Sydney Technical High School

Name: ....

Maths Class: ....

Year 11

Mathematics Extension 1

Preliminary Course

Assessment 3

September, 2020

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided.

Section I Multiple Choice
Questions 1-7
7 Marks

Section II Questions 8-13
53 Marks

Section I

7 Marks
Attempt Questions 1-7
Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1-7.

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1. How many solutions does $-3 = 3\sin x$ have when $0 \leq x \leq 360°$?

(A) 0

(B) 1

(C) 2

(D) 3

2. What is the solution to $|3x - 5| = 7$?

(A) $x = 4$ or $x = \frac{2}{3}$

(B) $x = -4$ or $x = -\frac{2}{3}$

(C) $x = 4$ or $x = -\frac{2}{3}$

(D) $x = -4$ or $x = \frac{2}{3}$

3. Given $\tan\theta = 0.75$ and θ is in Quadrant 3, $\sin\theta =?$

(A) -0.8

(B) 0.8

(C) 0.6

(D) -0.6

4. How many ways can the letters in the word QUACKED be arranged into 7 letter words if the consonants and vowels must alternate?

(A) $4!$

(B) $4! \times 3$

(C) $4! \times 3!$

(D) $4 \times 3!$

5. If $y = m$ is a horizontal tangent to the curve $y = 15x^2 + 3$, then:

(A) $m = 15$

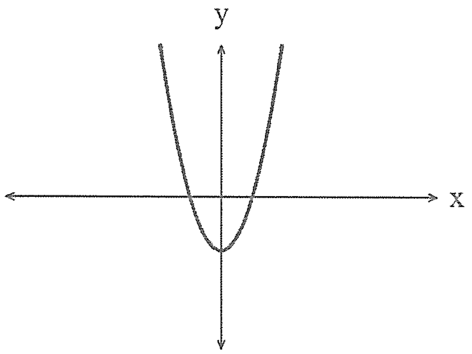
(B) $m = -15$

(C) $m = -3$

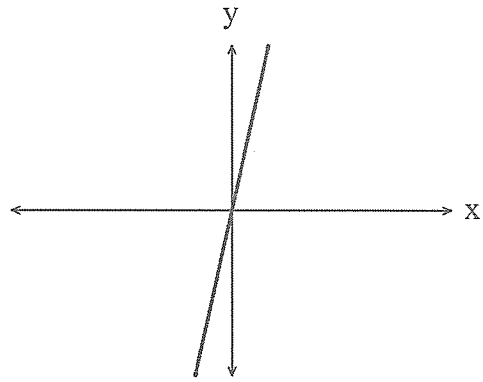
(D) $m = 3$

6. If $f'(x) = 3x^2 - 2$ which of the following graphs show a possible $y = f(x)$?

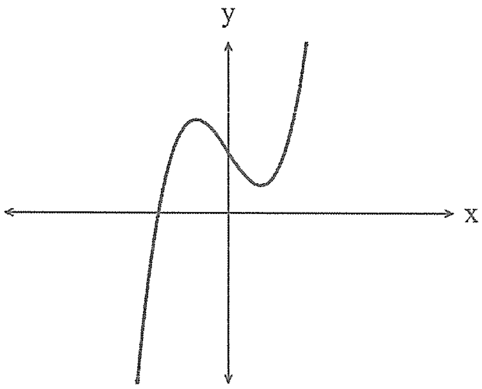
(A)



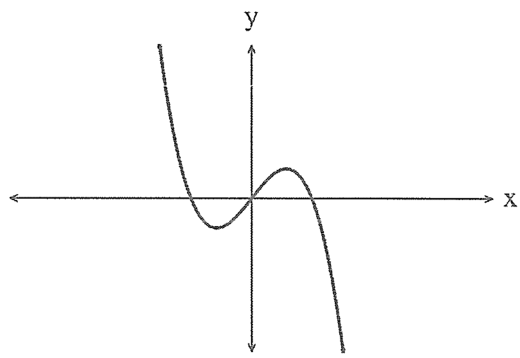
(C)



(B)



(D)



7. The graph of an odd polynomial $y = P(x)$ passes through the point $(3, -4)$. What is the remainder when $P(x)$ is divided by $(x + 3)$?

(A) 3

(B) 4

(C) -4

(D) -3

Section II

Total marks – 53
Attempt Question 8-13
Allow about 1 hour and 20 minutes for this section

Begin each question on a NEW page

In Questions 8-13, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (9 marks) Begin a NEW page.

a) Find the derivative of the following:

i. $y = \dfrac{1}{x\sqrt{x}}$ **1**

ii. $y = \dfrac{x+1}{x}$ **2**

iii. $y = (3x^2 - 1)^2$ **1**

b) Given $f(n) = n^2 - n$, simplify $f(n-1) + f(n+1)$ **2**

c) Given $f(x) = 2x - 1$

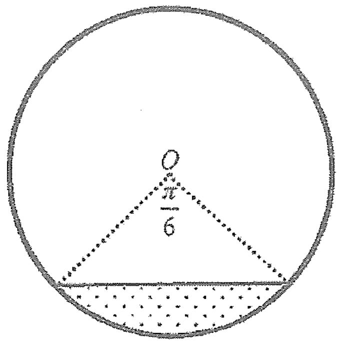
i. Sketch the graph $y = |f(x)|$, showing all important points **2**

ii. Hence solve $|2x - 1| = 1 - 2x$ **1**

End of Question 8

Question 9 (9 marks) Begin a NEW page.

a) The area of the minor segment of the circle pictured below is 100π m^2. **2**
What is the exact radius of this circle?



b) Consider $f(x) = \frac{|x|}{4} - \frac{x}{4}$ **3**

Find the midpoint of the line joining the points H and K with coordinates $(f(8), f(-8))$ and $(f(12), f(-12))$ respectively?

c) A team of six students is to be formed from a class of eight students. **1**
How many different teams can be formed if two particular students cannot be both selected for the team?

d) Find the Cartesian equation of the curve defined by the parametric equations: **3**

$$x = \frac{t}{t+1}, y = \frac{t}{t-1}$$

End of Question 9

Question 10 (9 marks) Begin a NEW page.

a) A bag contains a large number of coloured marbles. The ratio of pink marbles to other colours is 1:5.
Five marbles are selected at random from the bag.

i. What is the probability that no marbles are pink? **1**

ii. What is the probability that at least one of the marbles is pink? **1**

b) Express $x^2 + x$ in the form $a(x-1)^2 - b(x-1) - c$ **3**

c) The quantity of the population of mice is given by $M = 25 + 6t^2 - t^3$ (t in months)

i. What is the initial population of mice? **1**

ii. Find the average rate of change of mice over the first 3 months. **2**

iii. Find the instantaneous rate of change at the 3 month mark. **1**

End of Question 10

Question 11 (9 marks) Begin a NEW page.

a) Solve: **3**

$$\frac{4}{x+2} \geq \frac{1}{x}$$

b) Given $P(x) = 2x^3 - 5x^2 - 4x + 12$ has a root with multiplicity 2:

i. Fully factorise $P(x)$ **2**

ii. Sketch the graph $y = P(x)$ neatly, showing all intercepts **2**

iii. Hence or otherwise, neatly sketch on a separate number plane **2**

$$y = \frac{1}{P(x)}$$

End of Question 11

Question 12 (8 marks) Begin a NEW page.

a) Prove that: **2**

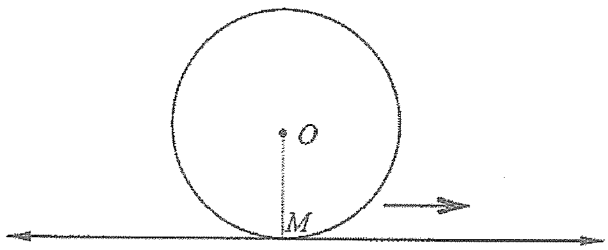
$$\cot\theta\,(1-\cos^2\theta) = \sin\theta\cos\theta$$

b) A polynomial $f(x)$ is given by the equation $f(x) = x(x+1) - a(a+1)$ for some constant a.

i. By using the factor theorem, find one factor of $f(x)$ in terms of a. **1**

ii. By division, or otherwise, express $f(x)$ as a product of linear factors. **2**

c) A train wheel of radius 40 cm and centre O rolls along a horizontal track as shown in the diagram below. M is a point on the wheel where the wheel touches the track before it starts to roll.



i. Through what angle does M rotate about O in radians after the wheel rolls 1 metre? **1**

ii. What would be the vertical height of M above the track after the wheel rolls 1 metre? **2**

End of Question 12

Question 13 (9 marks) Begin a NEW page.

a) There are four couples: **2**

Adam and Betty
Charlie and Dave
Ella and Fred
Gus and Heloise

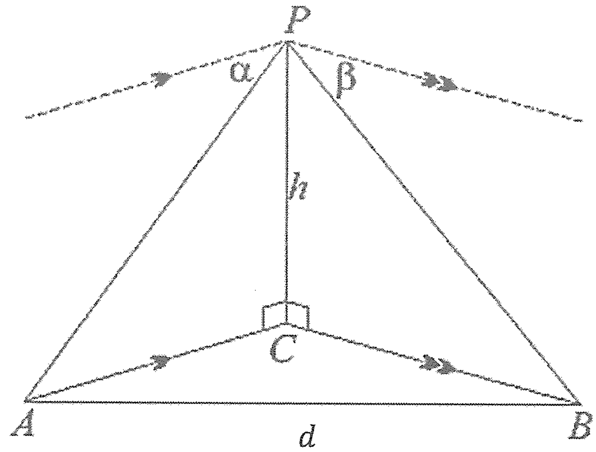
They go to a cinema and decide to sit in the last row. How many different arrangements are possible if Adam and Betty want to sit together but Ella and Fred do NOT want to sit together?

b) Solve the following for $0 \leq \theta \leq 2\pi$: **3**

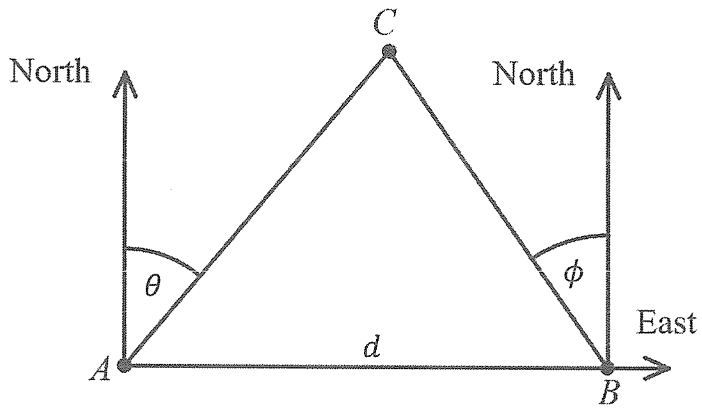
$$\tan^2 \theta = 2\sec^2 \theta - 3$$

Question 13 continues on the next page

c) The first diagram below shows the two points A and B on level ground. B is d metres due east of A. A tower of height h is also on the same level ground at C and its bearing is $N\theta°E$ and $N\phi°W$ from A and B respectively as shown in top view diagram below. From the tower P, the angle of depression of A is α and of B is β.



Top view



i. Prove that $h\sin(\theta+\phi) = d\cos\phi\tan\alpha$ **2**

ii. Prove that $h^2(\cot^2\alpha - \cot^2\beta) - 2hd\cot\alpha\sin\theta + d^2 = 0$ **2**

End of Exam

Section 1

1. B

2. C

3. D

4. C

5. D

6. B

7. B

1. $\sin x = -1$

   x only at $270°$

2. $|3x-5| = 7$

   $3x - 5 = \pm 7$

   $3x = -2$ or 12

   $x = -\frac{2}{3}$ or 4

3. $\tan\theta = \frac{3}{4}$

   opp $= 3$, 4 hyp $= 5$

   $\sin\theta = \frac{-3}{5}$, Q_3 $(\sin\theta < 0)$

4. $4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1$

   $= 4! \times 3!$

5. Horizontal at vertex

   $(0, 3)$

   $\therefore y = 3 \; : \; m = 3$

6. B $\rightarrow$ draw $f'(x)$ for A, B, C, D $\rightarrow$ upside down parabola.

   linear (not parabola)

   ☺

   horizontal line

7. $f(-x) = -f(x)$

   $f(-3) = -f(3) = -(-4) = 4.$

Section II

Q8. a) i. $-\frac{3}{2}x^{-5/2}$

ii. $y = 1 + \frac{1}{x}$

$\frac{dy}{dx} = -x^{-2}$

iii. $12x(3x^2 - 1)$

b) $f(n-1) + f(n+1)$

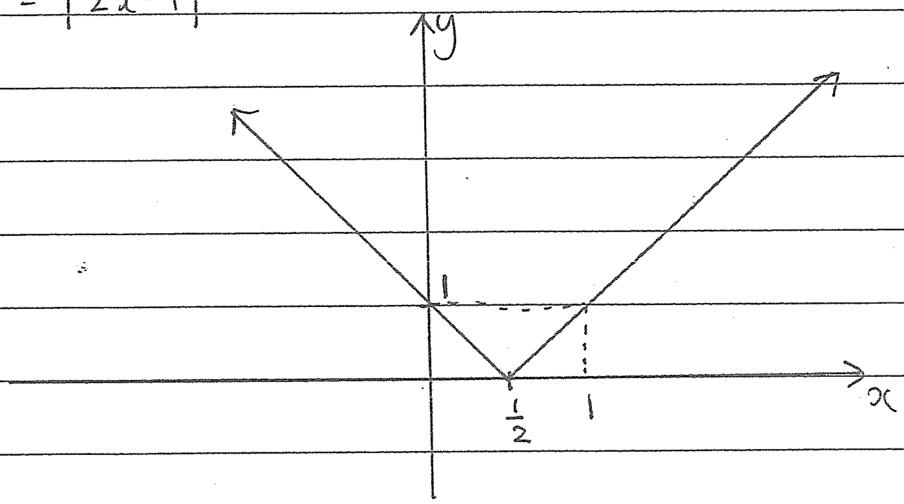
$= n^2 - 2n + 1 - n + 1 + n^2 + 2n + 1 - n - 1$

$= 2n^2 - 2n + 2$

$= 2(n^2 - n + 1)$

c) $y = |2x - 1|$

ii.



ii. $x \leq \frac{1}{2}$

Q9. a) Let r be radius of circle.

$$100\pi = \frac{1}{2}r^2 \times \frac{\pi}{6} - \frac{1}{2}r^2 \sin\frac{\pi}{6}$$

$$= \frac{\pi}{12}r^2 - \frac{1}{2}r^2 \times \frac{1}{2}$$

$$= r^2\left(\frac{\pi}{12} - \frac{1}{4}\right)$$

$$r^2 = \frac{100\pi}{\frac{\pi}{12} - \frac{1}{4}}$$

$$= \frac{100\pi}{\frac{\pi - 3}{12}}$$

$$= \frac{1200\pi}{\pi - 3}$$

$$r = \sqrt{\frac{1200\pi}{\pi - 3}}, \quad r > 0.$$

b) $H(0,4)$, $K(0,6)$

Midpoint $(0, 5)$

c) Select none, one or the other not both.

All − both.

$$^8C_6 - (^6C_4 \times {^2C_2})$$

$$= 13$$

d) $x = \frac{t}{t+1}$ OR $y = \frac{t}{t-1}$

$xt + x = t$ $\quad\quad$ $yt - y = t$

$x = t - xt$ $\quad\quad$ $-y = t - yt$

$t = \frac{x}{1-x}$ $\quad\quad$ $t = \frac{-y}{1-y}$

sub in $y = \frac{t}{t-1}$ $\quad\quad$ sub in $x = \frac{t}{t+1}$

$$y = \frac{\frac{x}{1-x}}{\frac{x}{1-x} - 1} \quad\quad x = \frac{\frac{y}{y-1}}{\frac{y}{y-1} + 1}$$

$$= \frac{x}{x-(1-x)} = \frac{x}{2x-1} \quad\quad = \frac{y}{y+y-1} = \frac{y}{2y-1}$$

Q10. a) i. $\frac{1}{6}$ pink $\frac{5}{6}$ not pink

$P(5 \text{ no pinks}) = \left(\frac{5}{6}\right)^5$

$= \frac{3125}{7776}$

ii. $1 - \frac{3125}{7776}$

$= \frac{4651}{7776}$ $\left[\text{or } 1 - \left(\frac{5}{6}\right)^5\right]$

b) $x^2 + x = a(x-1)^2 - b(x-1) - c$

let $x = 1$

$2 = -c$

$c = -2$

let $x = 0$

$0 = a + b + 2$ (1)

let $x = -1$

$0 = 4a + 2b + 2$ (2)

$0 = 2a + 2b + 4$ (1)$\times 2$

(2) $-$ [(1)$\times 2$]

$0 = 2a - 2$

$a = 1$

$b = -3$

$\therefore x^2 + x = (x-1)^2 + 3(x-1) + 2$

c) i. 25 mice

ii. ~~dM/dt~~ at $t = 0 \rightarrow M = 25$

$t = 3 \rightarrow M \doteq 52$

$\frac{52 - 25}{3} = 9$ mice / month

iii. $\frac{dM}{dt} = 12 \times t - 3t^2$

at $t = 3$

$\frac{dM}{dt} = 12 \times 3 - 3 \times 3^2 = 9$ mice/month.

Q11. a) $\dfrac{4}{x+2} \geq \dfrac{1}{x}$ $\quad x \neq 0$, $x \neq -2$

$\dfrac{4}{x+2} = \dfrac{1}{x}$

$4x = x+2$

$3x = 2$

$x = \dfrac{2}{3}$

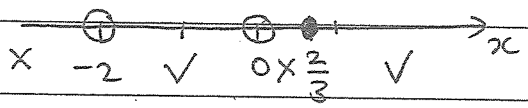
OR

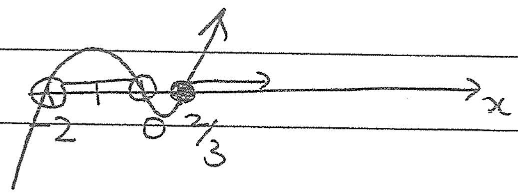
$4x^2(x+2) \geq (x+2)^2 x$

$4x^2(x+2) - (x+2)^2 x \geq 0$

$x(x+2)\left[4x - x - 2\right] \geq 0$

$x(x+2)(3x-2) \geq 0$





test values

$-2 < x < 0 \ \cup \ x \geq \dfrac{2}{3}$

$\therefore -2 < x < 0, \ x \geq \dfrac{2}{3}$

b) i $P(x) = 2x^3 - 5x^2 - 4x + 12$

$P'(x) = 6x^2 - 10x - 4$

$= 2(3x+1)(x-2)$

Test possible roots $-\dfrac{1}{3}, 2$

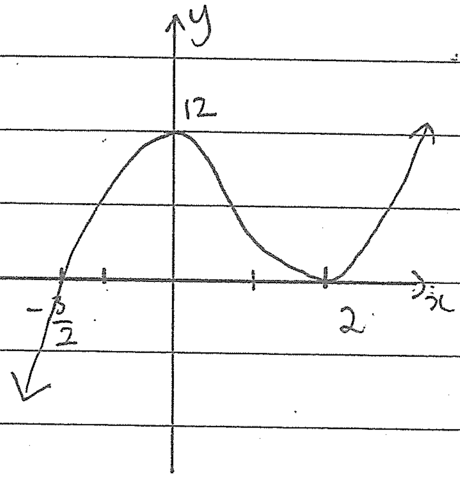
$P(2) = 0 \Rightarrow x = 2$ double root.

$P(x) = (x-2)^2(ax+b) \Rightarrow a = 2, \ b = 3.$

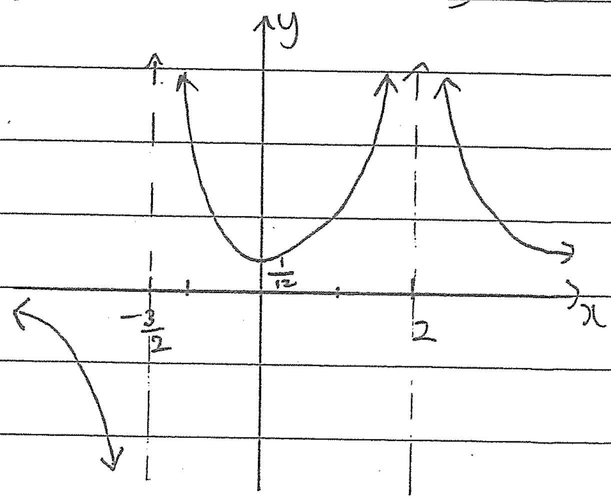
(by inspection or otherwise)

$P(x) = (x-2)^2(2x+3)$

ii.



iii.



Q12 a) $\cot\theta(1-\cos^2\theta) = \sin\theta\cos\theta$

LHS $= \cot\theta(1-\cos^2\theta)$

$= \cot\theta\sin^2\theta$

$= \dfrac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= \cos\theta\sin\theta$

$=$ RHS

b) i. $f(a) = a(a+1) - a(a+1)$

$= 0$

$\therefore$ one factor is $(x-a)$

ii.

$$\begin{array}{r l}
 & x + (1+a) \\
x-a \,\big) & x^2 + x - a^2 - a \\
 & \underline{x^2 - ax} \\
 & (1+a)x - a^2 - a \\
 & \underline{(1+a)x - a(1+a)} \\
 & 0
\end{array}$$

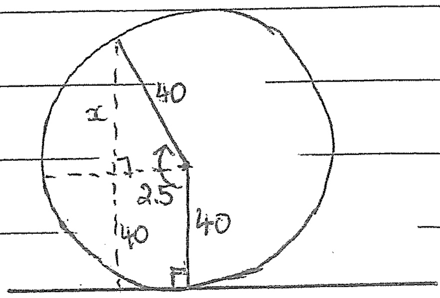
$\therefore f(x) = (x-a)(x+(1+a))$

c) i. $100 = 40\theta \quad (\ell = r\theta)$

$\theta = 2.5$ radians

ii. Turn $= 2.5$

height $= 40 + x$ (see $\rightarrow$)



$\frac{x}{40} = \sin(2.5 - \pi/2)$

$x = 40\sin(2.5 - \pi/2)$

height $= 40 + 40\sin(2.5 - \frac{\pi}{2})$

$= \cancel{72.04574462}\ldots$

≈ 72.04 cm or 0.720457 m

Q13. a) $\boxed{AB}$ C D $\underbrace{E\ F}_{\text{Separate}}$ G H

All arrangements with $\boxed{AB}$ $-$ $\boxed{AB}$ $\boxed{EF}$ arrangements

$7! \times 2 \qquad - \ 6! \times 2 \times 2$

$= 7200$

b) $\tan^2\theta = 2\sec^2\theta - 3$

$\tan^2\theta = 2\sec^2\theta - 2 - 1$

$\tan^2\theta = 2\tan^2\theta - 1$

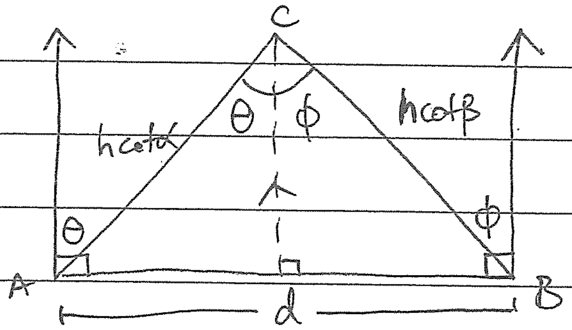
$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$

c) In $\triangle APC \rightarrow AC = \frac{h}{\tan\alpha}$

$\triangle BPC \rightarrow BC = \frac{h}{\tan\beta}$



i. $\frac{\sin(\theta+\phi)}{d} = \frac{\sin\angle CBA}{AC}$

$= \frac{\sin(90-\phi)}{h\cot\alpha}$, $\sin(90-\phi) = \cos\phi$

$h\sin(\theta+\phi) = d\cos\phi\tan\alpha$

ii. $CB^2 = AC^2 + d^2 - 2AC \times d \times \cos\angle CAB$

$h^2\cot^2\beta = h^2\cot^2\alpha + d^2 - 2dh\cot\alpha\cos(90-\theta)$

$0 = h^2\cot^2\alpha - h^2\cot^2\beta + d^2 - 2dh\cot\alpha\sin\theta$

$0 = h^2(\cot^2\alpha - \cot^2\beta) - 2hd\cot\alpha\sin\theta + d^2$

NAME: ______________________________

2020 EXTENSION 1 PRELIM YEARLY LATTICE

	ALGEBRA/ FUNCTIONS	TRIG	DERVIATIVES	POLYNOMIALS	PROBABILITY
1		/1			
2	/1				
3		/1			
4					/1
5			/1		
6			/1		
7				/1	
8a			/4		
8b	/2				
8c	/3				
9a		/2			
9b	/3				
9c					/1
9d	/3				
10a					/2
10b				/3	
10c			/4		
11a	/3				
11b				/6	
12a		/2			
12b				/3	
12c		/3			
13a					/2
13b		/3			
13c		/4			
TOTAL	/15	/16	/10	/13	/6

/60

Year 11 EXT 1 Assessment 3 Markers Comments

Question 8

a) (i) Must know the index laws! Makes it easy to change to negative fractional index first and then differentiate.

(ii) Easier to split the denominator first rather than using quotient or product rule. Many students did not check the reference sheet and wrote the formulas down wrong!

b) Some students incorrectly substituted into *f(n)* or made silly mistakes when simplifying the algebraic expression

c) (i) This is a simple absolute value graph. Some put the x-intercept as 1. Take a moment to carefully work out the x-intercept.

(ii) Very poorly done. HENCE means use the previous part to answer the question. If you sketched y = 1 - 2x on the same diagram as part i you would obtain the correct answer i.e. the inequality $x \leq \frac{1}{2}$. Those who tried to solve the equation algebraically got it wrong.

Question 9

a) It was as simple as finding the shaded area as A = sector – triangle (both of which are on the reference sheet) then sub in organise and solve for r = (in exact form) most students got zero out of 2.

b) Students should state the name of the coordinates they find and those who got it incorrect needed to actually find the x and y for each point then find the midpoint.

c) IT was no order (${}^nC_r$) poorly done across the cohort. Students needed to try and do no restrictions minus if BOTH were included.

d) VERY POOR UNDERSTANDING OF WHAT A CARTESAIN EQUATION ACTUALLY IS. The equation needed to have NO PARAMETERS (t) in the end equation just x and y only.

Question 10

b) Some students can't do the distributive law correctly when expanding brackets.

c) Students should know the difference between the average rate of change and instantaneous rate of change. Some students made mistakes when finding $\frac{dM}{dt}$.

Question 11

a) Poorly done. Whether using gatepost/squaring/ or graphing method, learn it, it is not that hard. Most of you assume you know how to solve these but you clearly don't!!

b) (i) As soon as "root of multiplicity" is mentioned DIFFERENTIATE!! See solutions.

(ii) and (iii) mostly well done although neatness is an issue. Don't draw a cusp on the x axis for a polynomial turning or touching, they don't exist. Y intercepts are easy to find so mark them, many lost a mark for failing to do so.

Question 12

a) Generally well done, a few select students need to take care with setting out their proofs.

b) This caused a lot of confusion. Many struggled to see that $f(a) = 0$, and either caught themselves in an algebraic mess, or incorrectly found that $f(1) = 0$. Moreover, several students did not connect $f(a) = 0$ to the factor of $(x - a)$.

Students also struggled with the algebraic difficulty of polynomial division with another pronumeral involved. However, they should know that if there is a remainder, the divisor is NOT a factor.

c) This caused a surprising amount of difficulty. Some students tried to redefine radians, rather than using $l = r\theta$. Moreover, some students seemed to use π as a unit of radians, and wrote it for no reason. Many errors in part i) caused difficulty in attaining full marks, as certain values made the question easier.

Otherwise, the two most common errors in ii) where to treat θ as an acute angle in a right angled triangle (it is obtuse), or to use the cosine rule in a way that implies that the radius of the circle extends beyond the circle.

Question 13

a) Most people were able to seat Adam and Betty together but were stuck when it came to Ella and Frank – please see solutions, it was most successfully done as a complementary events question.

b) $\tan^2 x + 1 = \sec^2 x$ is on THE REFERENCE SHEET and yet people still got this wrong as a substitution. If question asks for radians, don't use degrees!

c) This did not require double angle expansion CONSIDERING WE HAVE NOT DONE THEM YET! The prove questions needed to refer to the 3D trig diagram, not done as standalone proof questions for LHS=RHS (it made no sense to do them like this). Diagrams are not given to you simply for the fun of it, they are USEFUL.