TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT
TASK 4

Year 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 – 14
- Write your Board of Studies Student Number **and** Class Teacher on the writing booklet(s) **or** sheet(s) submitted
- **Weighting**: 40%

Total marks – 70

Section I Pages 3 – 7

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 8 – 14

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I **10 marks**

- Circle the correct response on the answer sheet provided
- Each question is worth 1 mark

---

1. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

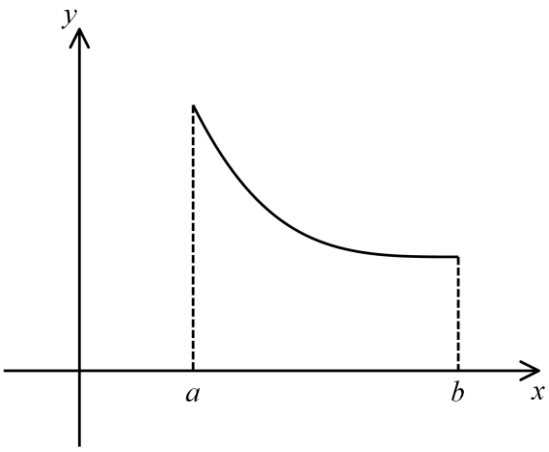
(A) $k \geq -3$

(B) $k \leq -3$

(C) $k \geq 3$

(D) $k \leq 3$

2. For the function $y = f(x)$, $a < x < b$ graphed below:



which of the following is true?

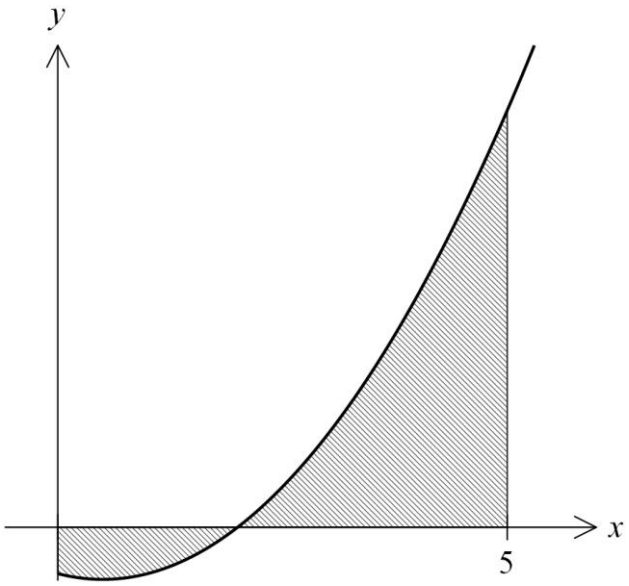
(A) $f'(x) > 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$

(C) $f'(x) < 0$ and $f''(x) > 0$

(D) $f'(x) < 0$ and $f''(x) < 0$

3. Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?



(A) $A = \left| \int_0^1 \left(x^2 - x - 2\right)dx \right| + \int_1^5 \left(x^2 - x - 2\right)dx$

(B) $A = \int_0^1 \left(x^2 - x - 2\right)dx + \left| \int_1^5 \left(x^2 - x - 2\right)dx \right|$

(C) $A = \left| \int_0^2 \left(x^2 - x - 2\right)dx \right| + \int_2^5 \left(x^2 - x - 2\right)dx$

(D) $A = \int_0^2 \left(x^2 - x - 2\right)dx + \left| \int_2^5 \left(x^2 - x - 2\right)dx \right|$

4. The solution to the equation $e^x = 4$, to three significant figures, is:

(A) 1.386

(B) 1.38

(C) 1.39

(D) 54.598

5. $\sum_{n=3}^{7} 2n + 3 =$

(A) 17

(B) 65

(C) 77

(D) 91

6. What are the coordinates of the focus of the parabola $4y = x^2 - 8$?

(A) (0, -8)

(B) (0, -7)

(C) (0, -2)

(D) (0, -1)

7. What are the domain and range of the function $f(x) = \sqrt{4 - x^2}$?

(A) Domain: $-2 \le x \le 2$; Range: $0 \le y \le 2$

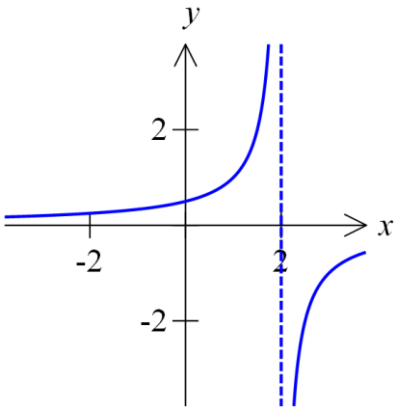
(B) Domain: $-2 \le x \le 2$; Range: $-2 \le y \le 2$

(C) Domain: $0 \le x \le 2$; Range: $-4 \le y \le 4$

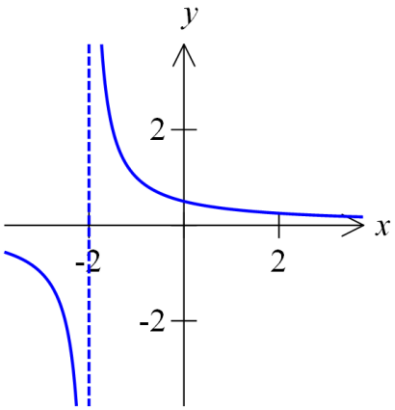
(D) Domain: $0 \le x \le 2$; Range: $0 \le y \le 4$

8. Which of the following represents the graph of $y = \dfrac{1}{2-x}$?

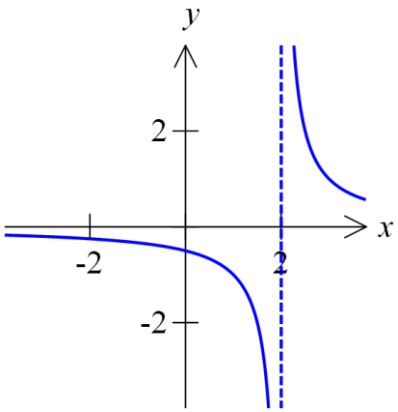
(A)



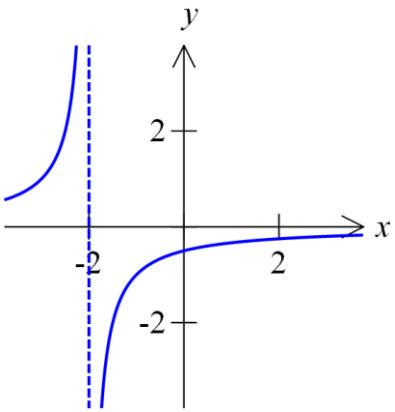
(B)



(C)



(D)



9. What is the value of $\displaystyle\int_0^1 \left(e^{3x} - 1\right)dx$?

(A) $\dfrac{e^3}{3}$

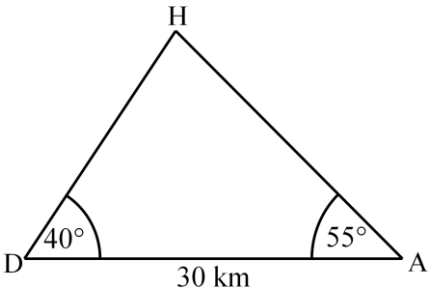
(B) $\dfrac{e^3}{3} - 1$

(C) $e^3 - 1$

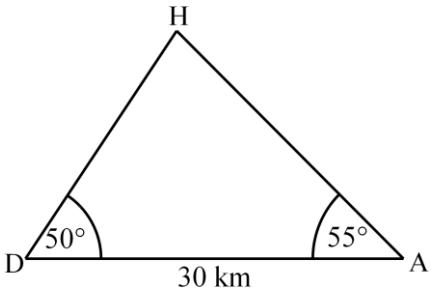
(D) $\dfrac{1}{3}\left(e^3 - 4\right)$

10. Archer (A) is 30km due east of Dulwich (D). Holwood (H) is on a bearing of 040° from Dulwich and 325° from Archer. Which of the following diagrams best represents this information?

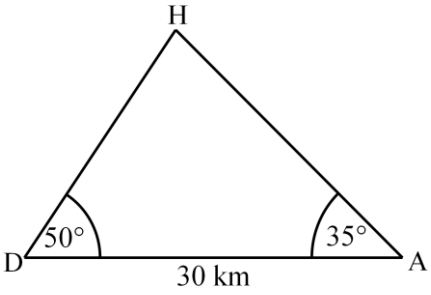
(A)



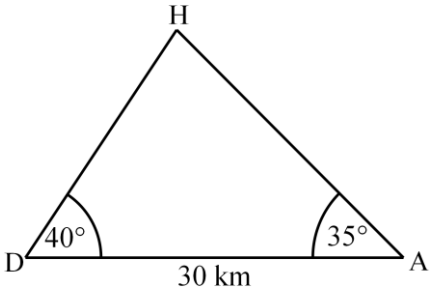
(B)



(C)



(D)



End of Section I

Section II 60 marks

- Begin each question in a new writing booklet or on a new answer sheet
- Show all necessary working
- Each question is worth 15 marks

Question 11 (15 marks)

a) Differentiate

i) $\sqrt{2x - x^2}$ **2**

ii) $e^{4x} + \frac{1}{x}$ **2**

iii) $\frac{3x - 7}{3 + 4x}$ **2**

b) Find:

i) $\int (5x - 1)^9 dx$ **2**

ii) the value of k such that $\int_1^k \frac{4}{x^2} dx = 3$. **2**

c) Solve $2\sin x + \sqrt{3} = 0$ for $0^\circ \leq x \leq 360^\circ$. **3**

d) Find the centre and radius of the circle $x^2 - 6x + y^2 + 2y - 6 = 0$. **2**

Question 12 (15 marks) Begin a NEW answer booklet or answer sheet

a) The sum of the first n terms of an arithmetic series is given by:

$S_n = n(38 - 2n)$

i) Calculate S_1 and S_2. **1**

ii) Find the first three terms of the series. **2**

iii) Find an expression for the n-th term. **2**

b) A parabola has the equation $2y^2 = 4x + 8$.

i) Express this equation in the form $(y - y_1)^2 = 4a(x - x_1)$. **2**

ii) Draw a neat sketch of the parabola showing the focus; vertex; axis of symmetry and x and y-intercepts. **2**

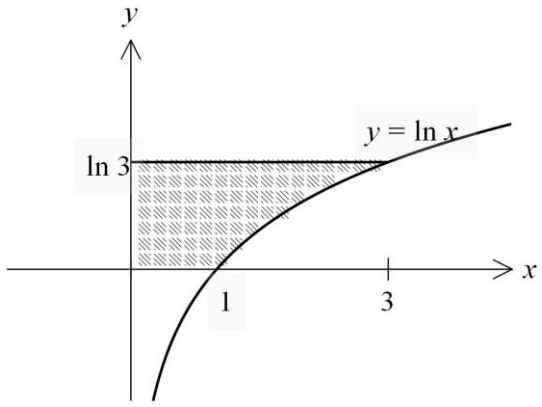
c) Use Simpson's Rule with 5 function values to find an approximate value of **3**

$$\int_2^4 \frac{x}{2} \ln x \, dx$$

correct to 3 decimal places.

Question 12 continues on the following page.

d) The diagram shows the area bounded by the graph $y = \ln x$, the coordinate axes and line $y = \ln 3$.

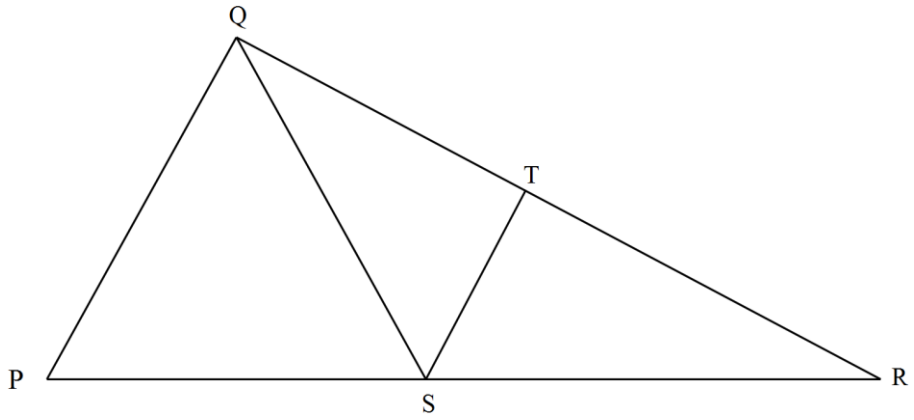


i) Find the shaded area. **2**

ii) Hence write down the exact value of $\int_1^3 \ln x \, dx$. **1**

Question 13 (15 marks) Begin a NEW answer booklet or answer sheet

a) In ΔPQR, point T lies on side QR and point S lines on side PR such that $QT = TR$, $QS = QP$ and $ST \perp QT$.



Copy or trace the diagram in your answer booklet.

i) Prove that $\Delta QTS \equiv \Delta RTS$. **2**

ii) Prove that $\angle QPS = 2 \angle TQS$. **2**

b) Consider the function $f(x) = x^4 - 4x^3$.

i) Find the stationary points on the curve and determine their nature. **4**

ii) Find any points of inflexion. **2**

iii) Sketch the curve $f(x) = x^4 - 4x^3$ showing all important features, including x and y-intercepts and stationary points. **2**

c) A normal is drawn to the curve $y = e^{2x}$ at the point $P(\log_e 2, 4)$. The normal cuts the x-axis at Q.

i) The equation of the normal at P is given by $x + 8y = k$ where k is a real number. Find the exact value of k. **2**

ii) Find the coordinates of Q. **1**

Question 14 (15 marks) Begin a NEW answer booklet or answer sheet

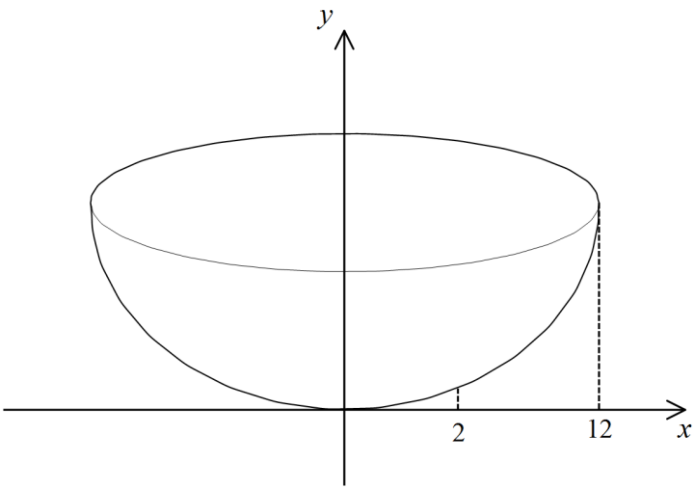
a) Given the equation $3x^2 + 4x - 3 = 0$ has roots α and β, evaluate the following:

i) $\alpha + \beta$ and $\alpha\beta$ **1**

ii) $(\alpha - 1)(\beta - 1)$ **1**

iii) $\alpha^3 + \beta^3$ **1**

b) A bowl was designed by rotating the section of the curve $y = \frac{1}{4}x^2$ between $x = 2$ cm and $x = 12$ cm, about the y-axis.



i) Calculate the volume of the bowl, leaving your answer in terms of π. **2**

ii) Hence, calculate the capacity of the bowl, correct to the nearest litre. **1**
($1000 \text{ cm}^3 = 1$ litre)

Question 14 continues on the following page.

c) Penny has just commenced work and is investigating superannuation funds. She calculates that she will need \$1 000 000 in her fund when she retires in 40 years. She finds a fund guaranteeing to pay interest on each deposit at the rate of 6% p.a. compounded monthly.

She intends to deposit \$$M$ into the fund at the beginning of each month for 40 years.

After 25 years she intends to double the amount she deposits each month. That is, she will deposit a further \$$M$ into the fund for the remaining 15 years.

i) Show that the value of M can be found using the formula **2**

$$1000000 = 201M\left(1{\cdot}005^{480} + 1{\cdot}005^{180} - 2\right).$$

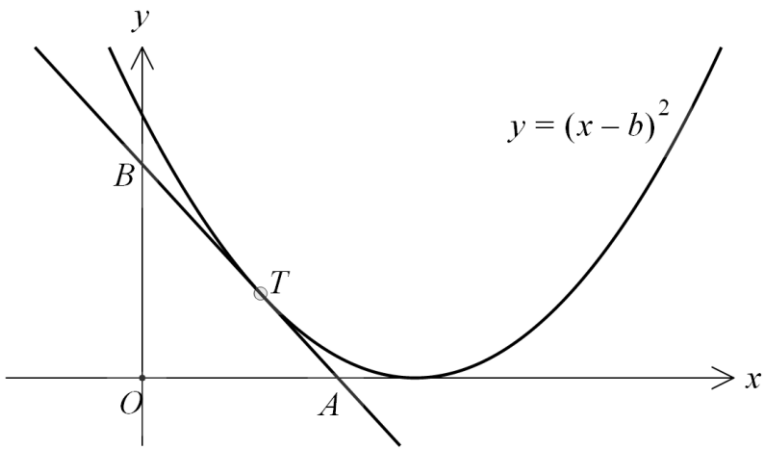
ii) Find the value of M. **1**

Question 14 continues on the following page.

d) The diagram shows the graph of $y = (x-b)^2$.

$T\ \left(t,\ (t-b)^2\right)$ is a point on the curve such that $0 < t < b$.

The tangent at T is drawn to intersect with the x and y axes at the points A and B respectively as shown.



i) Show that the tangent to the curve at T is given by **2**

$y = (2x - t - b)(t - b)$.

ii) Find the coordinates of the point T, in terms of b, which will **maximise** the area of ΔABO. **4**

End of Section II

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

Note $\ln x = \log_e x, \quad x > 0$

TRINITY GRAMMAR SCHOOL
2012, Year 11 Mathematics Extension 1
Yearly Examination
SECTION I
ANSWER SHEET

Student Number: …… Class Teacher: ……..

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D

YEAR 11 EXTENSION 1 ASSESSMENT 2 (1/3/12)

Question 1

a) $f(x) = x^2 + 1 + x^{-2}$

$f(1+\sqrt{2}) = (1+\sqrt{2})^2 + 1 + (1+\sqrt{2})^{-2}$ ✓

$= 1 + 2\sqrt{2} + 2 + 1 + \dfrac{1}{1 + 2\sqrt{2} + 2}$ ✓

$= 4 + 2\sqrt{2} + \dfrac{1}{3 + 2\sqrt{2}} \times \dfrac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})}$ ✓

$= 4 + 2\sqrt{2} + \dfrac{3 - 2\sqrt{2}}{9 - 8}$

$= 7.$

b) $x^2 - 10x + y^2 + 6y + 33 = 0$

$x^2 - 10x + 25 + y^2 + 6y + 9 = -33 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 1$

$\therefore$ centre $(5, -3)$ ✓ $\quad r = 1$ ✓

c) $y = \dfrac{x}{4 - x^2}$ $\quad D: \quad 4 - x^2 \neq 0$ ✓

$x^2 \neq 4$

$x \neq \pm 2$ ✓

$\therefore D:$ all real x, except $x \neq \pm 2$.

d) $f(x) = x - x^2$

$f(-x) = (-x) - (-x)^2$

$= -x - x^2$

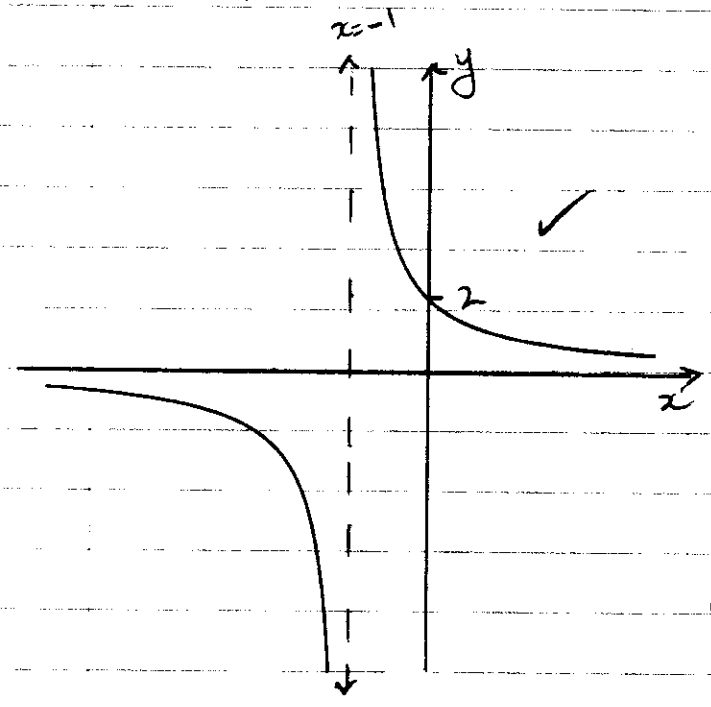
$-f(x) = -(x - x^2)$

$= -x + x^2$

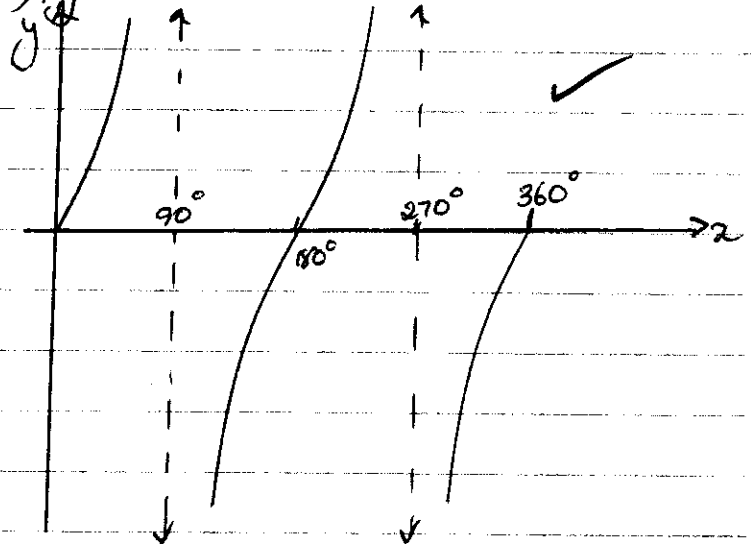
Since $f(x) \neq f(-x)$ or is $-f(x) \neq f(-x)$ the function is neither even or odd. ✓✓

Q2

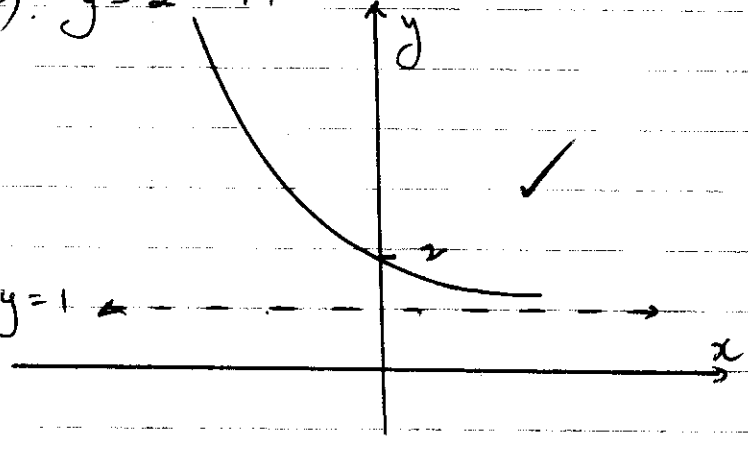
a) i) $y = \frac{2}{x+1}$



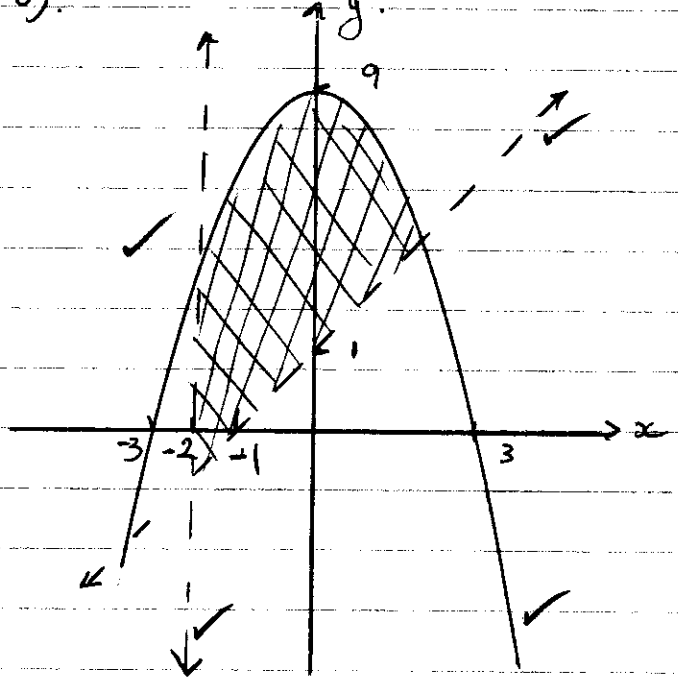
iii) $y = \tan x \quad 0° \leq x \leq 360°$



ii) $y = 2^{-x} + 1$



b)



Q2 c) i) $f(1) = b(1)^2 + 2$ a

$= b + 2$.

$f(-1) = (-1-1)^2$

$= 4$.

$\therefore b + 2 = 4$

$b = 2$ ✓

$f\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right) + 3$

$f\left(-\frac{1}{2}\right) = \left(\frac{1}{2} - 1\right)^2$

$= \frac{9}{4}$.

$\therefore \frac{a}{2} + 3 = \frac{9}{4}$

$2a + 12 = 9$

$2a = -3$

$a = \frac{-3}{2}$ ✓

ii) $f(x) = \begin{cases} (x-1)^2 & x < 0 \\ -\frac{3}{2}x + 3 & 0 \leq x < 1 \\ 2x^2 + 2 & x \geq 1 \end{cases}$

$\therefore$ Range: $y > 1$ ✓

Q3

a) $\cos 225 \times \tan 150$

$= \cos(180+45) \times \tan(180-30)$

$= -\cos 45 \times -\tan 30$

$= \frac{-1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{3}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$ ✓✓

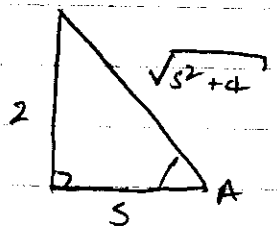
b) $\tan\theta = -1.44598$

$\theta = 180 - 55.33, \quad 360 - 55.33$

$= 124^\circ, \quad 304^\circ$ ✓✓

c) $\sec A = \frac{\sqrt{s^2+4}}{s}$

$\therefore \cos A = \frac{s}{\sqrt{s^2+4}}$



$\cos^2 A = \frac{s^2}{s^2+4}$ ✓

$\sin^2 A = \left(\frac{2}{\sqrt{s^2+4}}\right)^2 = \frac{4}{s^2+4}$

$\sin^2 A + \cos^2 A = \frac{4}{s^2+4} + \frac{s^2}{s^2+4}$

$= \frac{s^2+4}{s^2+4} = 1$ ✓

d) $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$ ✓

$= \frac{\sin^2\theta}{\sin^2\theta} = 1$ ✓

e) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

LHS $= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\tan^2\theta}{\sec^2\theta}$ ✓

$= (1-\tan^2\theta) \div \frac{1}{\cos^2\theta}$

$= \left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right) \cdot \cos^2\theta$ ✓

$= \cos^2\theta - \frac{\sin^2\theta \cos^2\theta}{\cos^2\theta}$

$= \cos^2\theta - \sin^2\theta$ = RHS.

Question 4.

a)

i) In ΔABC.

$$\frac{BC}{\sin 25} = \frac{20}{\sin 13} \quad \checkmark$$

$$BC = \frac{20 \sin 25}{\sin 13}$$

ii) $\sin 38 = \frac{CD}{BC}$.

$$CD = BC \sin 38 \quad \checkmark$$

$$= \frac{20 \sin 38 \sin 25}{\sin 13}$$

$$= 23.13301\ldots$$

$$\approx 23 \text{ m}.$$

b) ΔADC: $A = \frac{1}{2} ab \sin C$.

$$= \frac{1}{2} \cdot 9 \cdot 8 \cdot \sin 38.$$

$$= 22.16 \text{ cm}^2 \quad \checkmark$$

In ΔABC:

$$BC^2 = 6^2 - 4^2$$

$$= 20 \quad \checkmark$$

$$BC = \sqrt{20}.$$

$$\text{Area} = \frac{1}{2} \cdot 4 \cdot \sqrt{20}.$$

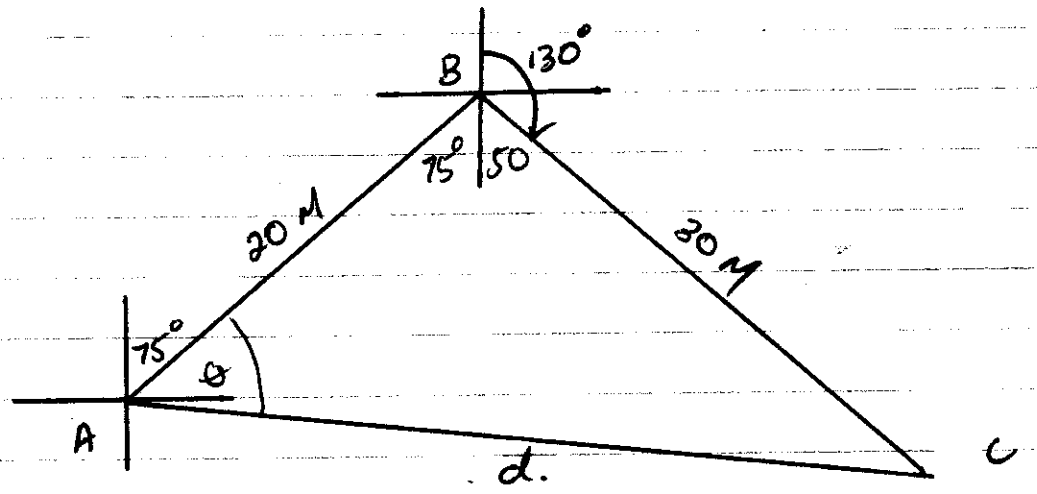
$$= 8.94 \text{ cm}^2 \quad \checkmark$$

$\therefore$ Total Area $= 22.16 + 8.94$

$$= 31.10808\ldots$$

$$= 31.11 \text{ cm}^2 \quad \checkmark$$

c)



i) $d^2 = 20^2 + 30^2 - 2(20)(30)\cos(125)$ ✓

$= 1988.29$

$d = 44.690264...$

$= 45$ nautical miles. ✓

ii) $\frac{\sin\theta}{30} = \frac{\sin 125}{45}$

$\sin\theta = \frac{30 \sin 125}{45}$

$\theta = 33°$ ✓

Bearing $= 33 + 75 = 108$ ✓

∴ Bearing from A to C. is 108°T.