DETUR GLORIA SOLI DEO

Trinity Grammar School

Mathematics Department

2014

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 4

Year 11

Mathematics Extension 1

Board of Studies Student Number
(Year 12 only)

Class Teacher:

....

Name:
(Year 11 only)

....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Preliminary Assessment Weighting:** 40%

Date of Examination: **Tuesday 2nd September**

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hour **45** minutes for this section

Section I **10 marks**

- Shade the correct response on the multiple-choice answer sheet (attached) for Questions 1 – 10.
- Each question is worth 1 mark.

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1 As a fraction, $2.0\dot{3}6\dot{4}$ is:

(A) $2\frac{91}{2500}$

(B) $2\frac{182}{4995}$

(C) $2\frac{91}{4995}$

(D) $\frac{182}{4995}$

2 Simplify $\frac{3x+2}{x+2}-\frac{x+3}{(2x-3)(x+2)}$.

(A) $\frac{x-5}{3-x^2}$

(B) $\frac{6x^2-6x-9}{(x+2)(2x-3)}$

(C) $\frac{x-5}{(x+2)(2x-3)}$

(D) $\frac{6x^2-6x-3}{(x+2)(2x-3)}$

3 Find the domain and range of $y=\sqrt{2x-6}$.

(A) Domain: $x \le 3$, range: $y \ge 0$

(B) Domain: all real x, range: $y \ge 0$

(C) Domain: $x \ge 3$, range: all real y

(D) Domain: $x \ge 3$, range: $y \ge 0$

4 If $a=\left(\frac{2}{3}\right)^7$ and $b=\left(\frac{3}{4}\right)^5$, evaluate $\frac{a^9(b^3)^2}{(a^2)^3 b}$.

(A) $\frac{3^4}{2^5}$

(B) $\frac{3^9}{2^{39}}$

(C) $\frac{3^4}{2^{29}}$

(D) $\frac{1}{2^{20}}$

5 Express $x-\frac{1}{x}$ with a rational denominator given that $x=3+\sqrt{2}$.

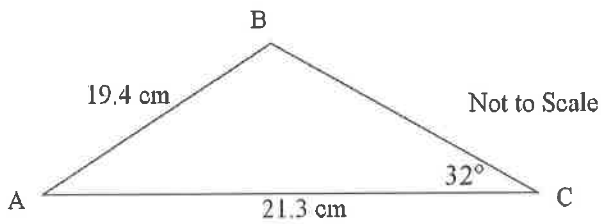
(A) $\frac{4\sqrt{2}+9}{5}$

(B) $\frac{6\sqrt{2}+27}{7}$

(C) 6

(D) $\frac{8\sqrt{2}+18}{7}$

6 In the diagram below, the **obtuse angle** ABC to the nearest degree is:



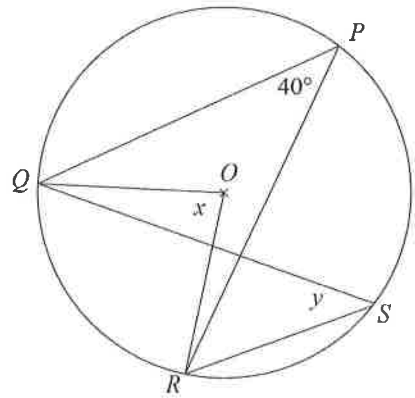
(A) 151°

(B) 116°

(C) 36°

(D) 144°

7 P, Q, R and S are points on a circle with centre O. $\angle QPR = 40^\circ$.



Why are the values of x and y?

(A) $x = 40^\circ$ and $y = 20^\circ$

(B) $x = 40^\circ$ and $y = 40^\circ$

(C) $x = 80^\circ$ and $y = 20^\circ$

(D) $x = 80^\circ$ and $y = 40^\circ$

8 For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

(A) $k \geq -3$

(B) $k \leq -3$

(C) $k \geq 3$

(D) $k \leq 3$

9 What are the coordinates of the **focus** of the parabola $4y = x^2 - 8$?

(A) (0, -8)

(B) (0, -7)

(C) (0, -2)

(D) (0, -1)

10 What is the acute angle between the lines $y = 2x - 1$ and $x - 3y + 6 = 0$, to the nearest degree?

(A) 45°

(B) 54°

(C) 79°

(D) 82°

End of Section I

Section II commences on the next page.

Section II **60 marks**

- Begin each question in a SEPARATE writing booklet or answer sheet
- Show all relevant mathematical reasoning and/or calculations

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Question 11 **(15 marks)** Use a SEPARATE writing booklet Marks

(a) If A is $(-2,-1)$ and B is $(1,5)$, find the co-ordinates of the point P that divides AB externally in the ratio 2:5. **2**

(b) Write $\dfrac{1}{2\sqrt{y+7}}$ in index form with a negative index. **1**

(c) Factorise $5x-x^2-3x+15$. **1**

(d) Solve $|2x+1|=3x-2$. **2**

(e) Show that $f(x)=4x-x^3$ is an odd function. **1**

(f) Find the exact value of $\cos 390^0$. **1**

(g) Find the equation of the straight line with a gradient of 5 that also passes through the intersection of the lines $x-2y-11=0$ and $2x-y-10=0$. **3**

(h) If α and β are the roots of $x^2-6x+3=0$ without solving, find the values of :

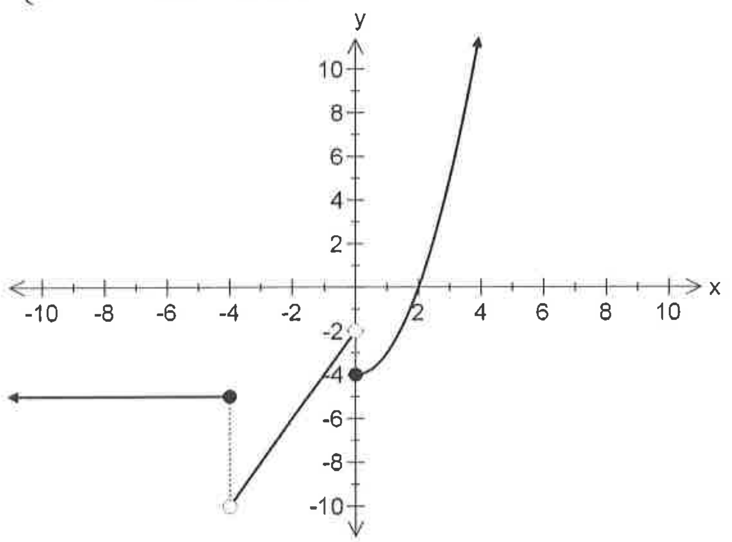
(i) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

(ii) $\alpha\beta^2+\beta\alpha^2$ **2**

End of Question 11

Question 12 **(15 marks)** Use a SEPARATE writing booklet Marks

(a) The function, $f(x)=\begin{cases} -5 & \text{for } \quad x \le -4 \\ 2x-2 & \text{for } \ -4 < x < 0 \\ x^2-4 & \text{for } \quad x \ge 0 \end{cases}$ has been graphed below.



(i) Evaluate $f(-4) - f(0)$. **1**

(ii) Find an expression for $f(a^2+1)$. **2**

(iii) For what value(s) of x is $f(x) = -5$? **2**

(b) Solve $\dfrac{2}{x-3} \le 3$. **3**

(c) Differentiate from first principles to find the gradient of the tangent at any point on the curve $f(x) = 3x^2 + 3x + 1$. **3**

(d) Find the centre and radius of the circle $x^2 - 10x + y^2 + 6y + 33 = 0$. **2**

(e) Simplify $\cot^2\theta - \dfrac{1}{\sin^2\theta}$. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet Marks

(a) Solve $sin2x = \frac{1}{\sqrt{2}}$ for $0^0 \leq 2x \leq 360^0$. **2**

(b) X is a point on side QR of ΔPQR. If $PQ = PR = QX$ and $PX = XR$:

(i) Draw a diagram illustrating this information. **2**

(ii) Calculate the size of $\angle PQR$, giving all reasons. **4**

(Hint: let $\angle PQR = x$)

(c) Differentiate with respect to x:

(i) $\sqrt[3]{x} + 5x^{-3}$ **1**

(ii) $(2x+5)^5$ **1**

(iii) $\dfrac{x+1}{3x^2-7}$ **2**

(d) Show that the following lines are concurrent. **3**

$$x - 5y - 17 = 0$$
$$3x - 2y - 12 = 0$$
$$5x + y - 7 = 0$$

End of Question 13

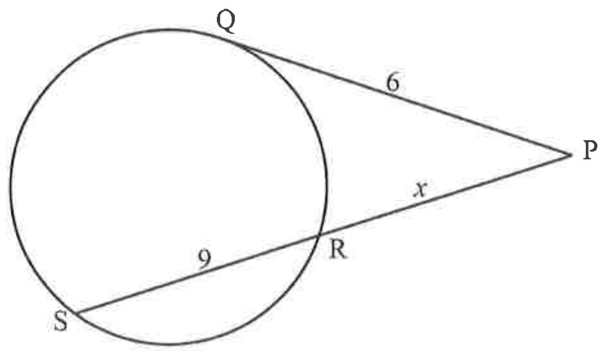
Question 14 **(15 marks)** Use a SEPARATE writing booklet Marks

(a) Prove that $\cot\theta + 2\sec\theta = \dfrac{1-\sin^2\theta + 2\sin\theta}{\sin\theta\cos\theta}$ **3**

(b) Solve for x: $3^{2x} - 10(3^x) + 9 = 0.$ **3**

(c) Football posts are 3.5 metres apart. If a footballer is standing 8 metres from one post and 11 metres from the other, find the angle within which the ball must be kicked to score a goal, to the nearest degree. **2**

(d) PQ is a tangent to a circle QRS, while PRS is a secant intersecting the circle in R and S, as in the diagram.



Given that $PQ = 6$, $RS = 9$, $PR = x$, find the length PR. **2**

(e) Find the equation of the normal to the curve $y = \sqrt{x}$ at the point (4,2). **2**

(f) Find the equation of the locus of a point P that moves so that PA is twice the distance of PB where A is (-3,1) and B is (2,-2). **3**

End of Section II
End of Examination

Solutions

Trinity Grammar School
Mathematics Department
2014
Year 11 Mathematics Extension 1
YEARLY EXAMINATION

Preliminary Assessment Task 4
Section I
Multiple Choice Answer Sheet

NSW Board of Studies Student Number
(Year 12 only)

Name: ______________________________
(Year 11 only)

Class Teacher: ______________________________

- Write your answers for **Section I** on this answer sheet.
- ***Shade completely only ONE answer for each question.***
- To change an answer, erase your previous mark completely and then record your new answer.

1	A	●	C	D
2	A	●	C	D
3	A	B	C	●
4	A	B	●	D
5	A	B	C	●
6	A	B	C	●
7	A	B	C	●
8	●	B	C	D
9	A	B	C	●
10	●	B	C	D

A table of Standard Integrals, if required, is printed on the reverse side of this answer sheet

2014 PRELIMINARY MAX TASK 4

SOLUTIONS

Question 11

a) Let the ratio be $2:-5$

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$= \frac{2(1) + (-5)(-2)}{2+-5}$$
$$= \frac{12}{-3}$$
$$= -4$$

$$y = \frac{my_2 + ny_1}{m+n}$$
$$= \frac{2(5) + (-5)(-1)}{2+-5}$$
$$= \frac{15}{-3}$$
$$= -5$$

$\therefore P$ is $(-4,-5)$

b) $$\frac{1}{2\sqrt{y+7}} = \frac{1}{2(y+7)^{\frac{1}{2}}}$$
$$= \frac{1}{2}(y+7)^{-\frac{1}{2}}$$

c) $$5x - x^2 - 3x + 15 = x(5-x) - 3(x-5)$$
$$= x(5-x) + 3(5-x)$$
$$= (x+3)(5-x)$$

d) $2x+1 = 3x-2$

$x = 3$

Check $x=3$

$LHS = |2(3)+1|$

$= 7$

$RHS = 3(3)-2$

$= 7$

$LHS = RHS$

OR

$2x+1 = -(3x-2)$

$2x+1 = -3x+2$

$5x = 1$

$x = \frac{1}{5}$

Check $x = \frac{1}{5}$

$LHS = |2(\frac{1}{5})+1|$

$= 1\frac{2}{5}$

$RHS = 3(\frac{1}{5}) - 2$

$= -\frac{7}{5}$

$LHS \neq RHS$

$\therefore x = 3$ is the only solution.

e) For an odd function, $f(-x) = -f(x)$

Now, $f(-x) = 4(-x) - (-x)^3$

$= -4x + x^3$

$= -(4x - x^3)$

$= -f(x)$

$\therefore$ $f(x)$ is an odd function.

f) $\cos 390 = \cos 30$

$= \frac{\sqrt{3}}{2}$

g) Solve simultaneously to find the point of intersection.

$2x - y - 10 = 0$ ①

$x - 2y - 11 = 0$ ②

② × 2 : $2x - 4y - 22 = 0$ ③

① − ③ : $3y + 12 = 0$

$y = -4$

Sub $y = -4$ into ②

$x - 2(-4) - 11 = 0$

$x = 3$

$\therefore$ The point of intersection is $(3, -4)$

Use $y - y_1 = m(x - x_1)$ where $m = 5$ at $(3, -4)$

$y + 4 = 5(x - 3)$

$y + 4 = 5x - 15$

$y = 5x - 19$ OR $5x - y - 19 = 0$

h) $\alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{1} = 6$

$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

(i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{6}{3}$

$= 2$

(ii) $\alpha\beta^2 + \beta\alpha^2 = \alpha\beta(\beta + \alpha)$

$= 3 \times 6$

$= 18$

Question 12

a) (i) $-5-(0^2-4)=-1$

(ii) For $f(a^2+1)$, we use x^2-4 since $a^2+1>0$

$$\therefore\ f(a^2+1) = (a^2+1)^2-4$$
$$= a^4+2a^2+1-4$$
$$= a^4+2a^2-3$$

(iii) We use $f(x)=2x-2$ for $-4<x<0$

$$\therefore\ -5 = 2x-2$$
$$2x = -3$$
$$x = -1\tfrac{1}{2}$$

We also use $f(x)=-5$ for $x\leqslant -4$

$\therefore$ The solution is $x=-1\tfrac{1}{2}$ and $x\leqslant -4$

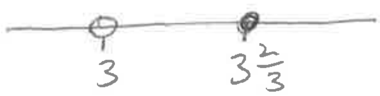
b) $\dfrac{2}{x-3} \leq 3$

$$2(x-3) \leq 3(x-3)^2$$
$$2x-6 \leq 3(x^2-6x+9)$$
$$0 \leq 3x^2-20x+33$$
$$0 \leq (3x-11)(x-3)$$

Now solve $(3x-11)(x-3)=0$

$\therefore\ x=3\tfrac{2}{3}$ OR $x=3$

but $x\neq 3$, as this would give 0 in the denominator



Check by testing $x=0$, $-\tfrac{2}{3}\leq 3$ is true.

$\therefore$ The solution is $x<3,\ x\geqslant 3\tfrac{2}{3}$

c) $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{[3(x+h)^2 + 3(x+h) + 1] - (3x^2 + 3x + 1)}{h}$

$= \lim_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 + 3x + 3h + 1 - 3x^2 - 3x - 1}{h}$

$= \lim_{h \to 0} \dfrac{6xh + 3h^2 + 3h}{h}$

$= \lim_{h \to 0} \dfrac{h(6x + 3h + 3)}{h}$

$= \lim_{h \to 0} 6x + 3h + 3$

$= 6x + 3$

d) $x^2 - 10x + y^2 + 6y + 33 = 0$

$(x-5)^2 - 25 + (y+3)^2 - 9 + 33 = 0$

$(x-5)^2 + (y+3)^2 = 1$

$\therefore$ The centre is $(5, -3)$ and radius $= 1$

e) $\cot^2\theta - \dfrac{1}{\sin^2\theta} = \dfrac{1}{\tan^2\theta} - \dfrac{1}{\sin^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$

$= \dfrac{\cos^2\theta - 1}{\sin^2\theta}$

$= \dfrac{-\sin^2\theta}{\sin^2\theta}$

$= -1$

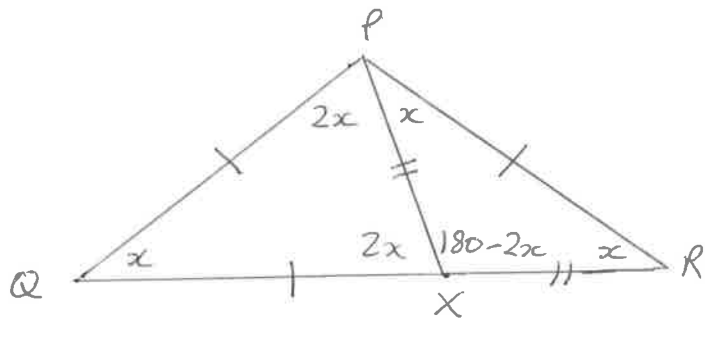
Question 13

a) $\sin 2x = \frac{1}{\sqrt{2}}$

$2x = 45°, 180 - 45°$

$\therefore x = 22.5°, 67.5°$

b) (i)



(ii) let $\angle PQR = x$

$\triangle PQR$ is isosceles ($PQ = PR$, given)

$\therefore \angle PRQ = \angle PQR = x$ (equal $\angle$'s opp equal sides in isosceles $\triangle$)

$\triangle PXR$ is isosceles ($PX = XR$, given)

$\angle XPR = \angle XRP$ (equal $\angle$'s opp equal sides in isosceles $\triangle$)

$\angle PXR = 180 - 2x$ (angle sum of $\triangle$)

$\angle PXQ = 180 - (180 - 2x)$

$= 2x$ (angle sum of straight line)

$\triangle QPX$ is isosceles ($PQ = QX$, given)

$\therefore \angle QPX = \angle QXP = 2x$ (equal $\angle$'s opp equal sides in isosceles $\triangle$)

$5x = 180$ (angle sum of $\triangle$)

$x = 36°$

$\therefore \angle PQR = 36°$

c) (i) $\sqrt[3]{x} + 5x^{-3} = x^{\frac{1}{3}} + 5x^{-3}$

$$\frac{d}{dx}\left(x^{\frac{1}{3}} + 5x^{-3}\right) = \frac{1}{3}x^{-\frac{2}{3}} - 15x^{-4}$$

$$= \frac{1}{3\sqrt[3]{x^2}} - \frac{15}{x^4}$$

(ii) $$\frac{d}{dx}\left[(2x+5)^5\right] = 5 \times 2 \times (2x+5)^4$$

$$= 10(2x+5)^4$$

(iii) $$\frac{d}{dx}\left[\frac{(x+1)}{(3x^2-7)}\right] = \frac{(3x^2-7)\times 1 - (x+1)\times 6x}{(3x^2-7)^2}$$

$$= \frac{3x^2 - 7 - 6x^2 - 6x}{(3x^2-7)^2}$$

$$= \frac{-3x^2 - 6x - 7}{(3x^2-7)^2}$$

d) $x - 5y - 17 = 0$ ①

$3x - 2y - 12 = 0$ ②

$5x + y - 7 = 0$ ③

①×3: $3x - 15y - 51 = 0$ ④

② − ④: $13y + 39 = 0$

$y = -3$

Sub $y = -3$ into ①

$x - 5(-3) - 17 = 0$

$x = 2$

∴ Point of intersection is $(2, -3)$

Sub $(2, -3)$ into ③

$LHS = 5(2) + (-3) - 7$

$= 0$

$= RHS$

∴ The lines are concurrent.

Question 14

a) $$\cot\theta + 2\sec\theta = \frac{1}{\tan\theta} + \frac{2}{\cos\theta}$$
$$= \frac{\cos\theta}{\sin\theta} + \frac{2}{\cos\theta}$$
$$= \frac{\cos^2\theta + 2\sin\theta}{\sin\theta\cos\theta}$$
$$= \frac{1-\sin^2\theta + 2\sin\theta}{\sin\theta\cos\theta}$$

b) $$3^{2x} - 10(3^x) + 9 = 0$$
Let $m = 3^x$
$$\therefore m^2 - 10m + 9 = 0$$
$$(m-9)(m-1) = 0$$
$m = 9$ OR $m = 1$

$3^x = 9$ $3^x = 1$

$x = 2$ OR $x = 0$

c) $$\cos\theta = \frac{8^2 + 11^2 - 3.5^2}{2\times 8\times 11}$$
$$\therefore \theta = 11^\circ$$

d) $$6^2 = x(x+9)$$
$$36 = x^2 + 9x$$
$$0 = x^2 + 9x - 36$$
$$0 = (x+12)(x-3)$$
Since PR is a length, $x = 3$ is the only solution.

e) $y = \sqrt{x} = x^{\frac{1}{2}}$

$$m = \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}(4)^{-\frac{1}{2}} = \frac{1}{4} \text{ at } x = 4$$

$\therefore$ Gradient of normal $= -\frac{1}{\frac{1}{4}} = -4$

The equation of the normal through $(4,2)$ is

$$y - 2 = -4(x-4)$$
$$y - 2 = -4x + 16$$
$$y = -4x + 18 \quad \text{OR} \quad 4x + y - 18 = 0$$

f)

$$PA : PB = 2:1$$
$$\frac{PA}{PB} = \frac{2}{1}$$
$$PA = 2PB$$
$$(PA)^2 = 4(PB)^2$$
$$(x - {}^-3)^2 + (y-1)^2 = 4\left[(x-2)^2 + (y - {}^-2)^2\right]$$
$$x^2 + 6x + 9 + y^2 - 2y + 1 = 4\left[x^2 - 4x + 4 + y^2 + 4y + 4\right]$$
$$x^2 + 6x + 9 + y^2 - 2y + 1 = 4x^2 - 16x + 16 + 4y^2 + 16y + 16$$

$\therefore 3x^2 - 22x + 3y^2 + 18y + 22 = 0$ is the required equation