Sydney Girls High School

MATHEMATICS EXTENSION 1

YEAR 11

Assessment Task 3

2018

Reading time :	5 minutes
Writing time :	60 minutes
Total marks :	60
Topics:	Harder algebra and inequalities, Polynomials, Permutations and Combinations, Circle Geometry, Further Trigonometry, Parametric Parabola, Division of an Interval in a given ratio and Iterative methods.

General Instructions:

- There are four (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.

Student Name:___________________________**Teacher Name:**____________________

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Question 1 (15 Marks)

a) Solve $(x+3)(x-5) \leq 0$ and sketch the solution on a number line. 3

b) Sketch $y=(x+5)(x-1)(3-x)$, showing all intercepts. 2

c) Find the exact value of $\tan 75^\circ$. 2

d) Solve $\dfrac{x}{x-2} \geq 3$. 3

e) The lines $x-2y-5=0$, $x+3ky=15$ and $4y=3x+20$ are concurrent.

Find the value of k. 2

f) Simplify $\dfrac{x^3-x^2}{8x^2+10x-3} \div \dfrac{4x^2-3x-1}{16x^2-1}$. 3

Question 2 (15 Marks)

a) A curve has parametric equations $x = \frac{t}{2}$, $y = 3t^2$. Find the Cartesian equation for this curve. 2

b) A committee of 6 people is to be formed from a group of 7 men and 5 women. Find the possible numbers of ways to form such a committee which contains :

i) an equal number of men and women. 1

ii) at least 2 women. 2

c) Given that $(x-3)$ and $(x+2)$ are factors of $x^3 - 6x^2 + px + q$,

find the values of p and q. 3

d) A chord of contact to the parabola $x^2 = 4y$, has the equation $y = x + 5$.

Determine the external point from which the tangents are drawn. 2

e) Given $t = \tan\frac{\theta}{2}$, show that $3\sin\theta + 4\cos\theta + 5 = \frac{(t+3)^2}{1+t^2}$. 2

f)

i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta+\alpha)$. 1

ii) Hence, solve $3\sin\theta + 4\cos\theta = 4$ where $0^\circ \le \theta \le 360^\circ$. 2

Question 3 (15 Marks)

a) α , β and γ are the roots of the equation $3x^3 - 4x + 6 = 0$. Evaluate :

i) $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ 2

ii) $\alpha^3\beta\gamma + \alpha\beta^3\gamma + \alpha\beta\gamma^3$ 2

b) Prove that $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$, provided that $\cos 2\theta \neq -1$. 2

c) When a polynomial $P(x)$ is divided by $(x-3)$ the remainder is 5 and when divided by $(x-4)$ the remainder is 9. Find the remainder when $P(x)$ is divided by $(x-3)(x-4)$. 3

d) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The equation of the chord PQ is $y = \left(\dfrac{p+q}{2}\right)x - apq$ (DO NOT PROVE THIS).

i) If PQ passes through the point $R(2a, 3a)$, show that $pq = p + q - 3$. 2

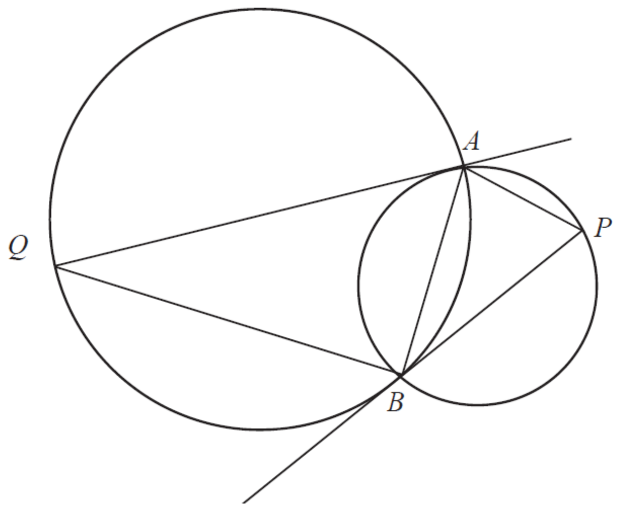
ii) If M is the midpoint of PQ, show that the coordinates of M are : 2
$\left(a(pq+3), \dfrac{a}{2}\left((pq+3)^2 - 2pq\right)\right)$.

iii) Hence, find the equation of the locus of M. 2

Question 4 (15 Marks)

a) At a dinner party Mr and Mrs Morrison and their six guests sit at a round table.
In how many ways can they be arranged if Mr and Mrs Morrison are to be separated? **2**

b) Two circles intersect at A and B, forming a common chord AB.
Tangents are drawn through A and B meeting the circles in Q and P respectively.



i) Prove that triangle AQB and triangle PBA are similar. **2**

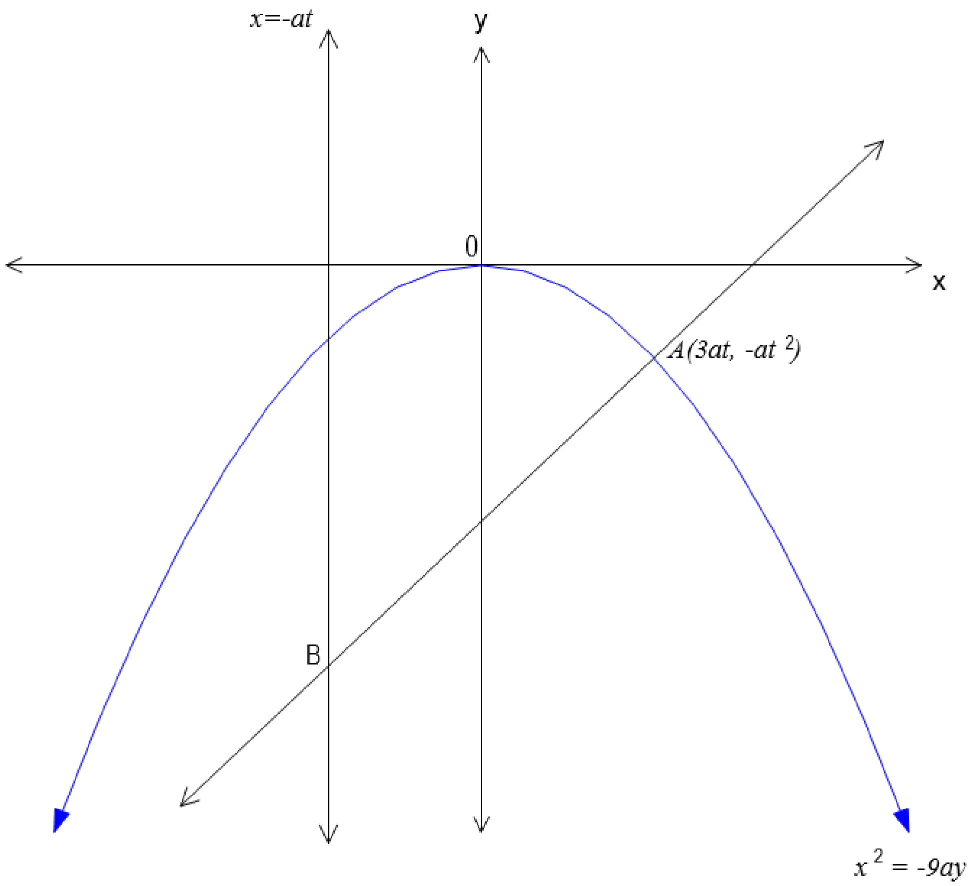
ii) Hence, or otherwise, prove $AB^2 = PA \times BQ$ **2**

c) The point $P(0,-2)$ divides the interval AB internally in the ratio $k : 1$.
If A is the point $(-3,7)$ and B is the point $(1,a)$, find the values of a and k. **2**

Question 4 (continued)

d) The point $A\left(3at,\ -at^2\right)$ is a variable point on the parabola $x^2 = -9ay$.

The normal at A meets the line $x = -at$ at point B. Point C lies on the normal and divides interval AB externally in the ratio $2:3$.



i) Show that the equation of the normal to the parabola at A is
$3x - 2ty = 2at^3 + 9at$. 2

ii) Find the coordinates of B in terms of a and t. 1

iii) Determine the coordinates of C in terms of a and t. 2

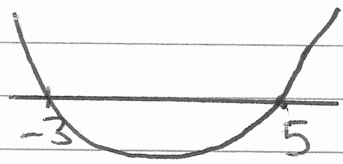
iv) Show that the equation of the locus of C is a parabola and state the equation of its directrix in terms of a. 2

The End.

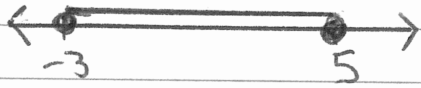
YEAR 11 2018 EXT TASK 3

QUESTION ONE

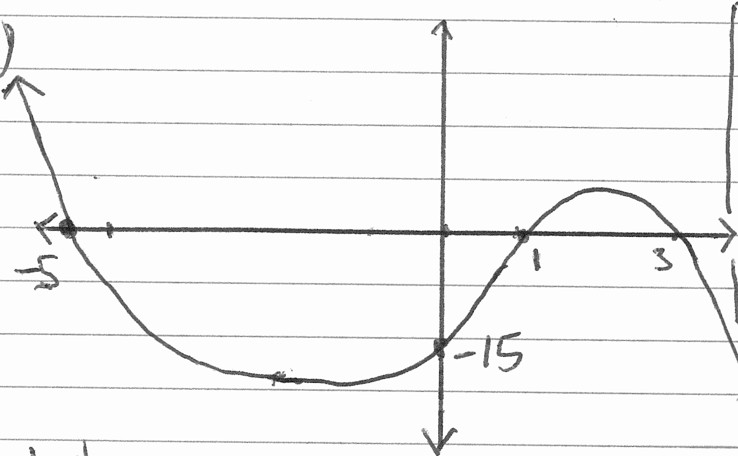
(a)



$-3 \leq x \leq 5$



(b)



The question asked for all intercept that includes the y-intercept

when $x=0$

$y=(5)(-1)(3)$

$y=-15$

(c) $\tan 75^\circ = \tan(30^\circ + 45^\circ)$

$$= \frac{\tan(30^\circ) + \tan(45^\circ)}{1 - \tan(30^\circ)\tan(45^\circ)}$$

$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\left(\frac{1}{\sqrt{3}}+1\right)^2}{1-\frac{1}{3}}$$

$$= \frac{\frac{1}{3}+\frac{2}{\sqrt{3}}+1}{\frac{2}{3}}$$

$$= \frac{4+\frac{6}{\sqrt{3}}}{2}$$

$$= 2+\frac{3}{\sqrt{3}}$$

$$= 2+\sqrt{3}$$

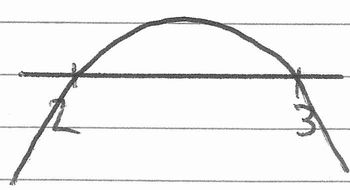
(d) $\frac{x}{x-2} \geqslant 3$ $x \neq 2$.

Many student forgot about point where $x=2$

$$x(x-2) \geqslant 3(x-2)^2$$

$$(x-2)(x-3x+6) \geqslant 0$$

$$-2(x-2)(x-3) \geqslant 0.$$



$$2 < x \leq 3$$

(e) $x - 2y - 5 = 0$ (A)

$3x - 4y + 20 = 0.$ (B)

Concurrent: means passing through the same point

$2\times$(A) $2x - 4y - 10 = 0$

(B)$-2\times$(A) $x + 30 = 0$

$x = -30.$

sub into (A)

$-30 - 2y - 5 = 0.$

$-2y = 35$

$y = -\frac{35}{2}.$

$(-30, -\frac{35}{2}).$

Sub into $x + 3ky = 15.$

$-30 - \frac{105k}{2} = 15$

$-\frac{105k}{2} = 45.$

$-105k = 90.$

$k = -\frac{90}{105}$

$k = -\frac{6}{7}$

(f) $$\frac{x^3-x^2}{8x^2+10x-3} \div \frac{4x^2-3x-1}{16x^2-1}$$

$$= \frac{x^2(x-1)}{(2x+3)(4x-1)} \times \frac{(4x-1)(4x+1)}{(x-1)(4x+1)}$$

$$= \frac{x^2}{2x+3}$$

2 a) $t = 2x$

$y = 3 \times (2x)^2$

$= 12x^2$

t is not the gradient of a tangent in this case.

b) i) ${}^7C_3 \times {}^5C_3 = 350$

ii) ${}^7C_4 \times {}^5C_2 + 350 + {}^7C_2 \times {}^5C_4 + {}^7C_1 \times {}^5C_5 = 812$

c) $P(3) = 0$

$3^3 - 6 \times 3^2 + p \times 3 + q = 0$

$-27 + 3p + q = 0$

$3p + q = 27$

$P(-2) = 0$

$(-2)^3 - 6 \times (-2)^2 + p \times -2 + q = 0$

$-8 - 24 - 2p + q = 0$

$-2p + q = 32$

$5p = -5$

$p = -1$

$3 \times -1 + q = 27$

$q = 30$

Those who used division of polynomials were not very successful.

d) chord of contact $x_1 x = 2a(y + y_1)$

$4a = 4$

$a = 1$

$x = y - 5$

$2x = 2(y - 5)$

$\therefore$ Pt is $(2, -5)$

Trying to find the pt of intersection of the chord and the equations of the tangents is far more difficult.

e) $3 \times \dfrac{2t}{1+t^2} + 4 \times \dfrac{1-t^2}{1+t^2} + 5$

$= \dfrac{6t + 4 - 4t^2 + 5 + 5t^2}{1+t^2}$

$= \dfrac{t^2 + 6t + 9}{1+t^2}$

$= \dfrac{(t+3)^2}{1+t^2}$

f) i) $R = \sqrt{3^2 + 4^2}$

$= 5$

$\tan \theta = \frac{4}{3}$

$\therefore \theta = \cancel{38^\circ 39' 35.31''}$ $53^\circ 7' 48.37''$

$\doteqdot \cancel{39^\circ}\, 53^\circ$

$\therefore 5 \sin(\theta + 53^\circ)$

ii) $5 \sin(\theta + 53^\circ) = 4$

$\sin(\theta + 53^\circ) = \frac{4}{5}$

$\theta + 53^\circ = 53^\circ, 127^\circ, 413$

$\therefore \theta = 0, 74, 360^\circ$

Many left out $\theta = 360^\circ$

Preliminary (Year 11) Mathematics Extension I
Assessment Task 3 - 2018

Q. 3

a) $3x^3 - 4x + 6 = 0$

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{4}{3}$

$\alpha\beta\gamma = -\frac{6}{3} = -2$

i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-\frac{4}{3}}{-2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$

(ii) $\alpha^3\beta\gamma + \alpha\beta^3\gamma + \alpha\beta\gamma^3$

$= \alpha\beta\gamma(\alpha^2 + \beta^2 + \gamma^2)$

$= \alpha\beta\gamma[(\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)]$

$= -2[0^2 - 2 \times -\frac{4}{3}]$

$= -2 \times \frac{8}{3} = -\frac{16}{3}$

Most students got this question correct, however small minority was confused about the sign of final product.

b) $\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$ Provided $\cos 2\theta = -1$

R.H.S

$= \frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \frac{1 - [2\cos^2\theta - 1]}{\cancel{1} + 2\cos^2\theta - \cancel{1}}$

$= \frac{1 - 2\cos^2\theta + 1}{2\cos^2\theta}$

$= \frac{\cancel{2}(1 - \cos^2\theta)}{\cancel{2}\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$=$ R.H.S.

Some students followed a long and complicated way to get to the final answer.

Year 11 Assessment Task-(2018)-III

Q-3

c) $P(x) = (x-3)(x-4) * Q(x) + Ax + B$ — ①

Sub $x = 3$ in ①

$P(3) = 3A + B$

But $P(3) = 5$

$5 = 3A + B$ — ②

Sub $x = 4$ in ①

$P(4) = 4A + B$

But $P(4) = 9$

$9 = 4A + B$ — ③

③ − ②

$4 = A$

Sub in ②

$5 = 12 + B$

$B = -7$

Hence Remainder $= 4x - 7$

Some students did not get the signs of A and B correct.

d) $Y = \left(\frac{P+Q}{2}\right)x - aPQ$ (Given)

i) It passes through $(2a, 3a)$

$3\cancel{a} = \left(\frac{P+Q}{\cancel{2}}\right)\cancel{2}\cancel{a} - \cancel{a}PQ$

$3 = P + Q - PQ$

$PQ = P + Q - 3$

Hence proved.

Most students got it correct.

Year 11 Assessment Task -(2018)- III

Q 3.

d

(ii) $M\left[\frac{\cancel{2}aP+\cancel{2}aQ}{\cancel{2}}, \frac{a(P^2+Q^2)}{2}\right]$

$M\left[a(P+Q), \frac{a}{2}((P+Q)^2-2PQ)\right]$

But $P+Q = PQ+3$ (Proved ~~abo~~ previously)

so

$M\left[a(PQ+3), \frac{a}{2}[(PQ+3)^2-2PQ]\right]$

Hence proved

Some students did not use $(a+b)^2-2ab = a^2+b^2$, and ended up getting very complicated approach

(iii) $x = a(PQ+3)$

$PQ+3 = \frac{x}{a}$

$Y = \frac{a}{2}\left[(PQ+3)^2 - 2PQ\right]$

so

$Y = \frac{a}{2}\left[\frac{x^2}{a^2} - 2\left[\frac{x}{a}-3\right]\right]$

$Y = \frac{a}{2}\left[\frac{x^2}{a^2} - \frac{2x}{a} + 6\right]$

$Y = \frac{x^2}{2a} - x + 3a$

Also $x^2 = 4aY$

$Y = \frac{\cancel{4}^{2}\cancel{a}Y}{\cancel{2a}} - x + 3a$

$Y = 2Y - x + 3a$

$Y - x + 3a = 0$

The most economical way is to get $PQ+3 = \frac{x}{a}$ and sub it in Y-coordinate of the ~~mid~~ point M and then to establish the relation between x and y.

Question 4

(a) $\#$ of arrangements $=$ total $-$ $\#$ where they are together

$$= 7! - 6! \times 2!$$

$$= 3600$$

Comment: Some students did not realise that there were 8 people in total. Also, some students did not consider that the Morrisons can be arranged in 2! ways.

(b)(i) In ΔAQB and ΔPBA:

(i) $\angle QAB = \angle BPA$ ($\angle$ in alt. segment)

(ii) $\angle AQB = \angle PBA$ ($\angle$ in alt. segment)

$\therefore \Delta AQB \parallel\!| \Delta PBA$ (equiangular)

(ii) $\dfrac{AB}{PA} = \dfrac{QB}{BA}$ (corr. sides of similar Δs in same ratio)

$\therefore AB \times BA = PA \times QB$ i.e. $AB^2 = PA \times BQ$

Comment: Students needed to identify correctly the two pairs of equal angles using correct geometrical reasoning. Some careless errors occurred in this question.

(c) $A(-3,7) \quad B(1,a)$

$k : 1$

$$\frac{k(1) - 3(1)}{k+1} = 0 \quad \underline{\text{and}} \quad \frac{k(a) + 1(7)}{k+1} = -2$$

$$k - 3 = 0$$

$$\therefore \underline{k = 3}$$

$$\frac{3a + 7}{4} = -2$$

$$3a + 7 = -8$$

$$3a = -15$$

$$\therefore \underline{a = -3}$$

Comment: Many mistakes made with the ordering for the calculation.

(d) (i) $y = \dfrac{-x^2}{9a} \qquad y' = -\dfrac{2x}{9a}$

at $A(3at, -at^2)$

$$m_T = \frac{-2(3at)}{9a}$$

$$= -\frac{2t}{3}$$

$$\therefore m_N = \frac{3}{2t}$$

Normal: $y - (-at^2) = \dfrac{3}{2t}(x - 3at)$

$$2t(y + at^2) = 3(x - 3at)$$

$$2ty + 2at^3 = 3x - 9at$$

$$\therefore 3x - 2ty = 2at^3 + 9at$$

Comment: Algebraic errors were not uncommon.

(d)(ii) when $x = -at$, $y = ?$

$$3(-at) - 2ty = 2at^3 + 9at$$

$$2ty = -12at - 2at^3$$

$$y = -6a - at^2$$

$$\therefore B = (-at, -6a - at^2)$$

(iii) $A(3at, -at^2)$ $B(-at, -6a-at^2)$

$-2 : 3$ (external division)

$$C = \left(\frac{3(3at) - 2(-at)}{-2+3}, \frac{3(-at^2) - 2(-6a - at^2)}{-2+3} \right)$$

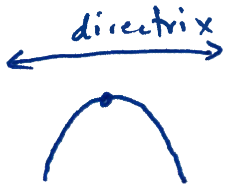
$$= (11at, -at^2 + 12a)$$

(iv) Using C from (iii) $x = 11at \Rightarrow t = \frac{x}{11a}$

$$\therefore y = -a\left(\frac{x}{11a}\right)^2 + 12a$$

$$= -\frac{x^2}{121a} + 12a \quad \therefore \frac{x^2}{121a} = -(y - 12a)$$

$$x^2 = -121a(y - 12a)$$



$\therefore$ A parabola with vertex at $(0, 12a)$.

$4 \times$ focal length $= 121a$ $\therefore$ focal length $= \frac{121a}{4}$

Directrix is $y = 12a + \frac{121a}{4}$ i.e. $y = \frac{169a}{4}$

Comment: (iii) Many mistakes due to ordering and/or calculating internal division instead of external.

(iv) Many students were unable to determine the equation of the directrix.