blue ball (the easiest approach is shown above)

Question 2 (continued)

(d) $y = 2x^3 - 6x^2 - 18x - 1$

(i) $y' = 6x^2 - 12x - 18$

$y'' = 12x - 12$

Stat. pts. when $y' = 0$

$6(x^2 - 2x - 3) = 0$

$6(x-3)(x+1) = 0$

$\therefore x = 3$ or $x = -1$

when $x = 3$, $y'' = 36 - 12$

$y'' = 24 > 0$ $\therefore (3, -55)$ is a minimum stat. pt.

when $x = -1$, $y'' = -12 - 12$

$= -24 < 0$

$\therefore (-1, 9)$ is a maximum stat. pt.

(ii) Inflexion points when $y'' = 0$ $12x - 12 = 0$

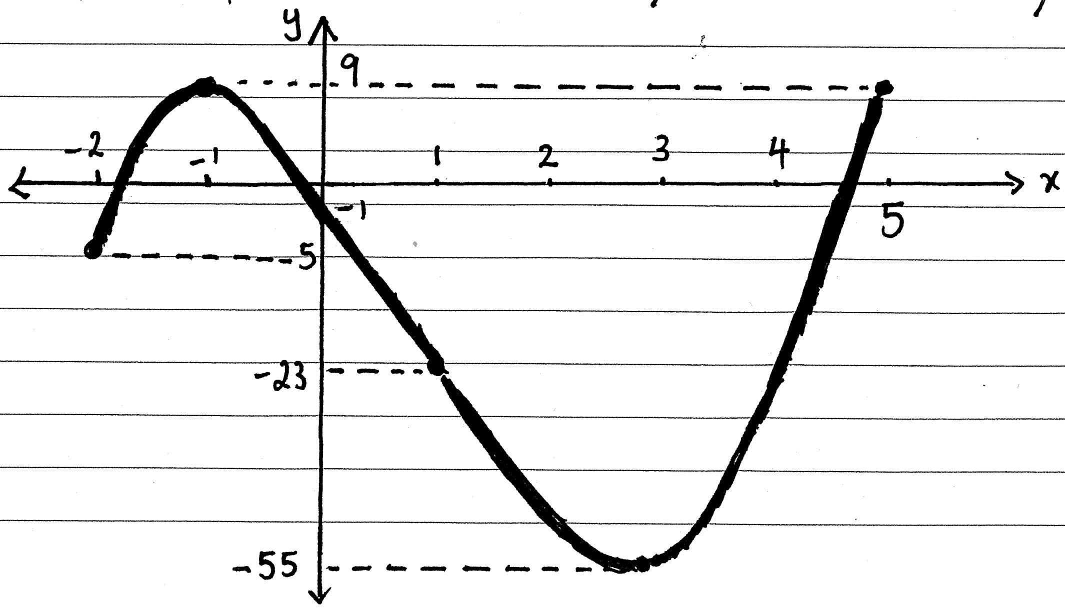
$\therefore$ Possible inflexion point at $x = 1$.

Check concavity

x	0	1	2
y''	-12	0	$+12$

Since concavity changes, $(1, -23)$ is an inflexion point.

(iii) Endpoints at $(-2, -5)$ and $(5, 9)$.



Comment on (d).

The main areas where responses could improve:

(i) clearly indicate the points and show the reasoning for their nature

(ii) You must check concavity to confirm it is an inflexion point

(iii) Many diagrams had issues with the shape (incorrect concavity in parts) and a reasonable scale and lack of endpoints.

3 a) $P(-1) = 8$

$1 + 3 + a - b - 6 = 8$

$a - b = 10$

$P(3) = 0$

$81 - 81 + 9a + 3b - 6 = 0$

$9a + 3b = 6$

$3a + b = 2$

$4a = 12$ [Students who tried division of polynomials were unsuccessful]

$a = 3$

$3 - b = 10$

$b = -7$

b) i) $-\frac{-9}{1} = 9$

ii) $\frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma} = \frac{\frac{1}{1}}{9}$

$= \frac{1}{9}$

iii) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= \left(-\frac{-3}{1}\right)^2 - 2 \times 1$ [Many students got the 1st line incorrect and hence 0 marks]

$= 7$

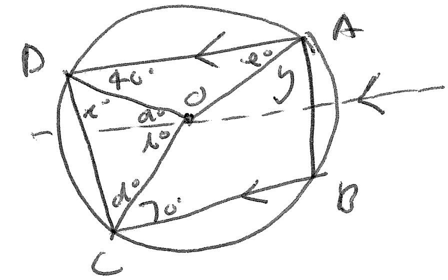
c) $m_{PA} \times m_{PB} = -1$

$\frac{y-1}{x-3} \times \frac{y+4}{x-5} = -1$

$y^2 + 3y - 4 = -(x^2 - 8x + 15)$

$x^2 + y^2 - 8x + 3y + 19 = 0$

[$PA^2 + PB^2 = 0$ is incorrect]

d) 

(i) $a° = 40°$ (alternate ∠s, || lines)

$b° = 70°$ (" " , " ")

$x° = a° + b°$

$= 110°$

(ii) $OD = OC$ (radii of a circle)

ΔOCD is isosceles (2 sides equal)

$c° = d°$ (base ∠s of an isosceles Δ)

$x° + 2d° = 180°$ (∠ sum of a Δ)

$2d° = 70°$

$d° = 35$

$OD = OA$ (radii of a circle)

ΔOAD is isosceles (2 sides equal)

$e° = 40°$ (base ∠s of an isosceles Δ)

$y° + e° = d° + 70°$ (opposite ∠s of a cyclic quad)

$\therefore y = 35°$

[Students tended to lose marks in this question because of incomplete reasoning]

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Question 4.

(a) $f'(x) = 2x + 7$ $\quad (1, 4)$.

$f(x) = x^2 + 7x + C$.

$f(1) = (1)^2 + 7(1) + C = 4$.

$8 + C = 4$

$C = -4$.

$\therefore f(x) = x^2 + 7x - 4$

(b) Let the roots be $-1, \alpha, \frac{1}{\alpha}$.

Sum of roots

$\Sigma\alpha = -1 + \alpha + \frac{1}{\alpha} = \frac{7}{6}$.

$\Sigma\alpha\beta = -\alpha - \frac{1}{\alpha} + 1 = \frac{a}{6}$.

So $-\frac{7}{6} = \frac{a}{6}$

Many students tried alternate methods by finding P(-1). Using the sum and product of roots was the most efficient and yielded less mistakes.

$\therefore a = -7$.

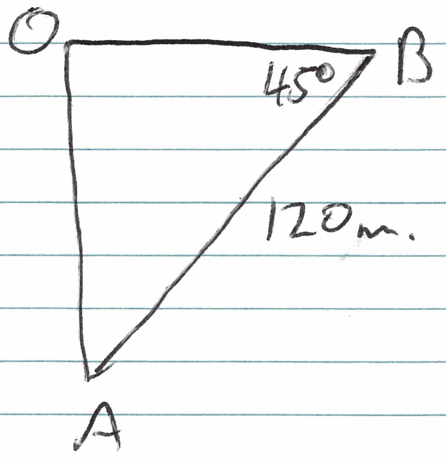
Product of roots

$-1 = -\frac{b}{6}$

$\therefore b = 6$.

$\boxed{\begin{array}{l} a = -7 \\ b = 6 \end{array}}$

(c) Base Δ OBA.



Most students did not use the Cosine Rule correctly which resulted in an equation for h^2 when the actual equation is quadratic with a h-term as you can see.

$$\tan 35° = \frac{h}{OB} \Rightarrow OB = \frac{h}{\tan 35°}.$$

$$\tan 28° = \frac{h}{OA} \Rightarrow OA = \frac{h}{\tan 28°}$$

Using cosine rule on the base triangle gives.

$$OA^2 = 120^2 + OB^2 - 240 \times OB \cos 45°.$$

$$\frac{h^2}{\tan^2 28°} = 120^2 + \frac{h^2}{\tan^2 35°} - \frac{240h}{\sqrt{2}\tan 35°}$$

$$h^2 \underbrace{\left(\frac{1}{\tan^2 28°} - \frac{1}{\tan^2 35°}\right)}_{a} + h\underbrace{\left(\frac{240}{\sqrt{2}\tan 35°}\right)}_{b} \underbrace{- 120^2}_{c} = 0.$$

This is a quadratic equation in terms of h, So

$$h = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{since } h > 0.$$

$$h \approx 46\text{m}$$

(d)(i) Given that $\Delta SRT ||| \Delta QPT$ all dimension are in the same ratio.

Side length   Altitudes

$$\frac{SR}{PQ} = \frac{VT}{UT}$$

All dimensions of similar triangles are in proportion but this needed to be explicitly stated. So just saying sides are in proportion does not cover the altitudes.

Or alternate solution for (i)

$\Delta SRT ||| \Delta QPT$

so $\frac{SR}{PQ} = \frac{RT}{PT}$

and $\Delta VRT ||| \Delta UPT$.

so $\frac{VT}{UT} = \frac{RT}{PT}$

$\therefore \frac{SR}{PQ} = \frac{VT}{UT}$

(ii)

Find SR. $VT = 3 - y$

with $PQ = 4$ $UT = y$.

$$\frac{SR}{4} = \frac{3-y}{y}$$

$$SR = \frac{12}{y} - 4.$$

(iii) area $\Delta PQT = 2y$

area $\Delta RST = \frac{1}{2}\left(\frac{12}{y} - 4\right)(3-y)$

$= \left(\frac{6}{y} - 2\right)(3-y)$

$= \frac{18}{y} - 6 - 6 + 2y$

Total area $A(y) = 4y - 12 + \frac{18}{y}$

(iv) $A'(y) = 4 - \frac{18}{y^2}$

$A''(y) = \frac{36}{y^3}$

Stationary points where $A'(y) = 0$.

$4 = \frac{18}{y^2}$

$2y^2 = 9$

$y = \pm\frac{3}{\sqrt{2}}$

Nature of Stationary points.

$A''\left(\frac{3}{\sqrt{2}}\right) = \frac{36}{\frac{27}{2\sqrt{2}}}$

$= \frac{72\sqrt{2}}{27}$

$= \frac{8\sqrt{2}}{3} > 0$

Students need to demonstrate that the stationary point is actually a minimum with numbers not just plus and minus symbols that confirm the question.

$\therefore y = \frac{3\sqrt{2}}{2}$ is when the area A is minimum