DETUR GLORIA SOLI DEO

# Trinity Grammar School

Mathematics Department

# 2013

**YEARLY EXAMINATION**

PRELIMINARY ASSESSMENT TASK 4

## Year 11

## Mathematics Extension 1

**Board of Studies Student Number**
(Year 12 only)

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|---|---|---|---|---|---|---|---|
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**Class Teacher:**

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**Name:**
(Year 11 only)

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***Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.***

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Preliminary Assessment Weighting:** 45%

**Date of Examination:** **Friday 30th August**

**Total marks – 70**

**Section I**

**10 marks**

- Attempt all Questions
- Allow about **15** minutes for this section

**Section II**

**60 marks**

- Attempt all Questions
- Allow about **1** hour **45** minutes for this section

**Section I 10 marks**

- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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**1** $sin\ 2A\ cosA - cos\ 2A\ sin\ A =$

(A) $\sin 3A$

(B) $\sin A$

(C) $\cos 3A$

(D) $\cos A$

**2** The simplification of $3\left(\log_a\left(\frac{m}{n}\right)+\log_a\left(\frac{n}{m}\right)\right)$ is

(A) 3

(B) $3\log_a\left(\frac{m+n}{n+m}\right)$

(C) $\log\left(\frac{m+n}{n+m}\right)^3$

(D) 0

**3** The complete factorization of $\boldsymbol{x^6 - y^6}$ is

(A) $(x^3 - y^3)^2$

(B) $(x^3 - y^3)(x^3 + y^3)$

(C) $(x - y)(x^2 + xy + y^2)(x^3 + y^3)$

(D) $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

4 The equation of the line passing through $(\sqrt{3}, -2)$ and parallel to the line $x-\sqrt{3}y+2=0$ in exact form is

(A) $x-\sqrt{3}\,y-3\sqrt{3}=0$

(B) $y=\sqrt{3}x-2$

(C) $y-2=\sqrt{3}x$

(D) $y-\sqrt{3}=1$

5 The midpoint of $(a, 3)$ and $(-4, b)$ is $(1, 2)$. The values of $a$ and $b$ are

(A) $a=6,\ b=1$

(B) $a=1,\ b=6$

(C) $a=-3\,, b=1$

(D) $a=-1,\ b=2$

6 The domain and range of $y=\frac{3}{2x-1}$ are respectively:

(A) $D_f: all\ real\ x, x \neq \frac{1}{2}.\quad R_f = all\ real\ y$

(B) $D_f: all\ real\ x.\quad R_f = all\ real\ y, y \neq \frac{1}{2}$

(C) $D_f: all\ real\ x. x \neq 0\quad R_f = all\ real\ y, y \neq 0$

(D) $D_f: all\ real\ x, x \neq \frac{1}{2}.\quad R_f = all\ real\ y, y \neq 0$

7 $cos^2 2x - sin^2 2x =$

(A) $sin\, 2x$

(B) $cos\, 2x$

(C) $sin\, 4x$

(D) $cos\, 4x$

8 If $\tan\frac{\theta}{2} = t$, then $3\sin\theta + 4\cos\theta$ expressed in terms of $t$ is

(A) $\frac{1+2t-t^2}{1+t^2}$

(B) $\frac{4+2t+t^2}{1+t^2}$

(C) $\frac{1+2t-4t^2}{1+t^2}$

(D) $\frac{4+6t-4t^2}{1+t^2}$

9 Find the $\lim\limits_{x\to\infty} \frac{3x^2-2x+7}{-2x^2+6x-9}$

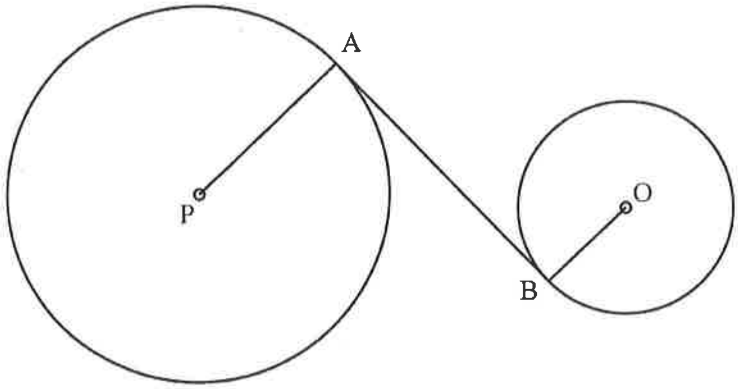
(A) $0$

(B) $-\frac{3}{2}$

(C) $\frac{1}{3}$

(D) $\frac{7}{9}$

**10** $AB$ is a tangent to two circles. If $O$ and $P$ are the centres of the circles, find the relationship between $OB$ and $AP$.




(A) $AP = OB$

(B) $AP = 2OB$

(C) $AP \parallel OB$

(D) $AP = 3OB$

**End of Section I**

**Section II commences on the next page.**

**Section II** **60 marks**

- Begin each question in a SEPARATE writing booklet or answer sheet
- Show all relevant mathematical reasoning and/or calculations

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**Question 11** **(15 marks)** Use a SEPARATE writing booklet

Marks

(a) Find the coordinates of the point $P\,(x, y)$ that divides $AB$ internally in the ratio $4 : 5$ given that $A$ has co-ordinates $(-3,2)$ and $B$ has the co-ordinates $(15,-10)$. **2**

(b) If $f(x) = (2x+1)(x^4-3)^2$ find $f'(-1)$. **3**

(c) Find the obtuse angle between between the curves $f(x) = x^2 - 4x$ and $g(x) = x^2 - 12$ at the point where they meet. **3**

(d) Find the value of $a$, $b$ and $c$ if $x^2 + 5x - 3 \equiv a(ax+1) + b(x+1)^2 + cx$. **3**

(e) If $\alpha$ and $\beta$ are the roots of $5x^2 - 4x + 3$, without solving , find the values of :

(i) $(\alpha+1)(\beta+1)$ **2**

(ii) $\alpha^2 + \beta^2$ **2**

**Question 12 (15 marks)** Use a SEPARATE writing booklet

Marks

(a) For what values of $M$ does the equation $x^2 + Mx + (3 - M) = 0$ have equal roots. **2**

(b) Solve $2^{2x} - 9(2^x) + 8 = 0$. **3**

(c) Prove that $3x - 4y + 25 = 0$ is a tangent to the circle with centre at the origin and radius 5 units. **3**

(d) Differentiate $\sqrt[5]{5 - 6x^3}$, giving your answer without negative or fractional indices **2**

(e) Let $y = x\sqrt{x}$.

(i) Find $\frac{dy}{dx}$, leaving your answer in surd form. **2**

(ii) Find the coordinates of the point on the graph of $y = x\sqrt{x}$ at which the normal has a slope of $-\frac{1}{4}$. **3**

**Question 13** **(15 marks)** Use a SEPARATE writing booklet Marks

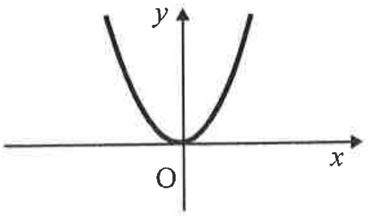
(a) **Figure 1** shows the graphs of the functions $f_1$, $f_2$, $f_3$, $f_4$

**Figure 2 includes** the graphs of the derivatives of the functions shown in **Figure 1**,

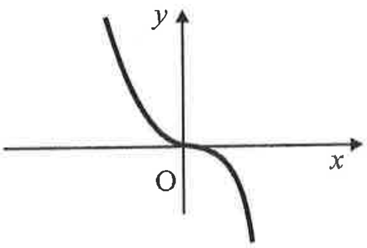
*eg* the derivative of $f_1$ is shown in diagram (d).

**Figure 1**

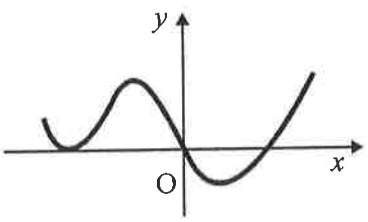
$f_1$




$f_2$




$f_3$




$f_4$

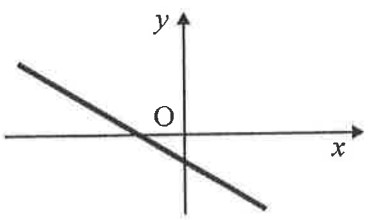
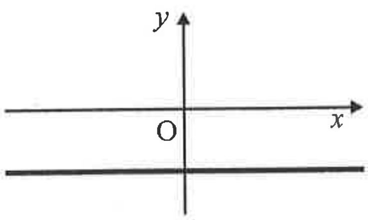
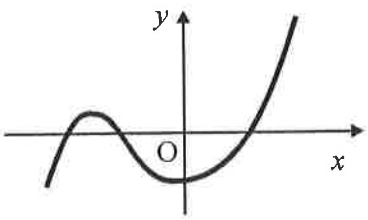



**Figure 2**

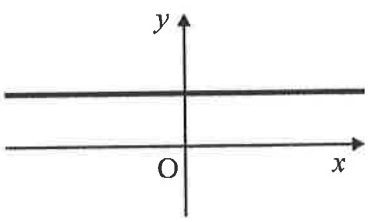
(a)



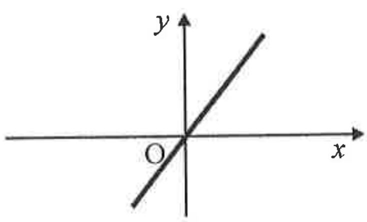

(b)



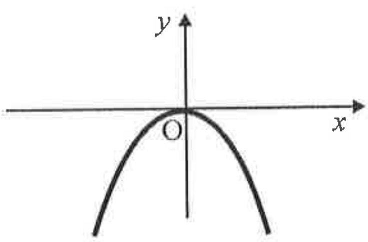

(c)




(d)




(e)




**Question 13 continues on next page.**

Marks

Copy the table below into your **writing booklet and complete** by matching each function with its derivative.

| Function | Derivative diagram |
|---|---|
| $f_1$ | (d) |
| $f_2$ | |
| $f_3$ | |
| $f_4$ | |

**3**

(b) Consider the function $y = \dfrac{3x-2}{2x-5}$.

The graph of this function has a vertical and a horizontal asymptote.

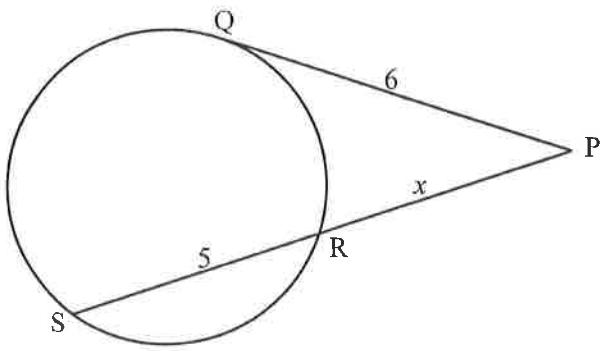
(i) Write down the equation of

($\alpha$) the vertical asymptote; **1**

($\beta$) the horizontal asymptote. **1**

(ii) Find $\dfrac{dy}{dx}$ simplifying the answer as much as possible. **2**

(c) $PQ$ is a tangent to a circle $QRS$, while $PRS$ is a secant intersecting the circle in $R$ and $S$, as in the diagram.




Given that $PQ = 6$, $RS = 5$, $PR = x$, find the length $PR$. **2**

**Question 13 continues on the next page.**

Q

Question 13 (continued)

Marks

(d) Show that $\dfrac{2\sin^3\theta + 2\cos^3\theta}{\sin\theta + \cos\theta} = 2 - \sin 2\theta$ provided, $\sin\theta + \cos\theta \neq 0$. **3**

(e) Find the equation of the straight line passing through $(3,6)$ that also passes through the intersection of the lines $x - 2y = 0$ and $3x + y + 7 = 0$. **3**

**Question 14 (15 marks)** Use a SEPARATE writing booklet Marks

(a) Let $f(x) = x^3 - 4x + 1$.

(i) Expand $(x + h)^3$. **1**

(ii) Use the formula $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. **3**

(iii) The tangent to the curve of $f(x)$ at the point $P(1, -2)$ is parallel to the tangent at a point $Q$ on the same curve. Find the coordinates of $Q$. **3**

(b) If the sum of the roots of $(1 + t)x^2 - (2t - 4)x - t^2 = 0$ is twice the product of the roots, where $t$ is a real number and $t \neq -1$, find the value(s) of $t$. **3**

(c) Solve $\sin 2x = \dfrac{1}{\sqrt{2}}$ for $0 \leq x \leq 360^\circ$. **2**

(d) Solve $\dfrac{2}{x-3} \leq 3$. **3**

**End of Section II**
**End of Examination**

Q11 - MR ROGERSON



Yr 11 Ext 1

Multiple choice.

1) B 2) D 3) D 4) A 5) A 6) D 7) D 8) D 9) B 10) C

Q11. a) A $(-3, 2)$ B $(15, -10)$ $4:5$.

$x_3 = \frac{4 \times 15 + 5 \times -3}{4+5} = 5$ (1) $y_3 = \frac{4 \times -10 + 5 \times 2}{9} = \frac{-30}{9} = \frac{-10}{3}$ (1)

b) $f(x) = (2x+1)(x^4-3)^2$

$f'(x) = 2(2x+1)(x^4-3).4x^3 + 2(x^4-3)^2$ (1) each part Total (2)

$f'(1) = 2.3.-2.4 + 2.(-2)^2 = -40$ ans (1)

c) $f(x) = x^2 - 4x$ $g(x) = x^2 - 12$

$x^2 - 4x = x^2 - 12.$ (1) putting equa

$-4x = -12$

$x = 3$ $y = -3.$ (1) for x (1) for y.

d) $x^2 + 5x - 3 \equiv a(x+1) + b(x+1)^2 + cx$

$b = 1$ $x^2$ term (1)

$x = 0$ $-3 = a + b$

$\therefore$ $a = -4$ (1)

$x = -1$ $1 - 5 - 3 = -4(4+1) + c$

$c = -13$ (1)

e) $5x^2 - 4x + 3$

$\left.\begin{aligned}\alpha + \beta &= 4/5 \\ \alpha\beta &= 3/5\end{aligned}\right\}$ (1)

i) $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$ (1)

$= \frac{3}{5} + 4/5 + 1 = 2\,2/5$

ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= \frac{16}{25} - \frac{6}{5} = \frac{-14}{25}$ (1).

MR Karakiozis



Q12. a) $x^2+Mx+(3-M)=0$

$\Delta = b^2-4ac=0$

$M^2-4\times(3-M)=0$ (1)

$M^2+4M-12=0$

$(M+6)(M-2)=0$

$M=-6$ or $2$ (1)

b) $2^{2x}-9\cdot 2^x+8=0$

$M=2^x$

$M^2-9M+8=0$ (1) or equival

$(M-8)(M-1)=0$

$M=8$ or $1$ (1)

$2^x=8=2^3$ or $2^x=1=2^0$

$x=3$ or $0$ (1)

c) Centre $(0,0)$ $\quad 3x-4y+25=0$

$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right| = \left|\frac{0+0+25}{\sqrt{3^2+4^2}}\right| = 5$ (1) Ans (1) Subst

since radius 5 & distant 5 only touches once ∴ tangent (1)

d) $y=(5-6x^3)^{1/5}$

$y'=\frac{1}{5}(5-6x^3)^{-4/5}\cdot -18x^2$ (1)

$=\frac{-18x^2}{5\sqrt[5]{(5-6x^3)^4}}$ (1)

e) $y=x\cdot x^{1/2}=x^{3/2}$ (1)

i) $\frac{dy}{dx}=\frac{3}{2}x^{1/2}=\frac{3}{2}\sqrt{x}$ (1)

ii) Normal $=-\frac{1}{4}$ $\quad$ tangent $=4$ (1)

$\frac{3\sqrt{x}}{2}=4$

$3\sqrt{x}=8$ $\quad y=\frac{64}{9}\times\sqrt{\frac{64}{9}}$

$\sqrt{x}=\frac{8}{3}$

$x=\frac{64}{9}$ (1) $\quad y=\frac{512}{27}=18\frac{26}{27}$ (1)

Mrs Geddes



Q13. a) $f_1 \to d$ $\quad f_2 \to e$ 1 $\quad f_3 \to b$ 1 $\quad f_4 \to c$ 1

b). $y = \frac{3x-2}{2x-5}$

i) $\alpha$) $x \neq \frac{5}{2}$ 1

$\beta$) $y \neq 3/2$. 1

ii) $\frac{dy}{dx} = \frac{3(2x-5) - 2(3x-2)}{(2x-5)^2}$ 1

$= \frac{-11}{(2x-5)^2}$ 1

c) $36 = x(x+5)$ 1

$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

$x = -9$ or $x = 4$

No soln

Solution $x = 4$ 1

d). $\frac{2\sin^3\theta + 2\cos^3\theta}{\sin\theta + \cos\theta} = 2 - \sin 2\theta$ cubic fact 1

$\frac{2(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\sin\theta + \cos\theta}$ 1 knowing $\sin^2\theta + \cos^2\theta = 1$

$= 2(1 - \sin\theta\cos\theta) = 2 - 2\sin\theta\cos\theta$ 1

$= 2 - \sin 2\theta$

e) $(3, 6)$

$x - 2y = 0$ ...1

$(3x + y + 7 = 0) \times 2$ ...2.

$+$

$6x + 2y + 14 = 0$ 1 Working

$7x + 14 = 0$

$x = -2$ $\quad y = -1$ 1 Ans

$m = \frac{-1-6}{-2-3} = \frac{7}{5}$

$y - 6 = \frac{7}{5}(x - 3)$ 1

$7x - 5y + 9 = 0$

MR WEREMY



Q14 a) $f(x) = x^3 - 4x + 1$

i) $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ (1)

ii) $f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$ (1) must have lim

$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1 - x^3 + 4x - 1}{h}$

$= \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2 - 4)}{h}$ (1)

$= \lim_{h\to 0} 3x^2 + 3xh + h^2 - 4$

$= 3x^2 - 4$ (1) Answer

iii) $P(1, -2)$

$m = 3 - 4 = -1$ ..... (1)

$\therefore 3x^2 - 4 = -1$

$x^2 = 1$

$x = \pm 1$ ... (1) $\therefore Q(-1, 3)$ ... (1)

b) $(1+t)x^2 - (2t-4)x - t^2 = 0$ for both

$\alpha + \beta = \frac{2t-4}{1+t}$ $\alpha\beta = \frac{-t^2}{1+t}$ (1)

$\frac{2t-4}{1+t} = 2 \times \frac{-t^2}{1+t}$

$(2t-4)(1+t) = -2t^2(1+t)$

$2t - 4 = -2t^2$ (1) since $t \neq -1$

$2t^2 + 2t - 4 = 0$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2$ or $1$ (1)

$\therefore t = -2$ or $t = 1$

c) $\sin 2x = \frac{1}{\sqrt{2}}$ $0 \le x \le 360$

$2x = 45, 135, 405, 495$ (1)

$\therefore x = 22.5^\circ, 67.5^\circ, 202.5^\circ, 247.5^\circ$ (1)

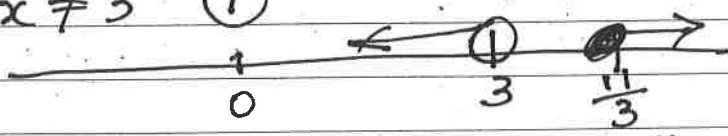
missing two answers only (1)

d) $\frac{2}{x-3} \le 3$ $x \neq 3$ (1)

$2 = 3x - 9$

$3x = 11$

$x = \frac{11}{3}$ (1)




$x < 3$ or $x \geq \frac{11}{3}$ (1) soln