TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 11 2010 ASSESSMENT TASK 4
Yearly Examination

MATHEMATICS

(EXTENSION 1)

Time Allowed - *two hours(plus5minutes reading time)*

WEIGHTING 40% towards final result

Outcomes referred to: P2-8, H1-9

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working.
3. **Begin** each question on a **new page.**
4. Write your name, your teachers initials and class at the top of each page.
5. Each question is of equal value. Mark values are shown beside each part.
6. Non-programmable silent Board of Studies approved calculators are permitted.
7. If requested, additional writing sheets may be obtained from the examinations supervisor upon request.
8. Total marks 84

Question 1. (12marks)(Start a new booklet) **Marks**

(a) Find the value of $\dfrac{(2.14)^2-1.72}{\sqrt{6.76+2}}$ correct to 2significant figures. **2**

(b) Solve for x: $2x^2-5x-3>0$ and graph it on the number line. **3**

(c) Solve for x: $3^x\times 9^{x+1}=\dfrac{1}{3}$ **2**

(d) Solve for x if $(2x+1)(x-1)=14$. **3**

(e) Find the Primitive of x^2+x^{-4} **2**

Question 2. (12marks)(Start a new booklet) **Marks**

(a) Find $\frac{dy}{dx}$ given (i) $y = x\sqrt{x} - \frac{1}{x^2}$ **2**

(ii) $y = \frac{3x}{x^2 - 7}$ **2**

(b) The line L_2 has the equation $2x + y - 2 = 0$.

(i) Find the gradient of L_1, if L_1 is perpendicular to L_2. **1**

(ii) The point (-4 , -1) lies on line L_1 . Show the equation of L_1 is $x - 2y + 2 = 0$. **1**

(iii) Find the coordinates of the point intersection of L_1 and L_2. **2**

(iv) Find the perpendicular distance between L_2 and the point P(-4 , -1) **2**

(c) Given $\frac{dy}{dx} = 4x - 3$. Find curve y given the curve passes through (2 , -8) **2**

Question 3. (12marks)(Start a new booklet) **Marks**

(a) For a certain arithmetic series the sum of the first six terms, S_6, is 84 and the sum of the first fourteen terms, S_{14}, is 420. **4**

(i) Find the first term and common difference.

(ii) Find an expression for the sum of n terms, S_n

(iii) Find an expression for the nth term, T_n

(b) The lengths of the sides of a triangle are 7 cm, 8 cm and 13 cm.

(i) Find the size of the angle opposite the longest side. **2**

(ii) Find the area of the triangle, correct to one decimal place. **1**

(c) The quadratic equation $x^2+(k+3)x-k=0$ has real roots.
Find all the possible values k can take. **3**

(d) Prove that $\dfrac{\cos^2 A}{1-\sin A}=1+\sin A$ **2**

Question 4. (12marks)(Start a new booklet) **Marks**

(a) Find the values of k in the equation

$x^2+(k+1)x+(\frac{k+1}{4})=0$ if:

(i) roots are equal in magnitude but opposite in sign. **2**

(ii) roots are equal **2**

(iii) roots are reciprocals of each other. **2**

(b) For the quadratic equation $x^2+4x-8y+12=0$ find:-

(i) the vertex **2**

(ii) the focus **1**

(iii) the equation of the directrix **1**

(iv) the equation of the tangent at $x=4$. **2**

Question 5. (12marks)(Start a new booklet) **Marks**

(a) Consider the curve $y = 4x^3 + 6x^2$.

(i) Find the coordinates of any turning points and determine their nature. **3**

(ii) Find the points of inflexion. **2**

(iii) Sketch the curve for the domain $-2 \le x \le 1$ Include x intersepts. **2**

(iv) What is the maximum value of $4x^3 + 6x^2$ in the domain $-2 \le x \le 1$ **1**

(b) Solve the equation for x

$$(x+\frac{1}{x})^2 - 5(x+\frac{1}{x}) + 6 = 0$$

Using the substitution $M = (x+\frac{1}{x})$ **4**

Question 6. (12marks)(Start a new booklet) **Marks**

(a) Solve $\sin^2 x = \frac{3}{4}$ for $0^0 \le x \le 360^0$ 3

(b) Prove that the line $3x+4y+21=0$ is a tangent to the circle $x^2-8x+y^2+4y-5=0$ 3

(c) Bill decides to borrow \$250 000 for a unit. The bank agrees that he does not have to make any payments for the first 3 months. The interest is 7.2%p.a. over 25 years compounded monthly. Find

(i) the amount he owes at the end of the first month. 1

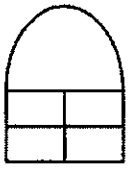
(ii) the amount he owes at the end of the fourth month. 1

(iii) how much is his monthly repayments over 25 years? 3

(iv) how much he pays back altogether 1

Question 7.(12marks)(Start a new booklet) **Marks**

(a) A metal frame consisting of four congruent rectangles and a semi-circle, as shown in the diagram, is to be constructed. The total amount of material for the whole frame is 36 m.



(i) If the length of each rectangle is x metres and the width is h metres, show that the total length of material L is $(6x+6h+\pi x)$ m. **2**

(ii) Show that the area A, enclosed by the frame may be expressed as $A=(24x-4x^2-\frac{1}{6}\pi x^2)\ m^2$ **2**

(iii) What is the length of the base of the frame if the frame is to enclose is to enclose a maximum area? **3**

(iv) Calculate the maximum area enclosed(2 decimal places) **1**

(b) The point $P(x,y)$ is equidistant from the lines $y=-2$ and $4y-3x+12=0$. Find the equation of the locus of P. **4**

Year 11 Ext 1 Yearly.

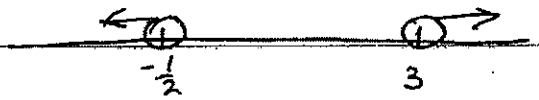
Q1a) $= 0.96616930\ldots$ (1)

$= 0.97$ (2 sign fig) (1)

b) $2x^2 - 5x - 3 > 0$

$(2x+1)(x-3) > 0$ (1)

Assume $=0$ $\quad x = -\frac{1}{2}$ or 3



(1)

$x < -\frac{1}{2}$ or $x > 3$ (1)

c) $3^x \times 9^{x+1} = \frac{1}{3}$

$3^x \times (3^2)^{x+1} = 3^{-1}$

$3^x \times 3^{2x+2}$ (1)

$3^{3x+2} = 3^{-1}$

$3x+2 = -1$

$3x = -3$

$x = -1$ (1)

d) $(2x+1)(x-1) = 14$

$2x^2 - x - 1 = 14$

$2x^2 - x - 15 = 0$ (1)

$(2x+5)(x-3) = 0$

$x = -\frac{5}{2}$ (1) or 3 (1)

e) $\int x^2 + x^{-4}\,dx = \frac{x^3}{3} - \frac{1}{3}x^{-3} + C$ (1) (1)

c optional.

Q2

a) (i) $y = x\sqrt{x} - \frac{1}{x^2} = x \cdot x^{1/2} - x^{-2}$

$= x^{3/2} - x^{-2}$ (1)

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} + 2x^{-3}$ (1)

(ii) $y = \frac{3x}{x^2-7}$

$\frac{dy}{dx} = \frac{(x^2-7).3 - 3x.2x}{(x^2-7)^2}$ (1)

$= \frac{-3x^2-21}{(x^2-7)^2}$ (1)

b) $2x+y-2=0$ $\quad y = -2x+2$

(i) $m = -2$

$m_\perp = \frac{1}{2}$ (1)

ii) $y - -1 = \frac{1}{2}(x - -4)$

$y+1 = \frac{1}{2}(x+4)$

$2y+2 = x+4$

$x - 2y + 2 = 0$

iii) $(2x+y-2=0) \times 2$

$+ \quad x-2y+2=0$

$4x+2y-4=0$

$5x - 2 = 0$

$x = \frac{2}{5}$ (1) $\quad y = \frac{6}{5}$ (1)

iv) $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|2(-4)+1(-1)+(-2)|}{\sqrt{2^2+1^2}}$ (1)

$= \frac{|-11|}{\sqrt{5}} = \frac{11}{\sqrt{5}}$ (1).

c) $\frac{dy}{dx} = 4x - 3$

$y = 2x^2 - 3x + C$ (1)

$(2, -8)$ $\quad -8 = 8 - 6 + C$

$C = -10$ (1)

$y = 2x^2 - 3x - 10$

Q3

a) $S_6 = 84$ $\quad S_{14} = 420$

(i) $84 = \frac{6}{2}[2a + (5)d] \rightarrow 3(2a+5d) = 84$ (1)

$420 = \frac{14}{2}[2a + 13d] \rightarrow 7(2a+13d) = 420$ (2)

Eqn (1) $2a + 5d = 28$

(2) $2a + 13d = 60$

$-8d = -32$

$d = 4$ (1)

Eqn (1) $2a + 20 = 28$

$\therefore a = 4$ (1)

ii) $S_n = \frac{n}{2}[8 + (n-1)\times 4] = \frac{n}{2}[4n+4]$

$= 2n^2 + 2n$ (1)

iii) $T_n = a + (n-1)d = 4 + (n-1)\times 4 = 4n + 4 - 4$

$T_n = 4n$ (1)

b) 7, 8, 13

i) $\cos\theta = \frac{7^2 + 8^2 - 13^2}{2\times 7\times 8}$ (1) $= \frac{-56}{112}$

$\theta = 120°$ (1)

ii) Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 7\times 8\times \sin 120$

$= 24.2\ cm^2$ (1)

(c) $x^2 + (k+3)x - k = 0$

Real $\Delta = b^2 - 4ac \geq 0$

$= (k+3)^2 - 4.1.(-k) \geq 0$

$= k^2 + 6k + 9 + 4k \geq 0$

Assume $= 0$ $\quad = k^2 + 10k + 9 = 0$

$(k+1)(k+9) = 0$

$k = -1$ or -9

$k \geq -1$ or $k \leq -9$.

Test 0

(d) $\frac{\cos^2 A}{1-\sin A} = \frac{1-\sin^2 A}{1-\sin A}$ (1) $= \frac{(1-\sin A)(1+\sin A)}{1-\sin A}$

$= 1 + \sin A$ (1).

Q4 a) $x^2 + (k+1)x + \left(\frac{k+1}{4}\right) = 0$

i) Roots are $\alpha, -\alpha$

Sum $\alpha + -\alpha = -\frac{b}{a} = -\frac{(k+1)}{1}$ (1)

$0 = -k - 1$

$k = -1$ (1)

ii) root are equal α, α

$\alpha + \alpha = -(k+1) = 2\alpha$ $\alpha = -\frac{1}{2}(k+1)$

$\alpha \times \alpha = \frac{k+1}{4}$ (1)

$-\frac{1}{2}(k+1) \times -\frac{1}{2}(k+1) = \frac{k+1}{4}$

$\frac{1}{4}(k^2 + 2k + 1) = \frac{1}{4}(k+1)$

$k^2 + 2k + 1 = k + 1$

$k^2 + k = 0$

$k(k+1) = 0$

$k = 0$ or -1 (1)

iii) roots are $\alpha + \frac{1}{\alpha}$

product $\alpha \times \frac{1}{\alpha} = \frac{(k+1)}{4} = 1$ (1)

$k + 1 = 4$

$k = 3$ (1)

b) $x^2 + 4x - 8y + 12 = 0$ $\boxed{x^2 = 4ay}$

$(x+2)^2 - 4 - 8y + 12 = 0$

$(x+2)^2 = 8y - 8 = 8(y-1)$ (1)

i) Vertex $(-2, 1)$ (1)

ii) focus $4a = 8$

$a = 2$

$S(-2, 3)$ (1)

iii) Dir $y = -1$ (1)

iv) $8y = x^2 + 4x + 12$

$y = \frac{1}{8}(x^2 + 4x + 12)$

$\frac{dy}{dx} = \frac{1}{8}(2x + 4)$

$x = 4$ $m = \frac{1}{8}(8+4) = \frac{12}{8} = \frac{3}{2}$ (1)

tangent $y - 6 = \frac{3}{2}(x - 4)$ (1)

Q5 a) i) $y = 4x^3 + 6x^2$

$\frac{dy}{dx} = 12x^2 + 12x = 0$ (1)

$12x(x+1) = 0$

$x = 0$ or -1 (1)

$y = 0$ $\quad\quad$ 2

$\frac{d^2y}{dx^2} = 24x + 12$

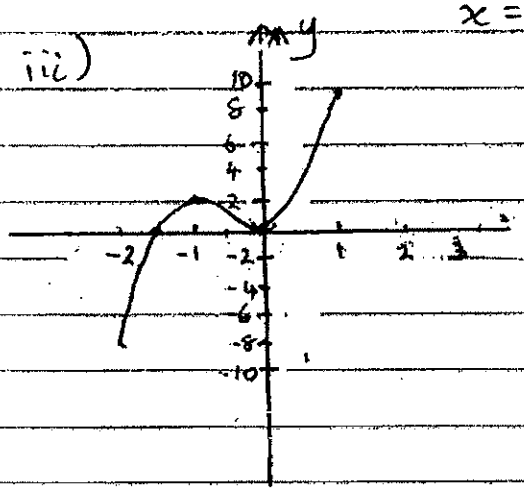
$f''(0) = 0 + 12 = +12 = +$ $\cup$ Min

$f''(-1) = -24 + 12 = -12 = -$ $\cap$ Max (1)

ii) Inflexion $24x + 12 = 0$ (1)

$x = -\frac{1}{2}$ $\quad$ $y = 1$ (1)

iii)



$-2 \leq x \leq 1$

(1) shape

(1) endpoints.

$y = 4x^3 + 6x^2 = 0$

$2x^2(2x + 3)$

$x = 0$ or $-\frac{3}{2}$

$x = 1$ $\quad$ $y = 10$

$x = -2$ $\quad$ $y = -8$

iv) Max = 10 (1)

b) $\left(x + \frac{1}{x}\right)^2 - 5\left(x + \frac{1}{x}\right) + 6 = 0$ $\quad$ $M = x + \frac{1}{x}$

$M^2 - 5M + 6 = 0$ (1)

$(M - 3)(M - 2) = 0$

$M = 3$ or 2 (1)

$x + \frac{1}{x} = 3$ or $x + \frac{1}{x} = 2$

$x^2 + 1 = 3x$ $\quad$ $x^2 + 1 = 2x$

$x^2 - 3x + 1 = 0$ $\quad$ $x^2 - 2x + 1 = 0$

$x = \frac{3 \pm \sqrt{9 - 4 \times 1 \times 1}}{2}$ $\quad$ $(x-1)(x-1) = 0$

$= \frac{3 \pm \sqrt{5}}{2}$ (1) $\quad$ $x = 1$ (1)

Q6 a) $\sin x = \pm \frac{\sqrt{3}}{2}$ (1) $0 \leq x \leq 360$

$x = 60, 120, 240, 300$

(1) 2 solns. (1) 4 solns.

b) $3x + 4y + 21 = 0$

$(x-4)^2 - 16 + (y+2)^2 - 4 - 5 = 0$

$(x-4)^2 + (y+2)^2 = 25$ (1)

Centre $(4, -2)$ $r = 5$ (1)

$d = \left| \frac{12 - 8 + 21}{\sqrt{3^2 + 4^2}} \right| = \frac{25}{5} = 5 = r$ (1)

$\therefore$ tangent

c) 250 000 3mm

(i) $A_1 = 250\,000(1.006)$ (1)

$A_2 = 250\,000(1.006)^2$

$A_3 = 250\,000(1.006)^3$

(ii) $A_4 = 250\,000(1.006)^4 - M$. (1)

$A_5 = 250\,000(1.006)^5 - M(1.006) - M$.

(iii) $A_n = 250\,000(1.006)^{300} - M(1.006^{296} + \ldots + 1)$ (1)

$A_n = 0$ (1) OR $250000(1.006)^{300} = M(1 + \ldots + 1.006^{296})$

$S_{297} = \frac{1(1.006^{297} - 1)}{1.006 - 1}$ (1)

$S_{297} = 818.3623487$

$1504299.2 = 818.36\ldots \times M$

$M = \$1838.18$ (1)

(iv) $\times 297 = 545940.15$ (1)

$C = 2\pi r$

Q7 a) $36x$ ✓ (1) (1).

i) $L = 6x + 6h + \pi x$ (2) showing working.

ii) $36 = 6x + 6h + \pi x$

$36 - 6x - \pi x = 6h$

$h = \frac{1}{6}(36 - 6x - \pi x)$ (1)

Area $= 2x \times 2h + \frac{1}{2}\pi x^2$

$= 2x \times \frac{1}{3}(36 - 6x - \pi x) + \frac{1}{2}\pi x^2$

$= 24x - 4x^2 - \frac{2}{3}\pi x^2 + \frac{1}{2}\pi x^2$ (1) Working.

$= 24x - 4x^2 - \frac{1}{6}\pi x^2$

iii) $\frac{dA}{dx} = 24 - 8x - \frac{1}{3}\pi x = 0$ (1)

$24 = x(8 + \frac{\pi}{3}) = x(\frac{24+\pi}{3})$

$\frac{72}{24+\pi} = x$ (1)

~~Base $= 2x = \frac{144}{24+\pi}$~~

$\frac{d^2A}{dx^2} = -8 - \frac{1}{3}\pi = -$ ∩. Max. (1)

iv) Area $= 24(\frac{72}{24+\pi}) - 4(\frac{72}{24+\pi})^2 - \frac{1}{6}\pi(\frac{72}{24+\pi})^2$

5.3055

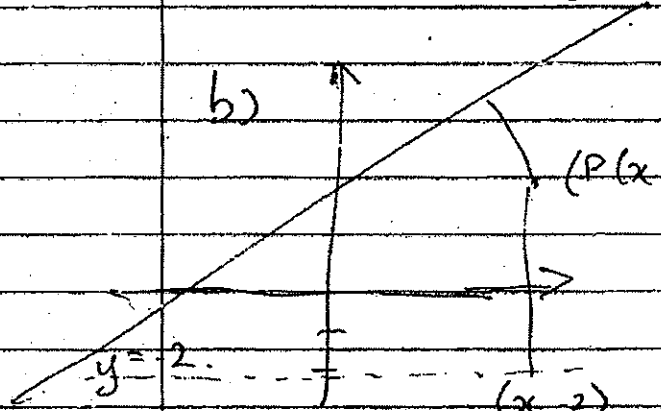
$= 127.332 - 4$ ~~127.33~~

~~$127.3322477 -$~~

$= 63.66612387 -$

$= 31.83$ (1)

b)



$d = \left|\frac{4y - 3x + 12}{5}\right|$ (1)

$d^2 = (y+2)^2$ (1)

$\left|\frac{4y - 3x + 12}{5}\right| = y + 2$

$4y - 3x + 12 = 5y + 10$

$3x - y - 2 = 0$ or (1)

$\frac{4y - 3x + 12}{5} = -y - 2$

$4y - 3x + 12 = -5y - 10$ (1)

$3x - 9y - 22 = 0$