TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2009 ASSESSMENT TASK 4

MATHEMATICS

EXTENSION 1

YEARLY EXAMINATION

Time allowed- *2 hours plus 5 minutes reading time.*

Date: Friday 4th September 2009

WEIGHTING: 40% towards final result

OUTCOMES referred to: P 2 – 8, H 1 – 9

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. There are 7 questions worth 12 marks each (Total marks: 84)
3. Show all necessary working.
4. Begin each question on a **new page.**
5. **Write your name, your teacher's initials and class at the top of each page.**
6. Mark values are shown beside each part.
7. The only calculators permitted are scientific calculators approved by the Board of Studies.
8. Responses must be in black or blue pen but diagrams are to be drawn in pencil.
9. If required, additional writing sheets may be obtained from a teacher upon request.
10. A table of integrals is attached to the back of this examination.

Name:_____

Start each question on a new page.

QUESTION 1 **12 Marks**

(a)	Evaluate to 3 significant figures	$\frac{\sqrt{5.2-4.73}}{3.8-2.1^2}$	2
(b)	Find the exact value for:	$2sin30°cos30°$	2
(c)	Find the primitive for:	$x^2+\frac{x^5}{6}-2$	2
(d)	Differentiate	$x^2(3x-4)^5$	3
(e)	Solve for x:	$x(x-4)\leq 12$	3

Start each question on a new page.

QUESTION 2 **12 Marks**

(a) The coordinates of the vertices A, B and C of a triangle are (-1, 3) (2, 5) and (1, -1) respectively. Find:

(i) The equation of the perpendicular line from A to BC. 2

(ii) The perpendicular distance from A to BC 2

(iii) Hence, or otherwise, the **exact** area of triangle ABC. 2

(b) Find the centre and radius of the circle 2

$$x^2 + y^2 - 12x - 4y + 15 = 0$$

(c) Solve 4

$$x + 3y - z = 13$$
$$3x + y - z = 11$$
$$x + y - 3z = 11$$

Start each question on a new page.

QUESTION 3 **12 Marks**

(a) Sketch a graph to show the information provided by the following statement:

"The amount of time spent by teenagers in sporting activities is increasing at a decreasing rate." 2

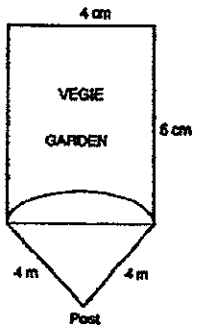
(b) Simplify $\frac{cos\theta}{1+sin\theta} - \frac{1-sin\theta}{cos\theta}$ 3

(c) Consider the parabola with equation $(y-1)^2 = 8(x+3)$

(i) Find the coordinates of the vertex of the parabola. 1

(ii) Find the equation of the directrix of the parabola. 2

(d) A goat is tethered to a 4 m long rope. The rope is tied to a post fixed at a point 4 metres from each of the 2 corners of a 6 metre by 4 metre rectangular vegetable garden.
This information is illustrated in the diagram below (diagram not to scale).



Calculate the exact area of the vegetable garden that the goat can eat. 4

Start each question on a new page.

QUESTION 4 **12 Marks**

(a) Given the function $f(x) = x^3 - x^2 - x + 4$ in the domain $-3 \leq x \leq 4$.

(i) Find the stationary points and determine their nature. **3**

(ii) Draw a sketch of the curve $y = f(x)$ in the domain $-3 \leq x \leq 4$, clearly showing all its essential features. **3**

(iii) Find the domain(s) for which the function is decreasing. **1**

(b) If $x^2 - 2kx + 1 = 0$ has two equal real roots. Find the value(s) of k. **2**

(c) If α and β are the roots of the equation $3x^2 + 4x + 7 = 0$, find the value of:

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

(ii) $(1 - \alpha)(1 - \beta)$ **1**

Start each question on a new page.

QUESTION 5 **12 Marks**

(a) Evaluate: $\int_0^3 (2x-5)^3\,dx$ **2**

(b) Solve the equation

$$(x^2-3x+4)^2-5(x^2-3x+4)+6=0$$

Using the substitution $m=(x^2-3x+4)$ **4**

(c) The positive numbers m and n are the second and fourteenth terms respectively of an arithmetic progression whose first term is 1.

(i) Write an expression for the common difference. **1**

(ii) Show that the expression for the fourteenth term is given by:

$$n=13m-12$$ **1**

(iii) Given also that m, 9, n are the first three successive terms of a geometric progression, calculate the values of m and n. **4**

Start each question on a new page.

QUESTION 6 **12 Marks**

(a) (i) On the same set of axes sketch the graphs of

$$y = \sqrt{4 - x^2} \text{ and } y = |x|. \quad 2$$

(ii) Hence, or otherwise find the enclosed area Abounded by: $y = \sqrt{4 - x^2}$ and $y = |x|$. 1

(b) A string of length 12m is cut into two portions, so that a circle of radius r metres, and a square of length x metres are formed. (HINT: Draw a diagram)

(i) Show that the length of the square, x metres, can be written as:

$$x = \frac{3 - \pi r}{2} \quad 1$$

(ii) Show area is given by:

$$A = \frac{36 - 12\pi r + 4\pi r^2 + \pi^2 r^2}{4} \quad 1$$

(iii) Find the radius of the circle if the sum of the two areas is a minimum. (Leave you answer in terms of π). 3

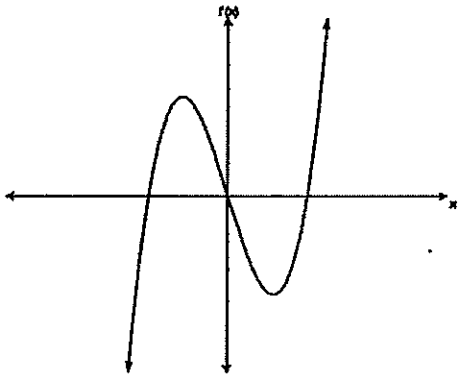
(c) Find as a relation between k, l and m, the condition for the quadratic equation in x to have real roots. Simplify your answer as far as possible. 4

$$(k^2 + l^2)x^2 + 2l(k + m)x + (l^2 + m^2) = 0$$

Start each question on a new page.

QUESTION 7 **12 Marks**

(a) The graph of $f'(x)$ is given below. Copy this diagram onto your answer sheets. Draw the graph of $f(x)$. 3



(b) Mr Jones on his 25th birthday started regular annual investments of \$500 into an investment fund which paid interest at a rate of 9% pa. After retiring on his 60th birthday he collected his investment payout with which he immediately repaid the balance owing on his housing loan. After that he was still left with a surplus of \$795 which he decided to give as a present to his grandchildren.

(i) Show that the amount Mr Jones owed on his housing loan just prior to his payout was \$116 767.36. 4

(ii) The surplus of \$795 was then divided amongst his 6 grandchildren. Each grandchild received a different amount, forming an arithmetic sequence with 6 terms. The oldest Clark (13yrs) received \$186, then Susan (11yrs), Jason (9yrs), Greg (7 yrs), Beatrice (5 yrs) received progressively less, with the youngest Debbie (3yrs) the smallest amount.

I. How much did Debbie receive? 3

II. How much did each of the other grandchildren receive? 2

END OF EXAMINATION

Student Number: Solutions Year 11 Ext 1 2009 Teacher's initials: LJR.

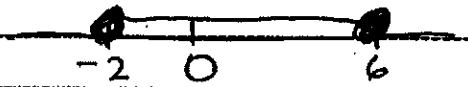
Q1 a) $= -1.123877\ldots$ ---(1) correct dec
$= -1.12$ (3 signfig) ---(1) correct rounding off (even from wrong answ)

b) $2\sin 30 \cos 30$
$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$
(1) for one correct
(1) Answer.

c) $\int x^2 + \frac{x^5}{6} - 2$
$= \frac{x^3}{3} + \frac{x^6}{36} - 2x + C$
(1) two parts corr.
(1) correct answ.

d) $y = x^2(3x-4)^5$
$\frac{dy}{dx} = x^2 \cdot 5(3x-4)^4 \cdot 3 + 2x(3x-4)^5$ (1) (leaving out part)
(1) one part corr.
$= 15x^2(3x-4)^4 + 2x(3x-4)^5$
or $x(3x-4)^4(21x-8)$
(1) correct answ. (simplify).

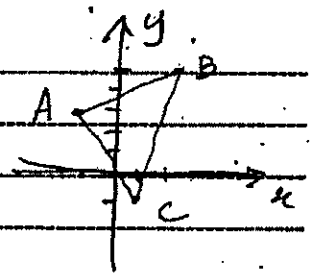
e) $x(x-4) \leq 12$
$x^2 - 4x - 12 \leq 0$ Assume =
$(x-6)(x+2) = 0$
(1) factorisation
$x = 6$ or -2
(1) Solving



check 0. (1) Soln.
$0 \leq 12$.
$\therefore -2 \leq x \leq 6$

Student Number: Soln. Teacher's initials: LJR

Q2 a) $A(-1,3)$, $B(2,5)$, $C(1,-1)$



i) $m_{BC} = \frac{-1-5}{1-2} = \frac{-6}{-1} = 6$

$m_\perp = -\frac{1}{6}$ (perp).

① $m_\perp = -\frac{1}{6}$

① line

equation $y - y_1 = m(x - x_1)$

$(-1,3)$ $y - 3 = -\frac{1}{6}(x+1)$

$6y - 18 = -x - 1$

$x + 6y - 17 = 0$

ii) Line BC

$(2,5)$ $m_{BC} = 6$ $y - 5 = 6(x-2)$

$y - 5 = 6x - 12$

$6x - y - 7 = 0$

① line

① perp dist.

$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$(-1,3)$

$= \left| \frac{6x-1 - 1x3 - 7}{\sqrt{6^2+(-1)^2}} \right|$

$= \left| \frac{-6-3-7}{\sqrt{37}} \right| = \frac{16}{\sqrt{37}}$

iii) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times \sqrt{37} \times \frac{16}{\sqrt{37}}$

$= 8$

$d_{BC} = \sqrt{(1-2)^2 + (-1-5)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

① d_{BC}

① Area.

b) $x^2 - 12x + y^2 - 4y + 15 = 0$

$(x-6)^2 - 36 + (y-2)^2 - 4 + 15 = 0$

$(x-6)^2 + (y-2)^2 = 25$

① completing square.

Centre $(6,2)$ $r = 5$.

① Answers.

Student Number:_____ Teacher's initials:____

Q2c) $x + 3y - z = 13$ ... ① $\times 3$ ④

$3x + y - z = 11$ ... ② $\times 3$ ⑤.

$x + y - 3z = 11$ ... ③

④ $3x + 9y - 3z = 39$

⑤ $9x + 3y - 3z = 33$

④ − ③ $2x + 8y = 28 \Rightarrow x + 4y = 14$ ... ⑥

⑤ − ④ $6x - 6y = -6 \Rightarrow x - y = -1$ ... ⑦.

⑥ − ⑦ $5y = 15$

$y = 3$

⑦ ... $x - 3 = -1$

$x = 2$

① $2 + 9 - z = 13$

$-z = 2$

$z = 2$

Marks ① eliminate one var one eqn.

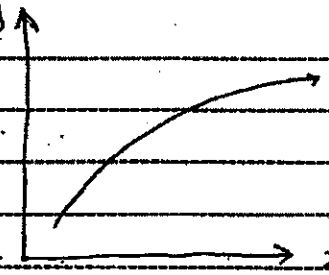
① eliminate second var.

① one correct value.

① correct answ.

Student Number: Soln Teacher's initials: LJR

Q3 a)



① curve going up.
① concave down.

b) $\frac{\cos\theta}{1+\sin\theta} - \frac{1-\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta - (1+\sin\theta)(1-\sin\theta)}{\cos\theta(1+\sin\theta)}$ ① common den

$= \frac{\cos^2\theta - (1-\sin^2\theta)}{\cos\theta(1+\sin\theta)}$ ① $1-\sin^2\theta = \cos^2\theta$ or equiv.

① Ans.

now $1-\sin^2\theta = \cos^2\theta$.

$= \frac{\cos^2\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)} = 0$

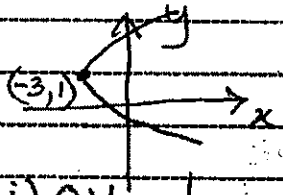
c) $(y-1)^2 = 8(x+3)$

i) V(−3, 1)

ii) Now $4a = 8$

$a = 2$

∴ Dir $x = -5$

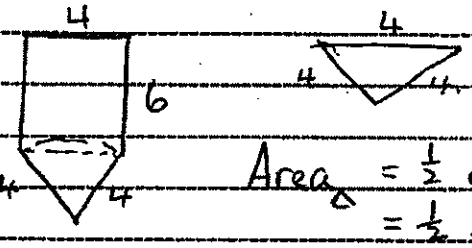


i) ① Vertex

ii) ① $a = 2$

① $x = -5$

d)



Angle 60°, ① Angle

① Area Δ

① Area sector

① Area of garden.

$\text{Area}_\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 4 \times 4 \times \sin 60.$

$= 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \approx 6.928$

$\text{Area of sector} = \frac{1}{6}\pi r^2$

$= \frac{1}{6} \times \pi \times 4^2 = \frac{8}{3}\pi \approx 8.378.$

Area of vege garden $= 8.378 - 6.928$

$= 1.45$

or $\frac{8}{3}\pi - 4\sqrt{3}$.

Student Number:_____ Teacher's initials:____

Q4. a) i) $f(x) = x^3 - x^2 - x + 4$ $-3 \le x \le 4$

$f'(x) = 3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$ ① factorisation = 0

$x = -\frac{1}{3}$ or 1 ① Values of x and y

$y = 4\frac{5}{27}$ 3 ① proving max and min

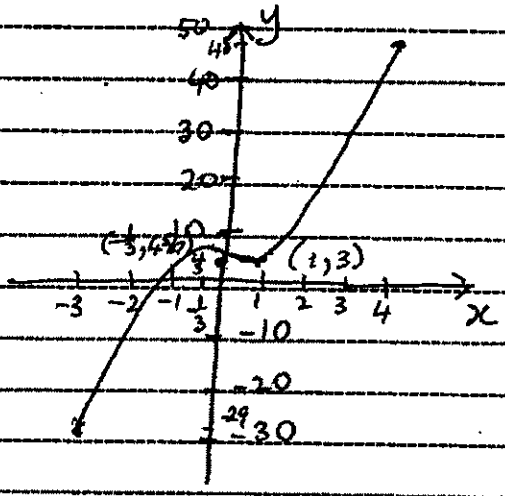
$(-\frac{1}{3}, 4\frac{5}{27})$ $(1, 3)$

$f''(x) = 6x - 3$

$f''(-\frac{1}{3}) = -2 - 3 = -$ ∴ Max. } need both.

$f''(1) = 6 - 3 = +$ ∴ Min. }

ii)



$x = 4$

$y = 64 - 16 - 4 + 4$

$= 48.$ ① End points

$x = -3$ ① shape

$y = -27 - 9 + 3 + 4$ ① y axis

$= -29$

$x = 0 \quad y = 4$

iii) Decreasing function $-\frac{1}{3} < x < 1$ ① Correct answer from graph.

not $-\frac{1}{3} \le x \le 1$

b) $x^2 - 2kx + 1 = 0$

$\Delta = b^2 - 4ac = 0$

$= 4k^2 - 4 = 0$ ① setting $\Delta = 0$

$4(k^2 - 1) = 0$ ① answer.

$4(k-1)(k+1) = 0$

$k = 1$ or -1

① $\alpha+\beta$ or $\alpha\beta$

c) $3x^2 + 4x + 7 = 0$ ① Answer(i)

$\alpha + \beta = -\frac{b}{a} = -\frac{4}{3}$ i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$\alpha\beta = \frac{c}{a} = \frac{7}{3}$ $= \frac{-\frac{4}{3}}{\frac{7}{3}} = -\frac{4}{7}.$

ii) $(1-\alpha)(1-\beta) = 1 - \alpha - \beta + \alpha\beta = 1 - (\alpha+\beta) + \alpha\beta$

$= 1 - -\frac{4}{3} + \frac{7}{3} = \frac{14}{3}$

① Answer(ii)

Student Number: _____ Teacher's initials: ____

Q5a) $\int_0^3 (2x-5)^3\,dx$

$= \left[\frac{(2x-5)^4}{4 \cdot 2}\right]_0^3$ ① Int.

$= \frac{1}{8}\left[1^4 - (-5)^4\right]$ ① Sub to get answer.

$= \frac{-624}{8} = -78$

b) $(x^2-3x+4)^2 - 5(x^2-3x+4) + 6 = 0$

Let $M = x^2-3x+4$ ① $M =$

$M^2 - 5M + 6 = 0$ ① Solving M.

$(M-3)(M-2) = 0$ ① Reachsolving

$M = 3$ or 2 ① for x.

$x^2-3x+4 = 3$ or $x^2-3x+4 = 2$

$x^2-3x+1 = 0$ $x^2-3x+2 = 0$

$x = \frac{3 \pm \sqrt{9-4}}{2}$ $x = \frac{3 \pm \sqrt{9-8}}{2}$

$x = \frac{3 \pm \sqrt{5}}{2}$ $x = \frac{3 \pm 1}{2} = 2$ or 1

c) $T_2 = m$ $T_{14} = n$

$a = 1$ A.P.

i) difference $d = T_2 - T_1$ accept $\frac{n-m}{12}$ or.

$= m - 1$ ① difference

ii) $T_{14} = a + 13d$

$= 1 + 13(m-1)$

$= 13m - 12 = n$ ① $n = 13m-12$

iii) $m, 9, n$ G.P.

$\frac{9}{m} = \frac{n}{9}$

$81 = mn$ ① $mn = 81$

$\therefore 81 = m(13m-12)$

$13m^2 - 12m - 81 = 0$ ① binomial.

$m = \frac{12 \pm \sqrt{144 - 4 \times 13 \times -81}}{26}$ ① finding m excluding neg.

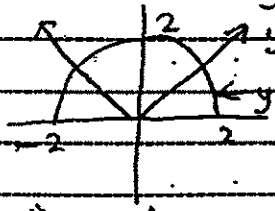
$= \frac{12 \pm 66}{26} = 3$ or $-2\frac{1}{13}$ ① finding n

but $m > 0$ $\therefore m = 3$ $n = 27$

bald $m = 3$ $n = 27$ ②

Student Number:_____ Teacher's initials:____

Q6 ai) $y = \sqrt{4-x^2}$ & $y = |x|$



① Drawing semi circle
① . V shape.

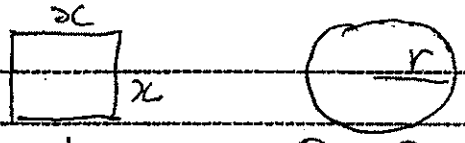
ii) Area 90° angle.

Area of sector $= \frac{1}{4} \times \pi \times 2^2$

$= \pi$ ① find area.

Student Number:_____ Teacher's initials:____

b)



i) $P = 4x$ $P = 2\pi r$

$4x + 2\pi r = 6$

$2x = 6 - \pi r$

$x = 3 - \frac{\pi r}{2}$

ii) $A = x^2 + \pi r^2$

$= \pi r^2 + (3 - \frac{\pi r}{2})^2$

$= \pi r^2 + 9 - \frac{6\pi r}{2} + \frac{\pi^2 r^2}{4}$

$A = \frac{4\pi r^2 + \pi^2 r^2 - 12\pi r + 36}{4} = \frac{36 - 12\pi r + 4\pi r^2 + \pi^2 r^2}{4}$

iii) $\frac{dA}{dr} = \frac{-12\pi + 8\pi r + 2\pi^2 r}{4} = 0$

$r(8\pi + 2\pi^2) = 12\pi$

$r = \frac{12\pi}{8\pi + 2\pi^2} = \frac{12}{8 + 2\pi}$

$\frac{d^2A}{dr^2} = \frac{8\pi + 2\pi^2}{4} = +$ ∴ Min ∪

Student Number:_____ Teacher's initials:____

(Q6c) $(k^2+\ell^2)x^2 + 2\ell(k+m)x + (\ell^2+m^2) = 0$

$\Delta = b^2 - 4ac \geq 0$

$4\ell^2(k+m)^2 - 4(k^2+\ell^2)(\ell^2+m^2) \geq 0$

$4\ell^2(k^2+2km+m^2) - 4(k^2\ell^2 + k^2m^2 + \ell^4 + \ell^2m^2) \geq 0.$

$4[\ell^2k^2 + 2\ell^2km + \ell^2m^2 - k^2\ell^2 - k^2m^2 - \ell^4 - \ell^2m^2] \geq 0.$

$4[-k^2m^2 + 2\ell^2km - \ell^4] \geq 0$

$-4[k^2m^2 - 2\ell^2km + \ell^4] \geq 0.$

$-4(km - \ell^2)^2 \geq 0.$

$(km - \ell^2)^2 \leq 0.$

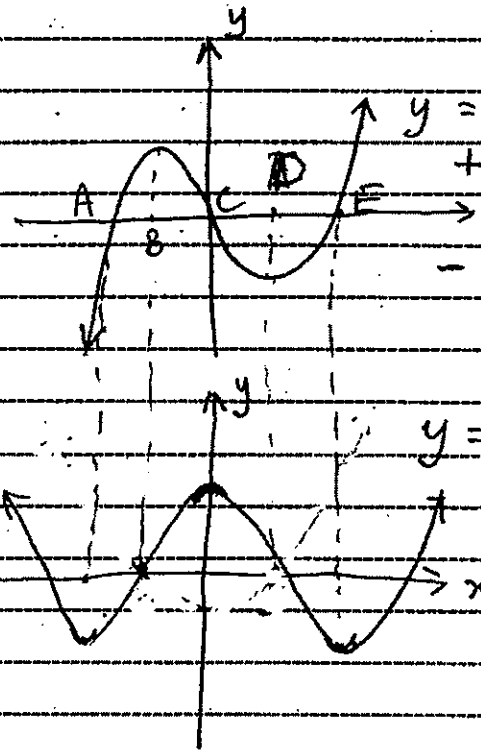
or

$km = \ell^2.$

① $\Delta \geq 0$
① expanding
① simplifying
① final two lines

Student Number:_____ Teacher's initials: ____

Q7. a)



① shape.
① zero's to max/min
① x + y axis.

b) i) $500. r = 9%.

n = 35 years.

$A_n = 500(1.09)^{35} + 500(1.09)^{34} + \dots + 500(1.09)$

$S_n = \frac{a(r^n-1)}{r-1}$ $a = 1.09$ $r = 1.09$ $n = 35$

$= 500\left(\frac{1.09(1.09^{35}-1)}{1.09-1}\right)$

$= 500 \times 235.1247226$

$= \$117\,562.36$

Amount owed $= 117\,562.36 - 795 = \$116\,767.36$

① 35 years.
① A_n =.
① G.P
① Ans.

ii) 795 6.

$a = 186$ $n = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$795 = \frac{6}{2}[372 + (n-1)d]$ now $n = 6$

$795 = 3[372 + 5d]$

$265 = 372 + 5d$

$5d = -107$

$d = -\$21.40$

$T_6 = a + (n-1)d$.

$= 186 + 5 \times (-21.40) = \$79$

① Forming AP
① $d = 21.40$
① $79

Susan = 164.60, Jason = 143.20 Greg = 121.80
Beatrice = $100.40

① Getting one other correct
① Answers.